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LOGIC IN THE LIGHT OF COGNITIVE SCIENCE

Abstract. Logical theory codifies rules of correct inferences. On the other hand, logical reasoning is typically considered as one of the most fundamental cognitive activities. Thus, cognitive science is a natural meeting-point for investigations about the place of logic in human cognition. Investigations in this perspective strongly depend on a possible understanding of logic. This paper focuses on logic in the strict sense; that is, the theory of deductive inferences. Two problems are taken into account, namely: (a) do humans apply logical rules in ordinary reasoning?; (b) the genesis of logic. The second issue is analyzed from the naturalistic point of view.

Keywords: *logica docens*, *logica utens*, rules of inference, genetic code, naturalism, information.

Logic is traditionally considered as very closely associated with cognitive activities. Philosophers and psychologists frequently say that human rationality consists in using logical tools in thinking and other cognitive activities, e.g. making decisions. In particular, reasoning and its correctness seem to have their roots in a faculty, which, by way of analogy with the language faculty, could be viewed as logical competence. On the other hand, one might also argue, logical competence itself is a manifestation of logic. Independently, whether logic is prior to logical competence or whether the latter has its grounds in the former, the concept of logic is indispensable for any analysis of the practice of inferences performed by humans as well as perhaps other species.¹ The above, a very sketchy introduction, justifies taking the concept of logic as fundamental for my further analysis of logical aspects related to the cognitive space.

First of all, we should distinguish cognitive acts or actions and cognitive products or results (see Twardowski 1912). For instance, constructing a theory consists of acts, but the theory (provisional or considered as fin-

ished) is a result. Consequently, reasoning and inferences can be viewed as acts or/and products. For my further analysis, let us agree that cognitive products are recorded in a language. Let us start with theories. A theory, let us say T , is an ordered set of sentences². This ordering divides elements of T into initial assumptions (for instance, axioms) and their consequences; that is, items deduced from accepted premises. Now, it is natural to say that logic is somehow involved in the operation of logical consequence. Let T be a theory and X ($X \subset T$) functions as a collection of initial assumptions of T . Thus, we can say that T is a set of logical consequences of X , symbolically $T = CnX$. This equality defines the predicate “being a theory” in an intuitive way. A more abstract approach comes from metalogic as a part of metamathematics. A theory (or deductive system) is defined as a set of sentences closed by the consequence operation. Formally speaking, T is a theory if and only if $CnT \subseteq T$ (a set of sentences is a theory if and only if consequences of T belong to T). Since the inclusion $T \subseteq CnT$ is justified by Tarski’s axioms for Cn (see below), we finally obtain the equality $T = CnT$, which says that a theory is a set of sentences identical with the set of its logical consequences.

What is the difference between both these accounts of theories, intuitive and metalogical? Although both are similar, they differ in some respects. According to the former (intuitive), theories are not infinite sets of sentences. Even if one says that the set of consequences of any set is always infinite, a straightforward answer points out that this fact follows from the general definition of Cn and is not particularly relevant for operating concrete theories. On the contrary, the argument continues, real theories are always finite and based on finite collections of initial assumptions. Moreover, no real theory, mathematical or empirical, is considered as identical with the collection of all the consequences of its initial assumptions. Clearly, theoreticians are usually interested only in some consequences regarded as particularly important, even if employers of theoretical systems admit that potential (yet unknown) consequences are always expected and can be qualified as important after discovering them. Just this circumstance allows us to use the strong inclusion \subset in the definition of being a theory in the ordinary sense. On the other hand, the metalogical account of theories discards several intuitive constraints; for example, it regards theories as an actually infinite set of sentences without deciding which items are interesting or lacking in theoretical importance. The equality $T = CnT$ also admits that for any $A \in T$, A is an assumption of T . Although this is a special and artificial case, from the metalogical point of view, it is fairly legitimate.

Logic can be also defined as a theory (I follow Woleński 2004, Woleński 2012)³. Assume that the operation Cn satisfies Tarski's general axioms:

- (1) denumerability of the language as a set of sentences;
- (2) $X \subseteq CnX$ (the inclusion axiom);
- (3) if $X \subseteq Y$, then $CnX \subseteq CnY$ (monotonicity of Cn);
- (4) $CnCnX = CnX$ (idempotence of Cn);
- (5) if $A \in CnX$, then there is a finite set $Y \subseteq X$ such that $A \in CnY$ (Cn is finitary).

Moreover, we assume that the deduction theorem:

- (6) if $B \in Cn(X \cup \{A\})$, then $(A \Rightarrow B) \in CnX$

holds in the theory of logical consequence.

Intuitively speaking, logic generates cases of inferring conclusions from some premises. In the simplest case (it can be easily generalized), we have $A \vdash B$ (meaning “ B is provable from the assumption A ”). By (6), we obtain $\vdash (A \Rightarrow B)$. Putting the symbol \emptyset (the empty set) before \vdash in the last formula gives $\emptyset \vdash (A \Rightarrow B)$. This means that if the formula $(A \Rightarrow B)$ is a theorem of logic, it is derivable from an empty set of assumptions. The above consideration motivates

- (7) $\mathbf{LOG} = Cn\emptyset$,

as the metalogical definition of logic as a theory⁴. By (3) and the definition of being a theory, we obtain that $\mathbf{LOG} \subseteq T$, for any theory T . The last assertion is equivalent to saying that \mathbf{LOG} is the only common part of the consequences of all sets of sentences (logic is the only deductive system belonging to all theories). Definition (7) considers logic as a set of theorems. An alternative treatment consists in defining logic as a set of inferential rules. To repeat a simple example, allow the scheme $A \vdash B$ as a rule. By the deduction theorem, it can be converted to the formula $\vdash (A \Rightarrow B)$. In consequence, logical rules and theorems are somehow equivalent. Although traditional typical codifications of mathematical logic employ axioms and rules of inference, many contemporary formalizations define logic by rules only (systems of sequents or what is called natural deduction)⁵. Theoretically speaking, logic can be also reduced to the stock of all its theorems, but this set is infinite and practically not usable as an instrument of deduction.

Although (7) looks extremely artificial at first light, very sound intuitions stay behind it. We expect that logic is prior to any other theory. This point is captured by the assertion that **LOG** belongs to any other theory. We also expect that logic as an instrument of deduction does not depend on particular subject matter; that is, universal. This intuition is supported by the completeness theorem for logic (if A is a theorem of logic – that is, provable within logic – it is universally valid – that is, true in every model). Hence, logical validity as a semantic phenomenon has its counterpart in derivability from an empty set of assumptions. In other words, no extralogical presupposition is required in order to prove a logical theorem. If there still occur doubts, we can also say that logical theorems are provable from arbitrary premises. This means that logic as such does not distinguish any extralogical content. Finally, the reverse of the completeness theorem (the soundness theorem) establishes that logical inferences cannot produce false conclusions from true assumptions. Summing up the above assertions, the consequences of the metalogical account of logic agree with the most relevant intuitive features of logic⁶.

It should also be stressed that (7) functions on the meta-level. This fact has relevance for the understanding of saying that logic does not depend on any extralogical content. In fact, any definition of logic is surrounded by various metalogical settings. In the case of (7), we have axioms for Cn . Their status is certainly not purely logical, but depends on intuitions dictated by deductive practices. For instance, (5), as I have already remarked above, displays our “human” finite inferential capacities. Further, (3) says that adding a new element to assumptions preserves already deduced conclusions, but this setting is dropped in non-monotonic logics dealing with inferences in which new premises refute previous conclusions. The way of defining Cn as a crucial metalogical concept depends on accepting a more or less definite view about our cognition and its functioning. It can be regarded as another meeting point of logic and cognitive science. Anyway, axioms for Cn are not exclusively dictated by logic itself, but also by empirical knowledge of human cognition.

Even if we accept the remarks in the previous paragraph, the above construction is very abstract and defines logic as so-called *logica docens*; that is, the theoretical science of logic⁷. *Logica docens* is traditionally contrasted or at least compared with *logica utens* (applied logic). Perhaps Petrus Hispanus’s words *ars artium scientia scientiarum ad omniam aliarum scientiarum methodorum principiam viam habent* (logic is the science of science, which establishes methodological principles for all other sciences) well present the idea of logic as applied in human cognitive enterprises. Yet there

is a controversial question about the scope of logic from this point of view. This issue is reflected by the following list of historical understanding of logic (see Risse 1980):

- (i) dialectic (analysis and synthesis of concepts; Plato);
- (ii) analytic (deduction; Aristotele);
- (iii) *organon* (methods of reasoning; Aristotele);
- (iv) canonic (norms of knowledge; Epicurus);
- (v) *medicina mentis* (Cicero);
- (vi) *Vernunftlehre* (rules of pure reason; the tradition of *philosophia rationalis*),
- (vii) *Kunstlehre* (the art of arguing; Husserl);
- (viii) *Wissenschaftslehre* (the theory of science);
- (ix) *Denklehre* (the theory of thinking; Arnauld, Nicole).

How to order this considerable variety of intuitions?

One way of looking for an answer consists in reflecting on how logic is (was) taught. According to the Polish tradition of teaching logic in secondary schools and universities (even at philosophical and mathematical faculties), a course of logic in the wider sense consists (see Trzęsicki 2016) of three parts: (a) semiotics or semantics *sensu largo* including syntax, (b) semantics in the narrow sense (as the logical theory of meaning and reference); and (c) pragmatics, formal logic (logic *sensu stricte*) and the methodology of science. Although this division is vague and principally motivated by pedagogical tasks (in particular, it combines descriptive as well as normative aspects of logical theories), it is still instructive. In particular, the third part of logic *sensu largo* deals with non-deductive reasoning; for instance, induction, abduction, analogy, etc. Moreover, it is a question whether all inferences used in science or in daily life are manifestations of the same pattern, or perhaps it is so that we have general logic as valid (or at least applicable) in all domains or/and several special logics restricted to concrete subject-matters; for instance, legal logic.

A typical problem that arises in the domain of logic *sensu largo* concerns the criteria of correctness of non-deductive inferences. If we say, as it is frequently asserted, that logic is the science of reasoning, we must indicate which inferences are taken into account, deductive ones or non-deductive as well. Reasoning is also a very favorite subject of cognitive science. The prevailing and fairly legitimate position is that cognitive science investigates all kinds of inferences (see Holoyak, Morrison 2005, Holoyak, Morrison 2012), but, as is simultaneously underlined, fallible inferential modes

(inferences with incomplete information) play a central role in our cognition. This position can motivate the view that cognitive science deals rather with *logica utens* than with *logica docens*, even if deduction is cognitively studied (see Rips 1994, Nickerson 2010, Celluci, Gillies 2005). However, I will restrict my remarks to formal logic (*logica sensu stricto*). This means that I skip non-deductive inferences except to make some comparative remarks. Not because I consider fallible inferences as not important or uninteresting for analysis. The reason is that I have nothing to say, for instance, about the criteria of “good” induction in order to take perhaps the most famous problem. As far as the issue concerns deductive inferences I plan to address two problems. Firstly, I will consider whether humans respect logical rules in performing ordinary deductions. Secondly, I will try to answer the question about the genesis of logical deductive competence. The first point definitely concerns *logica utens*, but the second one combines both outlined understandings of logic.

Do humans obey deductive rules? Wason’s famous selection task (see Stenning, Van Lambalgen 2008 for an extensive and critical analysis) motivates the answer not always. I will illustrate the point by two very simple examples; the Wason experiment concerned using the rules of modus ponens (if A implies B and A , then B) and modus tollens (if A implies B and not- B , then not- A). Firstly, consider four famous forms of categorical sentences: (a) every S is P ; (b) no S is P ; (c) some S are P ; (d) some S are not P . The selection task is as follows: which sentence is the negation of (a)? Nobody says (c), but most responders point out (b). The correct answer (following from the classical square of opposition) is that “some S are not P ” functions as the denial of “every S is P ”. Secondly, consider a concrete instance of (d), for instance, “some students are not ice-hockey players” and ask for its conversion. Most responders say “some ice-hockey players are not students”. It is important to note that experiments have been performed (by myself) with groups of students who participated in classes in which elementary logic was taught and who should have known logically correct answers. Incidentally, categorical sentences are somehow basic as is documented by the rise of Aristotle’s logic as the first logical theory invented in history.

A popular explanation for not preserving logically correct inferential rules in concrete situations stems from observations by Daniel Kahneman and Amos Tversky (see Kahneman 2011). They distinguish two cognitive systems: quick thinking and reflective thinking. In ordinary situations, people react via the first system, related to customs, habits, language intuitions, unconscious prejudices, etc. The deliberate and controlled usage of logical rules is associated with the second system which acts more slowly, but con-

trols reasoning from the point of view of its logical qualities. This dualism explains why even logically trained humans err in practically performed inferences. Some additional remarks are here in order. All experiments show that operating with negation causes serious troubles (see (b) and (d) or greater problems with modus tolens than with modus ponens). Reasoning agents consider inferences with premises known as false as ridiculous. An interesting fact is that if a premise is true, responders try to produce a structurally true conclusion, even if it does not follow from an assumption (see the conversion of “some S are not P ”). The above observations strongly suggest that concrete inferences are constrained by content, context, etc. Finally, I also performed an experiment which suggested that logical rules have an import in making concrete inferences. I explained to students of the legal faculty two ways of checking whether a formula of propositional calculus is tautologous or not, namely the method of truth-tables and using rules of natural deduction. The students were earlier informed that the former is mechanical and always leads to a result “yes or no”, whereas the latter is conclusive in the case of the answer “yes” but does not offer a refutation procedure in every case. Students could use either method, Interestingly, most students decided to solve tests by using natural deduction. This indicates that the adjective “natural” is properly applied with respect to such deductive procedures.

The above data motivate various accounts of logical (deductive) competence in action. I will only mention two such approaches. The first theory maintains that deduction goes via constructing so-called mental models (see Johnson-Laird 1983, Johnson-Laird 2014). According to this view, reasoning-agents quickly construct models of given cognitive situations and use them in deriving conclusions from accepted conclusions. The second theory assumes the existence of a mental logic (see Braine, O’Brien 1998), which generates real inferential processes. I do not feel myself to be sufficiently competent to evaluate both mentioned proposals. Nevertheless, I make two rather speculative (or philosophical, remarks). Firstly, a very important feature of both theories consists in considering deductive inferences as empirical cognitive processes. This approach agrees with some tendencies in the philosophy of mathematics underlying that mathematical reasoning is not purely logical (see Byers 2007). On the other hand, even if we remain on a very abstract level, the actual cognitive environment of real inferences, even if it directs reasoning to false conclusions or does not respect rigid deductive rules, agrees with earlier remarks about intuitive factors in the axiomatic definition of Cn . Secondly, both approaches try to formalize inferential performances by setting rules, similar to purely logical ones, but still

explaining errors in deductive steps. Thus, both theories explain the fact that humans are able to perform logically correct inferences and are able to learn logic, but also that they fail in making correct deductions. These remarks suggest that a strict formal approach to logic as *logica docens* is consistent with empiricism on the meta-level.

Contemporary cognitive science is opening new perspectives in answering the question concerning the genesis of logic. Consider *logica docens*; that is, the set of universally valid propositions. If we accept empiricism in epistemology, it is difficult to understand how such theorems became produced by humans, because experiential operations produce only empirical generalizations. Thus we have a deep tension between the metalogical characterization of logic by *Cn* and the epistemological basis of cognitive science, even if remarks made in the previous paragraph are accepted. One way to overcome this difficulty consists in adopting Platonism – that is, the view seeing logic as abstract to the highest degree and located in Plato’s heaven of forms as such – or arguing, as Descartes did, that abstract ideas are innate, or yet maintain that humans, created as *imago Dei*, obtained logic as a gift from God. All these views are anti-naturalistic or supra-naturalistic, some of them *ad hoc* (Platonism, Cartesianism), but others (theism of any kind) appear as based on speculative extra-scientific premises (theism). In any case, the position of the anti-naturalist appeals to secret beings (souls, spirits, ideas) and mysterious kinds of cognition (intellectual intuition, *Wesenschau*, direct perception of abstract ideas, etc.).

Naturalism is a good environment for empiricism in cognitive science. For the naturalist, logic has not fallen from heaven, platonic or other (I allude to the title of Hoimar von Ditfurth’s book *Der Geist fiel nicht vom Himmel – The Ghost Has Not Fallen From Heaven*). Leaving metaphors aside, empirical-naturalistic cognitive science shares three main assumptions, namely (a) all facts, mental or physical exist in space and time; (b) mental facts are embodied; (c) all phenomena of human cognition are empirically knowable, including their modeling by mathematical or logical methods into the domain of empirical procedures.⁸ Since the theory of mental models as well as the theory of mental logic accept (a), (b) and (c), both can serve as a background for proposals concerning the genesis of logic from the naturalistic point of view, provided that they are supplemented by evolutionary biology combined with genetics (this combination is frequently called “Neo-Darwinian evolution theory”).

Omitting details (see Woleński 2012), logic appears a result of the long development of the species. This process was essentially accelerated by the rise of human language and advanced counting ability (see Dehaene 1997,

Johansson 2005, Lieberman 2005, Hurford 2007, Talerman, Gibson 2012). All accessible empirical data suggest that speaking (linguistic competence in Chomsky's sense) and capacities for counting appeared earlier than making inferences using logical principles. On the other hand, some amount of logical competence is demonstrably present in newborns and is mastered relatively rapidly (see Langer 1980). Yet all these facts are consistent with the traditional empiricist account of logic (e.g. as in Mill), that is, seeing logic as consisting of firm empirical generalizations. An important factor consists in employing logical competence without being conscious of it. Consequently, logical competence belongs to the phylogenetic equipment of humans, although its manifestation is ontogenetic. Thus, any account of the genesis of logic must respect these settings. This observation leads to the idea that the deep biological structure of the human species is responsible for logical capacities. On the other hand, I guess that the metalogical definition of logic, that is, (7), cannot be dropped.

The Millian explanation of logic does not fit the metalogical features of logic. Take Kazimierz Ajdukiewicz's classification of inferences (see Ajdukiewicz 1955; I introduce minor changes to its original version). He distinguished deductive (infallible), inductive (increasing the initial probability) and worthless modes of inferences (this last category is illustrated by the following example "if today is Thursday, the Vistula is a river"). Speaking more formally, deduction is based on the operation Cn , induction on probabilistic connections between premises and conclusions (I omit any discussion of the question of how the concept of probability should be interpreted in this context) and the worthless inferences are lacking in any substantial links between the elements of such performances. Ajdukiewicz's treatment considers inferences as actions. This way of looking at inferential activities nicely contributes to the explanation of the fact that humans sometimes make good inferences and sometimes fail to be correct⁹. However, even if we assume that inductive and worthless inferential modes appeared earlier than deductive ones, the rise of logical rules looks like a very mysterious event in the history of human intellectual development. In other words, the transformation, if any, of inductive inferences into deductive ones has no epistemic justification.

For the sake of my further considerations, I will look at inferences as processes of information-transmission (this way of looking at reasoning is popular in cognitive science). Under this, Ajdukiewicz's classification takes the following form. Deductive inferences do not increase the amount of information contained in premises. Speaking formally, if $P \vdash^{\text{ded}} C$ (P – premises,

C – conclusion, \vdash^{ded} – deductively derivable), then $\text{Inf}(P) \geq \text{Inf}(C)$. In the case of inductive inferences, if $P \vdash^{\text{ind}} C$, then $\text{Inf}(P) < \text{Inf}(C)$. Finally, if $P \vdash^{\text{worth}} C$, then neither $\text{Inf}(P) \geq \text{Inf}(C)$ nor $\text{Inf}(P) < \text{Inf}(C)$ (information contained in P and information contained in C) are mutually independent. These settings fit our intuition, because deduction is infallible; that is conclusion cannot provide a counterexample to premises, induction is fallible, because counterexamples to premises are possible, but worthless inferences are such because there is no information link between premises and conclusions. Yet because nothing is free of price, fallibility of induction is the price for its role in increasing information. Perhaps it should be noted that a wrong deduction is a not a deduction at all, but a wrong induction remains an induction. This observation is not at odds with the fact that people err in deductive steps. Once again errors are made in inferences as actions.

At least two accounts of the rise of deduction are possible. The first possibility is based on the observation that humans are semantically prodigal. This means that some artifacts produced by humans are somehow redundant, although they can be cognitively relevant or interesting. In other words, we produce, as instances of the same species, various devices exceeding our direct biological needs stemming from contacts with the natural world. Perhaps the fine arts, surrealistic poetry, fictional novels, or highly abstract theoretical science like, for instance, pure mathematics, belong to this order of things. Logic is a natural candidate as the next element in the collection of this phenomenon. Some researchers argue (see Talmont-Kamiński 2014) that religion also appeared in human history as a cognitive by-product of more basic cognitive activities. In particular, humans are able to create content going beyond provided empirical information, to paraphrase Jerome Bruner’s famous phrase. Perhaps logic as a collection of universally valid theorems can be viewed as a cognitive by-product of fallible and worthless inferential performances, frequent in the acts of human cognition. In other words, formula (7) defines logic as a cognitive artifact, because our actual inferences never start from an empty set of premises.

Yet the above picture of the rise of logic can be questioned on similar grounds. First of all, it is *hoc* and too easy. One could argue that everything that arose in the process of biological evolution has significance. In the case of logic, it is enough to observe that (7) is equivalent to the assertion that logic is the only common part of all theories. Perhaps the human species possesses such a device as cognitively relevant, similarly as religion or abstraction fulfill quite important needs. In other words, perhaps it is so that (7) or its equivalents manifest something non-accidentally present

in our mental structure. Hence, we should look for a more complex explanation of the genesis of logic and not limit our theory to pointing out that logical rules are a cognitive luxury. Even if we agree that logical theorems go beyond empirically provided information, this fact requires a deeper justification.

The second proposal, consistent with the naturalistic point of view, assumes that logical competence has its roots in the molecular structure of DNA and the processes of transmitting genetic information. An important hint comes from the observation that neural networks can be modeled by logical and mathematical relations (see Perlovski 2001); also settings concerning cellular automata (see Ilachinski 2011) and biological computing (see Lamm, Unger 2011) are very helpful in the present context. A deeper picture stems from looking at DNA structure as a topological space (see Bates, Maxwell 2005). This space is open-closed. Now, the successful transmission of information must protect the existing informative amount, but, on the other hand, admit its increasing. The properties of topological closure are similar to Cn^0 . Thus, DNA-structure is partially closed, but it is also partially open. This openness is responsible for producing new information. This picture suggests that molecular structure can be regarded as a biological foundation of processes protecting possessed information as well as processes providing new information. To complete these considerations, molecular structure generates purely logical inferences (the closeness of DNA-structure) as well as inductive ones (the openness of DNA-structure). And because nothing is perfect, the same structure cannot block worthless inferences, perhaps as by-products of sound inferential performances. This account explains the existence of all kinds of inferences listed by Ajdukiewicz; that is, deductive, inductive, and worthless.

I do not say that logical competence or logic are ontologically present at the level of molecular structure. This thesis would imply that all species are equipped with an ability to use inference rule at least as habits. We know that this assertion is not true, although the transmission of genetic information is a universal phenomenon in all living components of the natural world. The argument for this thesis is very simple and points out that the molecular structure of many species, including humans, is very similar, but only we have advanced logical competence, and *logica utens* and *logica docens* as its outcome. *Logica utens* gradually developed in our phylogenetic history together with many other faculties (see above). The articulation of *logica docens* happened about 2500 years ago. but its metalogical codification was completed about 80 years ago. Anyway, logical competence, biologically grounded, precedes *logica utens* as well as *logica docens*. The point is that

not only temporally precedes it, but also that the latter are conditioned by the former. Schematically speaking, we have the following picture

- (*) DNA and its topological properties → genetic information and principles of its transmission → (potential) logical competence → ... → (actual) logical competence → *logica utens* → *logica docens*,

which lists the main steps of the process leading from the biological background of *Cn* to *logica docens*.

However, this scheme is still incomplete, which is indicated by the symbol → ... → pointing out an essential gap in (*). More specifically, the biological background (DNA and its topological properties, the method of transmission of genetic information) of *Cn* was not enough, although indispensable, for the rise of *logica utens* and its articulation in *logica docens*. In particular, we do not know at which stage of development humans began to use logical rules. Although we can conjecture that at the beginning it happened not consciously, the details remain unknown, as in the case of newborns (see above). The crucial factor probably consisted in transformation of genetic information into symbolic (semantic) information. This process has not been fully described and perhaps it remains mysterious as such in the future; personally, I am inclined to think that it is the deepest mystery in the entire history of our species, at least from the naturalistic point of view. The view that genetic information can be considered as being of a linguistic character is actually very tempting and executed by speaking of genetic codes as having alphabets, words, syntax, etc. On the other hand, passing from genetic information to semantic content cannot be explained in such a simple way based on analogies taken from formal grammar. Most linguists maintain that the evolution of signs passed from the expressive via the iconic to symbols. Grammatical structures evolved correspondingly (see Heine, Kuteva 2007, Hurford 2012) from nominal through sentential-extensional to sentential-intensional. Thus, the development of language proceeded by way of transition from a-semantic or poorly-semantic items to fully-semantic (intensionality, symbolism). This process had to precede the rise of logic as explicitly articulated rules or full-blooded theory. Omitting the biological background, at the beginning was the word; *logos* (as logic) appeared later.

N O T E S

¹ I will not enter into the problem of logical skills of other animals. See Aberdein 2008 for an interesting discussion in Cambridge (in the 17th century) about the logic of dogs.

² To avoid trivial cases, I assume that T is consistent. A non-trivial justification of this presupposition points out that only consistent theories have models.

³ I take classical logic as the point of reference. On the other hand, similar constructions can be executed for various non-classical logics. A fuller treatment of the definition of logic in the context of the existence of rival logics, require addressing such philosophical questions as, for instance, monism (there is the only correct system of logic) and pluralism (there are many correct systems of logic, relatively to different regions of reality). Actually, the situation is additionally complicated by new logics, like non-monotonic, fuzzy, etc. In my opinion, the word “logic” is employed to extensively, but it is a separate topic.

⁴ Not all axioms (1)–(6) are relevant for the given definition of logic. For instance, (5), although it does not participate in the construction of (7), possesses an interesting cognitive content, because it constrains that human deductive acts are reducible to inferences with finite sets of premises. This axiom is not proper for Cn in the frameworks of so-called infinitary logic.

⁵ In fact, Cn is a “battery” of means of deduction.

⁶ The plurality of logics is not restricted to so-called rival logics (see note 3). In particular, there is a question whether higher-order logics are logics in the proper sense, has different answers. Following Woleński 2004 I restrict logic to the first-order system (elementary logic), because it is universal in the literal sense.

⁷ The earlier thesis that logic is prior to any other science, holds only for *logica docens* and assumes (5) as well as the definition $T = CnT$.

⁸ It does not entail that mental facts are reduced to physical ones. This means that naturalism is not necessary a variant reductive physicalism.

⁹ I am fully aware of the fact that the borderline between wrong inductive (and even deductive ones) and worthless inferences is vague.

¹⁰ The main difference is that $Cn\emptyset \neq \emptyset$, but topological closure of the empty set is also empty. However, if we define **LOG** by saying that it is a part of any theory, this difference practically disappears.

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