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LOGICAL ANALYSIS OF EMPIRICAL EXPRESSIONS. WHAT IS WRONG WITH EMPIRICISM

Abstract. The following well-known problem motivated my handling more general problems. As we surely know, our pupils and even students are confronted with much more trouble when learning mathematics (and even physics) than when they learn 'empirical' sciences like biology, mineralogy etc. There are many factors that can at least partially explain this phenomenon. I would however mention one factor that is not too frequently adduced: mathematics, logic, and much of physics use concepts that are *abstract* while the empirical sciences seem to support understanding by using expressions concerning (denoting? expressing?) *concrete* objects.

Therefore the **first** topic to be explained (or explicated) is: *Abstract* vs. *concrete*. The **second** point will consist of applying the first point to explanation of the trouble with learning mathematics. The **third** point will ask **L**ogical **A**nalysis of **N**atural **L**anguage how to tell abstract expressions from concrete ones.

The **fourth** point will confront the concept described in the foregoing point with conceptions trying to abandon the distinction between analytic and empirical expressions. Here it will be shown that the *empiricism* representing this latter conception deprives semantics as applied to Natural language of important features of expressivity.

Keywords: abstract, concrete, empirical, sense, denotation, reference.

1. Abstract, Concrete

This pair of concepts has a very old tradition. To mention some rather modern names I have to adduce *Bernard Bolzano* and *Peter Fletcher*.

In his *Wissenschaftslehre* (1837) Bolzano illustrates the process of abstracting as follows. Consider a *real* sentence and disregard gradually such features of a real sentence as whether it is thought or written or pronounced. We have to state that disregarding such properties does not annihilate the sentence. What remains is Bolzano's *Satz an sich*, a typical abstract object. A similar process leads from the real non-sentential parts of a sentence

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to *Vorstellungen an sich* the important kind of which are *Begriffe*, concepts. All these objects *an sich* are no linguistic (real) objects; they are *extra-linguistic* objects. Bolzano was well aware of the fact that logic is not a linguistic discipline and his *Wissenschaftslehre* had no problem handling abstract (not real) objects. After all the only importance of expressions from the logical viewpoint is their *sense* and *denotation*, which were nearly defined about fifty years after *Wissenschaftslehre* by Frege. Bolzano's (and other logicians') discoveries would not come if the then rather popular psychologism had not been defeated by Frege.

No worries about the *existence* of abstract objects threatened Bolzano in his remarkable activities. True, abstract objects *do not exist* in the same sense as a real object but this circumstance does not prevent us from making proper inferences. Abstract objects are *objective*; logic *does* prevent us from creating arbitrary chains of thoughts which would not be truth-preserving.

A more recent analysis of *being abstract* can be found in the remarkable work by Fletcher (1998). "The idea is that we start with a physical object and selectively disregard some of its properties, thus creating an object that is like the original one but without the disregarded properties." (41) Thus, disregarding the color of a cat we get the abstract object "colorless cat", disregarding the number of branches of a tree we get a tree without a number of branches; better, we get rather the *idea* of a colorless cat, or a tree without a number of branches etc. Thus we can get *exact* concepts. Empirical concepts always admit some borderline cases.

2. Learning mathematics

Let us compare the situation when an ordinary pupil learns mathematics with the situation of learning, say, zoology or mineralogy. The former case is characterized by the fact that the concepts that should be understood and memorized are predominately abstract. The pupil *cannot imagine* many objects that are denoted/expressed by the respective expressions. (S)he will, e.g., complain that (s)he cannot imagine *cardinality* or *binary relation*.

We know what is usually recommended to save our pupils from maths: the teaching of abstract courses should be mostly graphic, or visual, so that the abstract factors are maximally suppressed. But irrespective of such recommendations – even authorized by such geniuses as Comenius – "*for any branch of sciences, each of which is defined by characteristic sets of abstractions*" it holds what Fletcher says: "Philosophers... would be better

employed putting abstraction and idealization on a firm foundation than trying to do without them." (43).

Yes, a certain role of visualization, illustrative examples etc. has to be admitted but we should not expect that whoever will be fascinated by great explosions mounted by propagators of learning physics will be successful in understanding the claims of this science. Not only that; those pupils who will not accept that they have to understand abstract ideas will accept the illusion that visualization can replace understanding, and so will joyfully state that, for example, zoology is easy. It's clear, we can *see* lions, dogs, spiders and so, we can *imagine* the objects studied by zoology! Some warning comes later. Free fall: Yes, we can imagine free fall! The physicist is amused: Do you really? Explain, please.

Evidently, visualization can help but is not omnipotent. It just makes it possible to be acquainted with singular concrete objects that may be instances of abstracts like properties or with singular representatives of some abstract object (paradigmatic examples of a triangle; better: a singular picture of a triangle).

So we should find some way how to handle abstract objects which are invisible but without which no knowledge would be possible. Clearly, even a simple primitive language could not exist without abstraction: we would be condemned to communicate like the 'scientists' from Swift's Gulliver (see Materna 2004).

So let us ask how a language can distinguish abstract objects from concrete ones. There are hundreds of theories of language and not each of them is interested to give relevant information in this regard. Only those ones which are *logical analyses* and are thus friendly to our way of asking can be asked.

3. Transparent Intensional Logic (TIL)

Among the LANL systems Tichý's Transparent Intensional Logic (Tichý 1988, Duží, Jespersen, Materna (2010)) is based on a conception of language which explicitly refuses any elements of subjectivism, being rather friendly oriented to Platonism, where the latter is defined as follows (read in (1988), vii): "...the view that over and above material objects, there are also functions, concepts, truth-values, and thoughts." Therefore, TIL does not need meta-language, where the names of the defined objects would be introduced and then interpreted. Tichý explains why, and his explanation is of key importance for our position so that a longer quotation is necessary.

To understand what Tichý means by the term *construction* has to be explicated. We first only suggest the idea of construction, afterwards we adduce the 'long quotation'.

TIL in general welcomes Frege's idea (1892) where the necessity of inserting *sense* between *expression* and *denotation*[1] is argued for. Unfortunately, Frege has not defined *sense*, he has just shown that *two distinct senses can determine one and the same denotation*. This means that *no set-theoretical object can be a Fregean sense* (See Materna 2016). Thus, functions obeying the principle of extensionality cannot be Fregean senses. Tichý exploited his early intuition (1968) that sense should be a procedure and this idea became the later (1988) definition of sense in TIL, where the Fregean sense of every expression of NL is a procedure defined as *construction*. Constructions are thus well–defined explications of sense and they realize the older idea that sense (meaning) should be structured (for example, Cresswell 1985). From the definition of constructions (Tichý 1988, 63–66, Duží, Jespersen, Materna 2010, 45, 52)) it follows that constructions are not set–theoretical objects: While the latter obey the principle of extensionality

EXT $$\forall x(\psi x = \chi x) \to \psi = \chi$$

the former do not:

> If *C as well as D* construct *a* then *C* may be distinct from D.

Observe:

$$m.(x + y) = m.x + m.y$$

What does this equality say? A *function* on the left side has the same value as the *function* on the right side (see EXT). BUT this *does not mean* that the *construction* whose result is the left side is *the same as the construction whose result is the right side*.

We can see that this corresponds to Frege's idea that one and the same result can be reached by distinct *senses*.

A basic property of constructions is that they are not linguistic expressions (unlike Montague's λ-terms). They may be, of course, expressed or denoted by expressions. Now we will enumerate – without important details – constructions used in TIL: This is just for recognizing the kind, not for understanding, which is given by the respective definitions.
Variables (they are not some letters, such letters are names of variables)
Trivialization (^0M constructs M without any change – it *displays* M)
Execution, Double execution (^{1}X, ^{2}X)

Closure ($\lambda_{x_1 \ldots x_m}$C) (*abstraction* in Montague)
Composition [$C_0 C_1 \ldots C_k$] (*application* in Montague)
Constructions 'work' within the type-theoretical setting given by a ramified hierarchy of types, where 1st order types are o (truth-values **T**, **F**), ι (iota, individuals), τ (time moments, doubling as real numbers), ω (possible worlds), functional types ($\alpha\beta_1 \ldots \beta_k$), and the ramified hierarchy introduces higher order types, i.e., types of constructions.

Some vague idea of constructions could be sufficient just for understanding the following long quotation from Tichý (1988, 71):

> There is no *intrinsic* relation between a formula and the construction it represents. Hence if anything said about the formula is to have a bearing on things mathematical, the relation of the formula as a whole, or of its constituents, to mathematical objects must be explicitly stipulated. In order to put such a stipulation into words, one has to *name* entities of both kinds: the mathematical objects and the linguistic expressions corresponding to them. Hence the need for a metalanguage, distinct and separate from the original notation in question. But the metalinguistic expressions themselves signify constructions. One thus faces a choice: one can either acquiesce in these higher-order constructions, or one can ignore them too and look instead at the meta-meta-expressions corresponding to them. If the first option is chosen the question arises why the same treatment cannot be applied at the bottom level, thus avoiding the original linguistic detour as well. And if the second option is taken one is obviously caught in an infinite regress of ever higher metalanguages.

Clearly, TIL is based on typed λ-calculus (similarly as Montague's IL) but due to some essential differences it became a *hyperintensional* logic, where hyperintensionality is given by the possibility of *displaying, not only executing, constructions themselves*. Thus, for example, we can systematically distinguish between the true claim that

$$(x + 4) - 2 = 2 + x$$

and the false claim that the left-hand and right-hand constructions are identical.

The respective constructions are

$$[^0{=}_\tau\ [^0{-}\ [^0{+}\ x\ {}^0 4]\ {}^0 2]]\ [^0{+}\ {}^0 2\ x]]$$
$$[^0{=}_*\ {}^0[^0{-}\ [^0{+}\ x\ {}^0 4]\ {}^0 2]]\ {}^0[^0{+}\ {}^0 2\ x]]$$

where $=_\tau / (o\tau\tau)$ is the identity of numbers while $=_* / (o{*_n}{*_n})$ is the identity of procedures.

In general, where C_1, C_2 are constructions typed to v-construct α-entities, the fact that C_1 and C_2 are *equivalent* in the sense of v-constructing

for every valuation v the same α-object or being both v-improper is expressed by $[^0{=}_\alpha C_1 C_2]$ while the fact that C_1, C_2 are *identical* procedures is expressed by $[^0{=}_*\,^0 C_1\,^0 C_2]$.

TIL is evidently a logical theory that surely is able to answer our questions concerning the relation between abstract and concrete expressions/objects. It is overtly based on some philosophical principles (unlike those systems which are strongly formalist and for which the term "*commitment*" heralds a kind of heresy) and while no claim formulated in TIL is an empirical claim, TIL can show that empirical sentences are analyzable in such a way that the difference between empirical and non-empirical is natural and hard and fast notwithstanding some famous trends.

4. What is wrong with empiricism

First we should formulate our pre-logical intuition concerning empirical expressions.

Let us consider the following list of expressions:

> *a prime*; *the third power of two*; *2 is a prime*; *to be divisible by 3*; *the number divisible by 0*; *the greatest prime is odd*

None of these expressions is empirical. Why?

Here we have to state that what our expressions mean and (therefore) denote is either dependent on what happens in the reality at the given time or independent of it. The expressions in the above list are all independent as to their meaning and denotation of what is happening in the reality at a given time. To name a prime is easily realizable without answering such questions as the actual weather or even the results of a physical experiment. The same holds in the next examples. We must however pay attention to the last two examples: They are both independent of the reality, but the special case is that there is no denotation here. There is no number divisible by 0 and the last sentence is neither true nor false. TIL is a partial system: it works with partial functions. It is however no many-valued logic: The two members of the type o are **T**, **F**, and *not possessing a value* is surely distinct from *possessing a third (fourth, etc.)* value.

Consider now the second list:

> *a horse*; *the richest man*; *Warsaw is the capital city of Poland*; *to rain in Europe*; *to be a man taller than Mt Blanc*; *the man who constructed Perpetuum mobile*

All these expressions are empirical. The property of being a hore is contingently instantiated by individuals; in other words, whether an individual happens to be a horse depends on the state-of-affairs. Whether some individual is or is not the richest man also depends on reality at the given time moment. Similarly the truth-value of the sentence about Warsaw is dependent on "world and time". The same holds about the rain in Prague. Now we have to carefully decide about the status of the last two expressions. Frege believed that the sense determines the denotation. This means that we can assume that what is denoted is what is given (we would say "constructed") by the sense. The sense of the expression *to be a man taller than Mt Blanc* is nothing other than the construction of that remarkable property; the sense itself does not say whether this property is occupied in reality. So what is the denotation of that expression? Just this property. Similarly the denotation of the last sentence is not some non-existent (!) individual but just that function whose value would be such an individual in the reality at some time.

TIL shows that logic can be endowed with tools that make it possible to distinguish between empirical and non-empirical expressions. The possibility to do so has been given since logicians recognized that the idea of possible worlds is not a fantastic (not scientific) idea but a well founded notion helping to solve semantic problems (see, e.g., Kripke 1963, Montague 1974).

Tichý defined intensions as functions not only from possible worlds but also from times. The variable of possible worlds is w, of time moments t. Thus any intension is a function from possible worlds to chronologies (i.e. functions from times) to a given type. So let α be any type; an intension is a function $((\alpha\tau)\omega)$, abbreviated as $\alpha_{\tau\omega}$. For $\alpha = o$ we get the type of propositions, for $\alpha = (o\iota)$ we get properties of individuals etc.

Intensions that are *constant* functions, i.e. having the same value in all possible worlds, are types of trivial intensions: these intensions lack the main property of empirical objects: they are not contingent. Thus analyzing the sentence *Every bachelor is a man* we get a trivial intension. This sentence is not empirical: In every possible world-time it is true. Similarly every intension false in all world-times and every intension undefined in every world-time are trivial, non-empirical.

The approach of TIL to analyzing empirical expressions can be described as follows: The sense of such an expression is the construction which is the result of the analysis. The denotation is simply that object (if any) that is constructed by the sense. Some of the results of such an approach are:

(1) *Every empirical expression denotes something.*

This holds because every intension is a function mostly constructed as $\lambda w \lambda t\ C$, where C is a construction. This closure always constructs a *function*, and such a construction is never improper, i.e., constructing nothing.

(2) *Some non-empirical expressions denote nothing.*

We know already an example: *the greatest prime*. The non-empirical expressions do not always express closure, and compositions may be *improper*, like *the number divisible by 0*.

(3) *We have to distinguish denotation of an empirical expression and its reference (if any).*

The link from the expression to its sense is analytic, necessary, derivable from analysis (no empirical steps). The link from the sense to the denotation is necessary as well. Therefore, the link from the expression to its denotation is necessary. There is a link connecting the denotation of an empirical expression with the value (*if any)* of this denotation in actual world-time. This link is *contingent*, not derivable from any computation. The contingent value of the denotation of an empirical expression at the actual world-time can be called *reference* and is so called in TIL.

This difference is important. The provably false opinion that an empirical expression may denote nothing stems just from not distinguishing between denotation and reference. Empirical expressions can, of course, lack a reference, but never a denotation (see (1)).

As an example of misunderstanding point (3) we can adduce an analysis that is as popular as erroneous: even Frege himself suggested a false analysis of the semantic relation between *"Morgenstern"* and *"Abendstern"*. The famous analysis consists in the claim that the denotation of *Morgenstern* as well as of *Abendstern* is Venus, the celestial body. We know that this is impossible, Venus is a contingent reference of both, and there is no common denotation: The sense of *M...* constructs another intension than the sense of *A...*.

(4) *With the possible exception of proper names no concrete object can be the denotation of an empirical expression.*

This is proved by the fact that empirical expressions denote intensions, i.e., *functions*.

More such interesting results could be adduced and, at the same time, we would see how many particular (albeit interesting) semantic problems should be solved. One important point has to be however emphasized before the standpoint to Quine's criticism is formulated:

The precise form of analytic notions and claims is made possible by the fact *that what is expressed and denoted by expressions in TIL are not the original intuitions but some surrogates for these intuitions.* Tichý (1988, pp. 194–5) says:

> The purpose of theoretical explication is to represent intuitions in terms of rigorously defined entities. It is to Frege that we owe the insight that the mathematical notion of function is a universal medium of explication not just in mathematics but in general. To explicate a system of intuitive, pre-theoretical notions is to assign to them, as surrogates, members of the functional hierarchy over a definite objectual base. Relations between the intuitive notions are then represented by the mathematically rigorous relationships between the functional surrogates. ... By representing intuitions with functional surrogates we can throw light on their logical interdependence and show how some of them can be defined in terms of others.

It is this notion of *explication*[2] which enables us to understand why the attempts by some LANL researchers to logically explain modalities used in NL were so harshly interrupted by Quine (e.g. in 1953).

Quine's attack in *Two dogmas of empiricism* (see (1953)) could be classified with criticism of those concepts which wanted to define sense in terms of such semantic notions as *synonymy* or *analyticity*. It was Pavel Tichý who in (1968) convincingly demonstrated that this effort is not viable (2004, 81). Tichý's arguments are similar to Quine's but Tichý continues doing logical analysis while Quine, speaking about senses (meanings), says that the theory of meaning concerns "simply the synonymy of linguistic forms and the analyticity of statements; meanings themselves, as obscure intermediary entities, may well be abandoned." (22).

Quine started a very fundamental critique, fundamental because it did not simply require some particular change within LANL: obeying Quine's proposals would mean to stop doing LANL and to begin doing *pragmatics*. This is no crime, of course, but implicitly it means that doing LANL is a useless (because actually impossible) activity.

The influence of Quine's criticism was immense from the very beginning. This fact shows that the philosophical community actually welcomed Quine's leaving the realm of logically necessary claims in LANL and evidently wants to prefer describing what actually happens when we use those operations that are connected with the concepts known from LANL. But as soon as we study *what actually happens...* we are no more doing logic (including LANL). It is this point where we have to ask: Are the problems that Quine tries to solve really problems of *logical* analysis? Are the claims that he formulates *logically*, or at least *analytically*, necessary?

Studying Quine's work we have to state that within his general view of language and science and his holistic conception of knowledge – where it is the distance of a claim/theory from a "sensory periphery" that determines the degree of being revised – we see that *no statement is immune to revision*. The maximum distance holds evidently for logic and mathematical disciplines. So even logical laws can be in principle revised. True, in practice Quine assumed that logical laws are fixed, at least if "logic" meant 1st order extensionalist logic, but he did not believe that, e.g. modalities could be logically explicable. From his general standpoint he inferred that there is no absolute distinction between the analytic and the synthetic. (See Materna 2007.)

All this Quinean revolution was essentially a refusal of the ontological foundation of the distinction between empirical and analytical factors. We can state that one of the reasons thereof was the inability (philosophically grounded) of accepting and applying *explication*: *any good explication means that some abstract entities* are introduced. Explications often face critical comments of this kind. A classical example is a remark by Bealer (1982), where he comments on the type-theoretical classification of properties as functions from possible worlds. He says that for example the aroma of coffee is a property but nobody will say that (s)he drinks a function. This "argument from aroma" (as it is called in TIL, see DJM, p. 6) is important: it shows what we stated above, viz. that *what is expressed and denoted by expressions in TIL are not the original intuitions but some surrogates for these intuitions*. Here, e.g., intensions as properties are surely not *mappings*, but they can be *modeled* as functions (here $(o\iota)_{\tau\omega}$) to capture "empirical variability". This step leads to leaving reality and creating a system of rigorous surrogates. So the original pre-theoretic intuitions must not disappear in the process of explication.

It seems to me that Quine and all his followers (including the later Wittgenstein), i.e., all philosophers and logicians who participate in replacing logical analysis by empirical description did not know or simply forgot the phenomenon of explication, which accompanies the development of every ripe science. (Take the development of physics – can you imagine how contemporary physics would look if it were not full of explications?)

The trend founded by Quine and followed by so many philosophers and logicians can be characterized as a *radical empiricism*. Empirical claims concern language (mainly English) and a rather fine analysis elaborates primarily *linguistic* data and their interrelations from the viewpoint of a clas-

sical 1st order predicate logic. (This kind of "linguistic turn" has been baptized "glossologism" by Tichý, who was alluding to the old "psychologism".) *Nothing like explication can we find in works of this kind of empiricism.*

When we compare the development of LANL and of Radical empiricism after 1953 we can dare to formulate a generalization: LANL after Carnap did not disappear and the results of Church's critical investigation of Frege (Church 1956) and Carnap (Church 1954 – Church 1993) belong to the most important milestones in the development of LANL building bridges to computer science. The end of the 20th century brought new results in the works of Montague and Pavel Tichý and TIL.

Radical empiricism led to a boom of pluralist conceptions which were all (for example, at least inferentialism (Peregrin)) hostile to representationalism.

What is typical for the development of radical empiricism is *a parsimony as for extralinguistic objects.* Tichý writes in his Preface (1988):

> It is one of the aims of the present work to propose a non-linguistic theory of variables and to give a consistently objectual version of Russell's Ramified Theory of Types. I will argue that the 'hierarchy of entities' which results from this rectification of Russell's system is not only a useful tool for diagnosing the flaws and ambiguities in Frege's logic but also the right medium for modelling our whole conceptual scheme.

So the above mentioned parsimony as to extralinguistic objects is strongly contrasted with a hierarchy of entities.

So what is wrong with empiricism?

Summarizing, its paranoic fear of invasion by extralinguistic elements into logic, replacing the normativist necessity for logical necessity, using the axiomatic before an analysis has been performed. Resulting in depriving logic of its objective expressivity.

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NOTES

[1] As for terminolgy, *Sinn* is *sense* (sometimes *meaning*), *Bedeutung* is translated *reference* (Geach, Black) or *denotation* (Church). Here I use *denotation* because in TIL *reference* is distinct from *denotation* when empirical expressions are analyzed.

[2] A more general definition of explication can be found already in Carnap (1947, 1950).

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