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CONTENTUAL APPROACH TO NEGATION[1]

Abstract. Interpretations of logics with only truth-functional connectives create a number of problems regarding the understanding of interpreted sentences. A particular problem is caused by the understanding of a sentence that is the negation of another. What is the meaning of sentence $\neg p$, for a particular sentence p? Even when we know what the semantic correlate of the sentence p is, we still do not know how to understand the semantic correlate of the sentence $\neg p$. The standard algebraic approach does not explain much. The problem is still open, because it is still unclear how to understand the element of algebra $-v(p)$. The propositional logic with the connective of the content implication, as it is a non-Fregean logic, proposes a simple solution to this problem. The key to understanding the content of the sentence $\neg p$ is to understand the content of the sentence p. Because, the content of the natural language sentence usually has a partial understanding, so it can be understood differently. Therefore, the negation $\neg p$ is here understood adequately to the current and partial understanding of the sentence p. It seems that the proposed approach is consistent with our daily thinking.

Keywords: content implication, sentential identity non-Fregean logic, Suszko, negation.

The number of various formally defined connectives of negation is impressive. Nevertheless, neither the syntactical nor semantic meaning of them explains how to "positively" understand that some sentence is not true. There is always one and the same problem which appears e.g. in the classical case. Everybody can say that "it is not raining". But what does it mean? Does it mean a sunny, cloudy, or snowy day or maybe, it is raining but not here, under this roof? How properly to understand that it is not raining? Contrary to semantic construction, there is no negative situation in the reality. A semantic correlate of the negation exists only in our thinking. This problem of the classical negation is inherited by other formal logics. As an example let us consider, especially well motivated and elegantly interpreted, the intuitionistic negation. Let us assume that intuitionistic $\neg p$ is true. In the light of the semantic interpretation, we know only that in all worlds accessible from our one, p is not true. Such an answer returns us

directly to the starting point: how to understand that a sentence p is not true or just false?

This problem comes from the fact that a connective of negation is extensional. In other words, a negation establishes a relation between p and $\neg p$ in the most general manner, according to the one criterion: is p true or not. Extensional connectives express only relations between the logical values of sentences. Meanwhile an understanding of the sentence $\neg p$ requires much more than a truthfulness of p – a sense of the sentence cannot be reduced to its logical value. Thus, as far as formal tools will be reduced to extensional connectives, a successful expressing of a meaning of the negation will be unworkable. From this point of view, it seems reasonable to extend a formal language by such a connective which would give the chance to express relations between the senses of sentences. Of course, no one should expect that in general for a sentence p there is the only one sense of the sentence $\neg p$. Usually, there are many possibilities of interpretation of a negated sentence. For a given sentence p it should be possible to express some wider or narrower, but some range of possible ways of understanding $\neg p$, i.e. a scope of those situations which exclude a situation expressed by p. Of course, this scope depends on the way of understanding the given sentence p. Senses of $\neg p$ and p are strictly correlated. Since the sentence $\neg p$ denies what is said by p, it means that to know what is said by $\neg p$ we previously need to know what is said by p.

For example, let us consider a sentence $p =$ "There are exactly three fresh apples on the tree in my garden near Łódź". Then, among others, p says what is said by:

$q_1 = $ "there is some fruit tree"
$q_2 = $ "this tree is an apple-tree"
$q_3 = $ "there are no k apples on the tree, where the natural number $k \neq 3$"
$q_4 = $ "the tree grows near Łódź"
$q_5 = $ "there is no December"
$q_6 = $ "there is no winter"

Of course, p says what is said by q_1, q_2, q_3, q_4, q_5, q_6 or, shortly, p says $q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6$. Certainly, this list of sentences is not complete.[2] However, our standard understanding of a natural language sentence is also incomplete and to some extent, arbitral – e.g. it depends on the current conversational situation, a knowledge of the interlocutors and the degree of their linguistic competences.[3] In our consciousness, a negation $\neg p$ can deny only this meaning of the sentence p we are thinking about. In the case of the example, a sense of the sentence $\neg p$ should be given by the sentence

$\neg(q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6)$, and consequently, by disjunction $\neg q_1 \vee \neg q_2 \vee \neg q_3 \vee \neg q_4 \vee \neg q_5 \vee \neg q_6$. Thus, p is false even when 1. it is autumn, and neither winter nor December, 2. there is a fruit tree in my garden near Łódź, 3. there are exactly three fruits but 4. these fruits are pears not apples. Let us notice, that such an interpretation coincides with our everyday thinking: some sentence is false, if it says something false. More precisely, a false sentence can say something true, and it usually says something true. Indeed, p is true only, if all sentences q_1, q_2, q_3, q_4, q_5, q_6 are true, because a true sentence cannot say anything false. Thus, the sentence p is false, if at least one of the sentences q_1, q_2, q_3, q_4, q_5, q_6 is false, and consequently, the sentence $\neg p$ $(= \neg q_1 \vee \neg q_2 \vee \neg q_3 \vee \neg q_4 \vee \neg q_5 \vee \neg q_6)$ is true, if at least one of the sentences $\neg q_1$, $\neg q_2$, $\neg q_3$, $\neg q_4$, $\neg q_5$, $\neg q_6$ is true. That is why, the sentence p may be false in many ways, but there is only one possibility for the truth of p. As we can see, the proverb

"There are many lies but barely one truth"

is always up to date as well as logically justified.

From the analysis above it follows that a correct formal recognition of the sense of the negation $\neg p$ must be preceded by also formal recognition of the sense of the sentence p. Naturally, extensional connectives are not sufficient for expressing the sentence's sense. As was already explained, truthfulness criterion has no use in this case. That is why many intensional connectives are also useless. Just sensible is to use some contentual connective, i.e. such a connective which establishes a relation between the senses of sentences.

1. Formal construction

In Łukowski 1997[4] there is presented a connective which solves a liar antinomy in a surprisingly easy way. What is worth noticing, this solution was only possible thanks to the contentual nature of the connective ":", due to its symbol previously called a colon, now known as the content implication. Although originally, solution of the liar problem was the only reason for the creation of the connective, its sense was clear from the very beginning: to express the fact that the sense of some sentence is a part (proper or not) of the sense of another sentence. That is why the desired pronounce of the implication $p : q$ was always the same: "p says q" or "p says what is said by q". In the standard case, every sentence says more than it is said by the sequence of words composing the sentence.[5] This means that, usually, for any sentence p, there are sentences $q_1, \ldots, q_n$, such that $p : q_i$, for $i \in \{1, \ldots, n\}$.

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In such a way, the sense of any sentence is developed/explained by the senses of other sentences. The difference between a content implication connective and any other – extensional or intensional – one is analogous to the difference between collective and distributive understanding of sets. In our case, every sentence is an ingredient, in Leśniewski's sense, of another sentence – it is a whole or a proper part of some sentence. If $p : q$ and $q : p$, then $p = q$, which means that, p is the whole q as well as q is the whole p. If $p : q$ and not $(q : p)$, then q is a proper part of p.

Syntactically, a connective of the content implication is given by the following axioms

A_1: $((\alpha : \beta) \wedge (\beta : \delta)) \rightarrow (\alpha : \delta)$

A_2: $(\alpha \wedge \beta) : \alpha)$

A_3: $(\alpha \wedge \beta) : (\beta \wedge \alpha)$

A_4: $\alpha : (\alpha \wedge \alpha)$

A_5: $((\alpha : \beta) \wedge (\beta : \alpha)) \rightarrow ((\neg\alpha : \neg\beta) \wedge (\neg\beta : \neg\alpha))$

A_6: $((\alpha : \beta) \wedge (\beta : \alpha) \wedge (\delta : \gamma) \wedge (\gamma : \delta)) \rightarrow (((\alpha\S\delta) : (\beta\S\gamma)) \wedge ((\beta\S\gamma) : (\alpha\S\delta)))$, for $\S \in \{\rightarrow, \leftrightarrow, :\}$

A_7: $((\alpha : \beta) \wedge (\delta : \gamma)) \rightarrow ((\alpha \S \delta) : (\beta \S \gamma))$, for $\S \in \{\wedge, \vee\}$

A_8: $(\alpha : \beta) \rightarrow (\alpha \rightarrow \beta)$

extending the syntax of the propositional classical calculus.[6] Modus Ponens $\{\alpha \rightarrow \beta, \alpha\} \vdash \beta$ remains the only inference rule. Syntactic inference is defined in a standard way.

A semantic adequate for *Contentual Classical Calculus* (*CCl*) is the class of all *CCl*-models, i.e., matrices $\mathcal{M} = (\mathcal{A}, D)$, such that $\mathcal{A} = (A, -, \cap, \cup, \Rightarrow, \Leftrightarrow, \supset)$ is an algebra similar to $\mathcal{L}_{CCl} = (For, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, :)$, D is a non-empty subset of A and for all $a, b \in A$,

1. $a = a \cap a$
2. $a \cap b = b \cap a$
3. $a \cap (b \cap c) = (a \cap b) \cap c$
4. $-a \in D$ iff $a \notin D$
5. $a \cap b \in D$ iff $a \in D$ and $b \in D$
6. $a \cup b \in D$ iff $a \in D$ or $b \in D$
7. $a \Rightarrow b \in D$ iff $a \notin D$ or $b \in D$
8. $a \supset b \in D$ iff $a = b \cap c$,   for some $c \in A$

Semantic inference is defined in a standard way:

$X \models_{CCl} \alpha$ iff   for any *CCl*-model $\mathcal{M} = (\mathcal{A}, D)$ and $v \in Hom(\mathcal{L}_{CCl}, \mathcal{A})$
$v(\alpha) \in D$, if for any $\beta \in X$, $v(\beta) \in D$.

In the light of this semantic interpretation it is clear, why "p says what is said by q" is the desired reading of the implication "$p : q$". For a given sentence p, the content of this sentence seems to be the most natural interpretation of its semantic correlate $v(p)$. Even "situation" seems to be a less appropriate understanding of a semantic correlate of the sentence.

As was already mentioned, the truthfulness of the sentence $p : q$ means that the content of the sentence q is a part of the content of p. However, this part can be not proper. Such a case holds when also the sentence $q : p$ is true. Then both sentences p and q have the same semantic correlate, $v(p) = v(q)$. Unfortunately, there is no possibility to express this fact in the language $\mathcal{L}_{CCl}$. That is why, let us extend this language as well as *CCl*, by another connective $\equiv$, given by the axiom

A_9: $(\alpha \equiv \beta) \equiv ((\alpha : \beta) \wedge (\beta : \alpha))$

There is a strict relation between our identity and identity defined by Suszko.[7] Let us recall that Suszko's identity is given by the following axioms

$A_{1_\equiv}$: $\alpha \equiv_s \alpha$

$A_{2_\equiv}$: $(\alpha \equiv_s \beta) \rightarrow (\neg\alpha \equiv_s \neg\beta)$

$A_{3_\equiv}$: $((\alpha \equiv_s \beta) \wedge (\gamma \equiv_s \delta)) \rightarrow ((\alpha \,\S\, \gamma) \equiv_s (\beta \,\S\, \delta)), \quad \text{for } \S \in \{\wedge, \vee, \rightarrow, \leftrightarrow, \equiv_s\}$

$A_{4_\equiv}$: $(\alpha \equiv_s \beta) \rightarrow (\alpha \rightarrow \beta)$

A real identity should be trivial in this sense, that, without additional assumptions, α is the only sentence identical with α. Suszko's identity satisfies this criterion. Consequently, $\alpha \equiv_s \beta$ is a true sentence, if and only if α has the same semantic correlate as β. It is not difficult to notice that our identity is not trivial. Indeed, our connective $\equiv$ is Suszko's identity $\equiv_s$ extended by three axioms

$(\alpha \wedge \alpha) \equiv_s \alpha$

$(\alpha \wedge \beta) \equiv_s (\beta \wedge \alpha)$

$(\alpha \wedge \beta) \wedge \gamma \equiv_s \alpha \wedge (\beta \wedge \gamma)$

It seems quite reasonable to assume that the content of the conjunction does not depend on the order of its conjuncts, just as the repetition of any content does not add anything new.

2. De Morgan extension of *CCl*

From the point of view of the reconstruction of the sense of negation it seems necessary to extend the classical propositional calculus with content implication by two "de Morgan" axioms

A_{10}: $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$

A_{11}: $\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$

Indeed, a consequent understanding of the negation of the sentence whose sense is formed by the conjunction of some sentences needs an axiom's translating sense of a negated conjunction onto the appropriate disjunction.

Let "$\mathcal{L}_{CCl+}$" and "CCl^+" be names for, respectively, the *CCl*-language with the new connective of identity $\equiv$ and the classical propositional calculus extended by axioms $A_{1:}$–$A_{11:}$. Thus, the CCl^+-model is a matrix $\mathcal{M}^+ = (\mathcal{A}^+, D)$, such that $\mathcal{A}^+ = (A^+, -, \cap, \cup, \Rightarrow, \Leftrightarrow, \supset, \approx)$ is an algebra similar to $\mathcal{L}_{CCl+} = (For, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, :, \equiv)$.

CCl-model satisfying conditions

9. $-(a \cap b) = -a \cup -b$

10. $-(a \cup b) = -a \cap -b$

11. $a \approx b \in D$ iff $a = b$

for $a, b \in A^+$.

3. Sense of sentence

It is easy to see that the content implication connective precisely satisfies previously presented preliminary assumptions. Let us return to our example. The sentence $p =$ "There are exactly three fresh apples on the tree in my garden near Łódź" says what is said by all sentences $q_1, q_2, q_3, q_4, q_5, q_6$. Since $p : q_1$, so $v(p : q_1) \in D$, for some CCl^+-model $\mathcal{M}^+ = (\mathcal{A}^+, D)$ and for some valuation $v \in Hom(\mathcal{L}_{CCl+}, \mathcal{A}^+)$. Thus,

$$v(p) = v(q_1) \cap c_1,$$

for some $c_1 \in A^+$. In other words, the content of the sentence q_1 is a part of the content of the sentence p. Let us notice, that, by assumptions, the sense of q_1 is not the whole sense of p. There are some other components of the sense of p. This fact is expressed by $c_1 \in A^+$, consisting of the senses of q_2, q_3, q_4, q_5, q_6 and probably some additional sense $c \in A^+$ we did not think of yet. Thus,

$$c_1 = v(q_2) \cap v(q_3) \cap v(q_4) \cap v(q_5) \cap v(q_6) \cap c,$$

for some $c \in A^+$. Indeed, since at the same time $v(p : q_i) \in D$, for $i = 2, \ldots, 6$, so

$$v(p) = v(q_i) \cap c_i,$$

for some $c_i \in A^+$. The same result is obtainable due to axioms $A_{1:}$, $A_{2:}$, $A_{4:}$, by which, from six formulas $p : q_i$ $(i = 1, \ldots, 6)$, we easily infer

$$p : (q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6).$$

Since, $v(p : (q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6)) \in D$, then by the fifth and eighth conditions of the CCl^+-model, we have

$$v(p) = v(q_1) \cap v(q_2) \cap v(q_3) \cap v(q_4) \cap v(q_5) \cap v(q_6) \cap c,$$

for some $c \in A^+$. In the case of natural language sentences, it seems impossible to present all senses composing the sense of a given sentence. If q_1, q_2, q_3, q_4, q_5, q_6 would be a complete set of sentences whose senses all together compose the whole sense of p, then we should accept $(q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6) : p$, as a true sentence, and consequently, we would obtain an equality $v(p) = v(q_1) \cap v(q_2) \cap v(q_3) \cap v(q_4) \cap v(q_5) \cap v(q_6)$. Unfortunately, usually, for the equality between $v(p)$ and $v(q_1) \cap v(q_2) \cap v(q_3) \cap v(q_4) \cap v(q_5) \cap v(q_6)$ it is necessary to add some unknown content c. This additional, unknown content c represents every part of the meaning of p that we did not think about yet. Although, we do not know what is a content c, we know very well, that such a content c exists. This means that the sense of any sentence p is of the form

$$v(p) = v(q_1) \cap \ldots \cap v(q_k) \cap c,$$

where $v(q_1) \cap \ldots \cap v(q_k)$ is a part of the sense we are aware of, and c is a part we are not aware of.

4. Sense of negation

The just presented formal approach to the sense of a sentence generates some consequences for, also formal, presentation of the sense of the sentence's negation. There are two possibilities depending on the formalization of the meaning of a given sentence.

4.1. The first case

The content of a sentence is defined completely, i.e. it is given by a finite set of contents of sentences. In this case, we believe that the sense of some sentence can be expressed/explained by an utterance of sentences composing a finite set. Thus, there is such a natural number k, that

$$c(p) = c(q_1) \cap \ldots \cap c(q_k),$$

where $c(p)$ and $c(q_i)$ are contents of sentences p and q_i. Of course, $c(p) = v(p)$ and $c(q_i) = v(q_i)$, for some (our) interpretation v. In this case, the content of the negation of the sentence p is quite precise

$$-c(p) = -c(q_1) \cup \ldots \cup -c(q_k).$$

Now, one can formally explain the meaning of the false sentence. A given sentence is false, when its content contains at least one false component being the content of such a sentence q_i about which p says. If $\neg p$ is true, the conjunction of negations all these false components of the content of p form a sense of $\neg p$. If $\neg p$ is false, its meaning is identical with the sense of the disjunction $\neg q_1 \vee \ldots \vee \neg q_k$.

In the case of our example, $p : (q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6)$. Moreover, assume that $(q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6) : p$. This means that $p \equiv (q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6)$, and so, $v(p) = v(q_1) \cap v(q_2) \cap v(q_3) \cap v(q_4) \cap v(q_5) \cap v(q_6)$. By the assumption and $A_{5:}$, we have $\neg p : \neg(q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6)$ and $\neg(q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6) : \neg p$, and by $A_{9:}$, $\neg p \equiv \neg(q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6)$. Then, $-v(p) = -v(q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6)$, and in the light of the 9th condition of the CCl^+-model, $-v(p) = -v(q_1) \cup -v(q_2) \cup -v(q_3) \cup -v(q_4) \cup -v(q_5) \cup -v(q_6)$. Thus, the meanings of p and $\neg p$ are given by

$$c(q_1) \cap c(q_2) \cap c(q_3) \cap c(q_4) \cap c(q_5) \cap c(q_6)$$

and

$$-c(q_1) \cup -c(q_2) \cup -c(q_3) \cup -c(q_4) \cup -c(q_5) \cup -c(q_6),$$

respectively. In other words, the senses of $q_1, q_2, q_3, q_4, q_5, q_6$, taken together, form a sense of p. Thus, p is true, if and only if all $q_1, \ldots, q_6$ are true, and the same the meaning of p is one: a conjunction of senses $c(q_1) \cap \ldots \cap c(q_6)$. A sense of $\neg p$ is much more complicated. The easiest answer would be as follows: the meaning of $\neg p$ is given by disjunction $-c(q_1) \cup \ldots \cup -c(q_6)$. However, $\neg p$ is true if at least one disjunct $-c(q_i)$ is true, for $i = 1, \ldots, 6$. In fact, there are many various meanings of the negation $\neg p$. A complete list of them follows

$$-c(q_i),$$

$$-c(q_i) \cap -c(q_j),$$

$$-c(q_i) \cap -c(q_j) \cap -c(q_k),$$

$$-c(q_i) \cap -c(q_j) \cap -c(q_k) \cap -c(q_r),$$

$$-c(q_i) \cap -c(q_j) \cap -c(q_k) \cap -c(q_r) \cap -c(q_s),$$

$$-c(q_1) \cap -c(q_2) \cap -c(q_3) \cap -c(q_4) \cap -c(q_5) \cap -c(q_6),$$

for $i, j, k, r, s \in \{1, \ldots, 6\}$.[8]

Now, assume that among all q_i only two sentences q_3 and q_4 are false. Thus, p is false and the sense of its negation is identical with the meaning of the conjunction $\neg q_3 \wedge \neg q_4$. In other words, $c(\neg p) = -c(q_3) \cap -c(q_4)$.

4.2. The second case

The content of a sentence cannot be defined completely, i.e. it cannot be given by a finite set of contents of sentences. In this second case, we know that the sense of some sentence cannot be expressed/explained by an utterance of sentences composing a finite set. Always something important for the sense of a given sentence will be unspoken. Thus, for any natural number k, there must be some $\boldsymbol{c}$ such that

$$c(p) = c(q_1) \cap \ldots \cap c(q_k) \cap \boldsymbol{c},$$

where $\boldsymbol{c}$ is the content of some unspoken sentence about which a given sentence says. This sentence is not only unspoken, we even do not think at the moment about the sentence and its content. In this case, the content of the negation of the sentence p is not precise, because although it is given by equality

$$-c(p) = -c(q_1) \cup \ldots \cup -c(q_k) \cup -\boldsymbol{c}$$

the last component of this disjunction is unknown at the moment. Moreover, every disjunction, this one or even longer, will contain such an indeterminate element. It means that we actually know several conditions under which p is a false sentence, but we cannot be sure that in this situation p is false because of something else. Moreover, usually we even suppose that there is a condition unknown to us responsible for the falsehood of p. Even when the list of known conditions of falsehood of p be longer and longer, we will never be certain that we know everything about the conditions of the falsehood of p.

Returning to our example, $p : (q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6)$. However, instead of $(q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6) : p$, we have $(q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6 \wedge s) : p$, where s is an unspoken sentence or, more probably, the conjunction of all unspoken sentences, such that $p : s$. Thus, $p : (q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6 \wedge s)$, and so $p \equiv (q_1 \wedge q_2 \wedge q_3 \wedge q_4 \wedge q_5 \wedge q_6 \wedge s)$. Avoiding predictable steps of reasoning, let us formulate a conclusion: the meanings of p and $\neg p$ are given by

$$c(q_1) \cap c(q_2) \cap c(q_3) \cap c(q_4) \cap c(q_5) \cap c(q_6) \cap c(s)$$

and

$$-c(q_1) \cup -c(q_2) \cup -c(q_3) \cup -c(q_4) \cup -c(q_5) \cup -c(q_6) \cup -c(s),$$

respectively. In other words, the senses of q_1, q_2, q_3, q_4, q_5, q_6, s taken together, form the sense of p. Since s is permanently unknown, the meaning of p is still partial only. That is why the sense of the negation $\neg p$ is also partial. Thus, p is true, if and only if all $q_1, \ldots, q_6$ are true and when some unknown condition is satisfied. Thus, the meaning of p cannot be reduced to the conjunction of senses $c(q_1) \cap \ldots \cap c(q_6)$. Similarly, no one can say that the meaning of $\neg p$ is given by disjunction $-c(q_1) \cup \ldots \cup -c(q_6)$. However, $\neg p$ is true if at least one disjunct $c(q_i)$ is false, for $i = 1, \ldots, 6$, but also when some other condition does not hold. Of course, as in the previous case, there are many various meanings of the negation $\neg p$. A complete list of them follows

$$-c(s),$$

$$-c(q_i),$$

$$-c(q_i) \cap -c(s),$$

$$-c(q_i) \cap -c(q_j),$$

$$-c(q_i) \cap -c(q_j) \cap -c(s),$$

$$-c(q_i) \cap -c(q_j) \cap -c(q_k),$$

$$-c(q_i) \cap -c(q_j) \cap -c(q_k) \cap -c(s),$$

$$-c(q_i) \cap -c(q_j) \cap -c(q_k) \cap -c(q_r),$$

$$-c(q_i) \cap -c(q_j) \cap -c(q_k) \cap -c(q_r) \cap -c(s),$$

$$-c(q_i) \cap -c(q_j) \cap -c(q_k) \cap -c(q_r) \cap -c(q_s),$$

$$-c(q_i) \cap -c(q_j) \cap -c(q_k) \cap -c(q_r) \cap -c(q_s) \cap -c(s),$$

$$-c(q_1) \cap -c(q_2) \cap -c(q_3) \cap -c(q_4) \cap -c(q_5) \cap -c(q_6),$$

$$-c(q_1) \cap -c(q_2) \cap -c(q_3) \cap -c(q_4) \cap -c(q_5) \cap -c(q_6) \cap -c(s),$$

for $i, j, k, r, s \in \{1, \ldots, 6\}$. In every case there is the unknown component $-c(s)$.

Now, again assume that among all known sentences q_i only two are false, q_3 and q_4. Thus, p is false but the sense of its negation can be not identical with the meaning of the conjunction $\neg q_3 \wedge \neg q_4$ but with the meaning of $\neg q_3 \wedge \neg q_4 \wedge \neg s$. Then, $c(\neg p) = -c(q_3) \cap -c(q_4) \cap -c(s)$. This means that a sense of $\neg p$ is partially known because of unknown component $-c(s)$. Usually, we believe that we know very well, thanks to which component of

the content, p is false at the moment. However, the analysis above shows that even in such a situation we should doubt whether we have taken into account all the conditions of the current falsehood of p.

4.3. Common conclusions from both cases

Although both cases seem to be significantly different, they can be summarized with one common conclusion: p is false, if there is a false q such that $p : q$ is true; and so a sense of the falsehood of q becomes a falsehood of p or maybe a falsehood of p together with something more. In other words, a falsehood of q explains, maybe not exhaustively, why p is false.

5. Final remarks and perspectives

An approach proposed in the paper has some consequences for the understanding of such a formally presented negation of a sentence.

Firstly, it may seem that our formal presentation of the negation is burdened with the reasoning to infinity. Actually, the sense $-c(p)$ of the negation $\neg p$ is defined by sense $-c(q_i)$ of some other negation $\neg q_i$, which probably should be clarified by the sense of another sentence, etc. However, this is a quite natural and, thus, desirable situation. First of all, it would be too strong to assume that there are atomic and complex senses, in other words, that some senses are atomic, while other, not. Meanwhile, even an atomic sentence can be correctly interpreted by a complex content. Much more reasonable is to suppose that every sentence p can be simultaneously in two opposite positions. In one, p says something that is said by another sentence q; then $p : q$ is a true sentence. In the second position, some other sentence s says what is said by p, and so, $s : p$ is true. Such a holistic perspective fits the facts about natural languages, and so coincides with the nature of these languages. On the other hand, the practice of using natural language shows that searching for the real reason of the falsehood of some sentence, we finally find enough of a simple condition, like the proper number of apples on a tree or the proper species of fruit. It may happen, and usually it does, that every next explanation is simpler and more precise then the previous one. For the falsehood of the example sentence $p = $ "There are exactly three fresh apples on the tree in my garden near Łódź", one explanation can be given by the falsehood of the sentence $q_3 = $ "there are no k apples on the tree, where the natural number $k \neq 3$". But also the falsehood of q_3 can be explained. Let, $s_n = $ "there are exactly n apples on the tree", where $n \in \{0, 1, 2, 3, \ldots, n_0\}$ and $n_0 = 10000$.[9] Then,

$q_3 : \neg s_n$, for any $n \neq 3$, and, $q_3 : (\neg s_0 \wedge \neg s_1 \wedge \neg s_2 \wedge \neg s_4 \wedge \ldots \wedge \neg s_{10000})$. Thus, $\neg q_3 : (s_0 \vee s_1 \vee s_2 \vee s_4 \vee \ldots \vee s_{10000})$, and so on – the further procedure is already known. Of course, a similar clarification can be made for the species of the tree, or for something else.

Secondly, by the construction it is obvious that although a true sentence must say only the truth, a false sentence can say something true. It seems not too easy to find a sentence whose complex content would express only the false. And so,

Thirdly, thanks to the content implication connective it is easy and quite natural to show a commonly shared and well known opinion that the truth is one, but this only truth can be deformed in many ways: *there are many lies but barely one truth.*

Finally, our daily, ordinary using of the negation of sentences is based on a, similar to the above, way of understanding. However, the connective of the content implication allows a precise formalization of this common and standard practice. Moreover, our approach proposes an answer to the question on the borders of understanding of a sentence's negation. Can a sentence "there is no tree" be a negation of "there are exactly three apples on the tree"? The answer to this question is coupled with the answer to another one: Is the content of the sentence "there is no tree" a part of the content of the sentence "there are exactly three apples on the tree"? In other words, an understanding of the negation $\neg p$ of the sentence p depends on an understanding of the content of p.

Perspectives. It seems that a logic extended with the content implication connective has references to some important ideas in the philosophy of language, especially the theories of meaning. First of all, as was already noted in the first remark, on the ground of the logic with the content implication, the *Duhem-Quine thesis* is obviously satisfied thanks to the holistic nature of language.[10] Moreover, a fundamental thesis of *pluripropositionalism*, also known as the *multiple-proposition approach* or *multipropositional approach* is here satisfied.[11] Indeed, since p says $q_1, \ldots, q_k$ ($p : q_1, \ldots, p : q_k$), p does not express the only one meaning being a precise resultant of the meaning of words of the sentence p. In our approach, every sentence says more than is expressed by the words composing the sentence. On the other hand, $p : (q_1 \wedge \ldots \wedge q_k)$ is not the only formula acceptable on the ground of the logic with the new contentual connective. For some, probably unknown, sentence q an opposite implication also should be accepted, $(q_1 \wedge \ldots \wedge q_k \wedge q) : p$. Thus, a satisfaction of the Duhem-Quine thesis does not exclude some modified version of *verificationism*. The meaning of every sentence p[12] is constituted by a conjunc-

tion of senses $c(q_1) \wedge \ldots \wedge c(q_k) \wedge c(q)$ of some sentences $q_1, \ldots, q_k$, and maybe some still unknown sentence q. It means that to know the meaning of a sentence p we must know the meaning of other sentences $q_1, \ldots, q_k, q$. All these sentences explaining the meaning of a given sentence p can be understood as conditions of the truthfulness of p. Thus, the meaning of the sentence is given by truthfulness conditions for the sentence. It is important that in our approach, some unknown sentence q, probably also being a conjunction of some other sentences, is a factor necessary for completing the meaning of the sentence p. That is why an even apparently well understood sentence p is permanently imprecise, just like in our thinking.

N O T E S

[1] This research is supported by the National Science Center of Poland – grant No. 2015/17/B/HS1/02332.

[2] It is an interesting question, for which sentences, beside mathematical, such a list could be complete.

[3] Many books and articles. For example, papers presenting speech act theories, e.g. Austin [1962], Searle [1969], Searle and Vanderveken [1985], or conversational act theory, e.g. Grice [1975].

[4] See also, Łukowski 2006, 2011.

[5] For example, see the already considered sentence $p =$ "There are exactly three fresh apples on the tree in my garden near Łódź".

[6] Of course, there are possible extensions of other propositional calculi, for example, intuitionistic, or Heyting-Brouwer logic.

[7] Suszko [1975].

[8] In light of the two first conditions of the CCl^+-model, it is not necessary to assume that every index differs from another one.

[9] It is here assumed that the amount of apples on the tree cannot be bigger than 10000.

[10] Cf. Duhem (1906).

[11] Korta and Perry (2008).

[12] Of course, the sentence p can be simple, i.e. without any sentential connective.

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