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## THE PROBLEMS OF THE MENTAL LOGIC WITH THE DOUBLE NEGATION: THE NECESSITY OF A SEMANTIC APPROACH

**Abstract.** The double negation has always been considered by the logical systems from ancient times to the present. In fact, that is an issue that the current syntactic theories studying human reasoning, for example, the mental logic theory, address today. However, in this paper, I claim that, in the case of some languages such as Spanish, the double negation causes problems for the cognitive theories mainly based on formal schemata and supporting the idea of a universal syntax of thought in the human mind. Thus, I propose that, given those problems, semantic frameworks such as that of the mental models theory seem to be more appropriate for explaining the human inferential activity.

*Keywords:* double negation, formal rules, mental models, semantics, syntax.

### Introduction

From the Stoicism to the modern logical systems, including, of course, the one presented by Gentzen (1935), the issue of the double negation has always been taken into account in the history of logic. In this way, it can be said that it continues to be considered even by the most current theories trying to explain human reasoning based on logical rules. This is the case, for example, of the mental logic theory (e.g., Braine & O’Brien, 1998a; O’Brien, 2009, 2014; O’Brien & Li, 2013; O’Brien & Mafrinati, 2010). This theory states that there is a syntax of thought, and its arguments hence are somehow related to the idea of ‘language of thought’ and to theses coming from Fodor (1975) and Macnamara (1986). The chapter authored by Braine and O’Brien (1998b) is very enlightening in this regard. However, the most important aspect of the mental logic theory for this paper is that it claims that the human mind has a number of formal schemata that are applied when the sentences in natural languages include certain words or expressions. In particular, the schemata are used when the words or expressions

are such that the structures of the sentences in natural languages match the structures of the schemata.

As far as the double negation is concerned, this means that, whenever two words denoting negation appear in a sentence in natural language, the schema that, according to the mental logic theory, corresponds to the double negation will be used. Nevertheless, while this is not a problem if English language is considered, it does become a problem in other languages, for example, Spanish, in which the double negation does not always mean exactly double negation. Indeed, in this last language, there are several cases in which the double negation actually indicates only one and simple negation.

From this perspective, it seems that it can be problematic to try to describe human reasoning just resorting to syntax, and that not only pragmatics (which, as shown below, is also deemed by the mental logic theory) but also semantics plays an important role in the human inferential activity. This in turn leads us to think that maybe a semantic framework such as that of the mental models theory (e.g., Johnson-Laird, 2006, 2012, 2015; Johnson-Laird, Khemlani, & Goodwin, 2015; Khemlani, Lotstein, Trafton, & Johnson-Laird, 2015; Khemlani, Orenes, & Johnson-Laird, 2012, 2014; Orenes & Johnson-Laird, 2012) can be a better alternative to explain the real way the human mind works, irrespective of the language spoken by the particular individuals.

To show all of this is the main goal of this paper. To do it, I will begin by commenting on the historical importance that the double negation had in systems such as Stoic logic in Ancient Greece or standard propositional calculus and Gentzen's (1935) framework in more recent times. Then I will explain, on the one hand, how the mental logic theory assumed the theses of systems such as those mentioned about the double negation, and, on the other hand, the characteristics of the mental formal rule that the theory attributes to that kind of negation. Thirdly, I will describe the difficulties that can be found if that formal rule is accepted in the case of certain languages such as, for example, Spanish. Finally, after accounting for the general ideas of the mental models theory, I will argue why, given those facts, this last theory can be a more adequate approach to expound the real way the human mind makes inferences.

### **The double negation in Stoic logic and in standard calculus**

Indeed, the double negation is an issue that has been addressed from ancient times. In particular, as indicated, for example, by Barnes, Bobzien,

and Mignucci (2008, p. 102), the Stoics were concerned with a concrete type of denial: the ὑπὲρ ἀποφατικόν, expression translated by them as ‘super-negation.’ According to these writers, this kind of negation was clearly what we understand as ‘double negation’ today, and, to support their idea, they resort to Diogenes Laërtius (*Vitae Philosophorum* VII, 69), where it is stated that a sentence such as ‘it is not possible that it is not day time’ does not say something different from a sentence such as ‘it is day time.’ So, there is no doubt that Greek philosophers were already interested in the characteristics of this particular type of negation.

Nevertheless, as it is well known, it has been also an important element in the course of the history of logic. In this way, perhaps it is enough to remember that it is crucial in Gentzen’s (1935) system, and that, as shown by Deaño (1999, p. 153), it refers to one of the primitive rules of standard logical calculus, the negation elimination rule, which can be expressed as follows:

$$\neg\neg p / \text{Ergo } p$$

where ‘ $\neg$ ’ stands for logical negation.

It is absolutely obvious that this is an essential rule in logic, since, for instance, it can be involved in many demonstrations of proofs carried out via *Reductio ad Absurdum*. But what is important now is that this rule was assumed by the mental logic theory. This point is developed in the next section.

### **The double negation in the mental logic theory**

Actually, the mental logic theory took a further step. As presented by Braine and O’Brien (1998c, p. 80; Table 6.1), this theory proposes that a rule akin to the previous one is one of the ‘Core Schemata’ of human reasoning, which means that people usually apply it to any sentence in which that is possible. This is a relevant point, since the theory does not consider all of the rules that it accepts to be of the same kind. Its proponents continuously insist on that their framework is not the same as that of standard propositional calculus, which can be checked in any of their main works (e.g., Braine & O’Brien, 1998a; or any of the papers or chapters cited above) and in several studies describing or explaining the theory (e.g., López-Astorga, 2015a, 2015b, 2015c, 2016). In texts such as those ones, it can be seen that the mental logic theory rejects some of the rules of standard logic. Those rules are the ones that, if the results of empirical experiments are taken into ac-

count, individuals seem not to use. Nonetheless, on the other hand, those works also show that all of the rules that the theory accepts are not, as said, of the same type.

Although there are more types of schemata in the mental logic theory, perhaps the most important ones here are only the Core Schemata, which have already been mentioned, and the ‘Feeder Schemata.’ The Core Schemata are the most basic and, as indicated, people use them whenever they can. The main requirement is that the sentences spoken or written in any natural language have the same structure as a particular Core Schema. When this happens, that schema tends to be applied. A very illustrative example can be that of the Core Schema 7 (Braine & O’Brien, 1998c, p. 80; Table 6.1), which is a version of one of the ἀναπόδεικτοι (*indemonstrables*) that, according to Diogenes Laërtius (*Vitae Philosophorum* VII, 80), was proposed by Chrysippus of Soli. It is often expressed in English in this way:

$$\begin{array}{l} \text{If } p \text{ then } q \\ p \\ \hline \text{Ergo } q \end{array}$$

As it is well known, the Latin name of this rule in logic is *Modus Ponendo Ponens*, but what is interesting here is that, as stated, according to the mental logic theory, this is a Core Schema. This means that people will use it whenever they find a conditional such as ‘if p then q’ and its if-clause, i.e., p, and that, by virtue of it, they will always conclude q.

But another important type of schemata is, as also said, that of the Feeder Schemata. These schemata have an important restriction. They are only used when their application can be productive, that is, when their application can lead to further conclusions. A typical example of Feeder Schema, which matches Schema 8 in Braine and O’Brien (1998c, p. 80; Table 6.1), is the conjunction introduction rule in standard calculus. As it is known, this rule can be expressed in English as follows:

$$\begin{array}{l} p \\ q \\ \hline \text{Ergo } p \text{ and } q \end{array}$$

This said, what is important for this paper is that the double negation is related to these two types of schemata. Indeed, if expressed as in standard logic, i.e., as  $\neg\neg p$  / Ergo p, it is a Core Schema (in particular, Core Schema 1 in Braine & O’Brien, 1998c, p. 80; Table 6.1). On the other hand, if expressed

in the opposite direction, that is, as  $p / \text{Ergo } \neg\neg p$ , it is a Feeder Schema (see Braine & O'Brien, 1998c, p. 81; Footnote b).

However, this account of the double negation, which seems to be undoubtedly obvious, and appears to describe adequately the human syntax of thought, presents a great problem when not only English but also other languages are considered. That problem is linked to the fact that there are languages such as Spanish in which a sentence can include two words denoting negation without necessarily having a positive sense or meaning. This is clearly a difficulty for the mental logic theory, since, as far as I understand it, it claims that, when a sentence has two terms meaning denial or negation, a logical form such as  $\neg\neg p$  must be attributed to it. This is so because the mental logic theory is, as I have commented, a formal and syntactic theory. Therefore, it can be thought that, from the framework of the mental logic theory, it is hard to explain the reasoning processes in languages other than English, and that, for this reason, another cognitive approach with a more universal scope can be more appropriate. But, before taking this last idea into account, it can be opportune to better explain the exact role of the double negations in certain languages. My example is, as mentioned, Spanish.

### **Spanish and the double negation**

True, there are sentences in Spanish with two negations whose meaning refers to only one negation and that, however, are absolutely correct. A clear example can be the following:

*No viene nadie*

Given that the sentence includes two words denoting negation (*no*, which can be translated as 'not', and *nadie*, which can be translated as 'nobody'), a literal translation could be this one:

Nobody does not come

Nevertheless, this sentence, in addition to seeming very rare in English, does not reflect the actual sense of the Spanish sentence, which is really:

Nobody comes

Of course, there is a grammatical reason why the first Spanish sentence is correct in this language. In it, if words such as *nadie* (nobody), *nada* (nothing), or *nunca* (never) are after the verb of the sentence, the adverb

*no* must be before the verb. Thus, the final result is that the sentence has two negations, or, if preferred, that it includes a double negation. Clearly, this is a very interesting linguistic phenomenon that has been studied in many works (e.g., Camus, 1992; Jespersen, 1917; López García, 1977). But what is relevant here is that, if the human mind mainly reasons based on syntax (which is what the mental logic theory seems to claim), then a Spanish native speaker should understand the first Spanish sentence in this section as an expression whose correct logical form is  $\neg\neg p$ , since that sentence, as said, has two negations. Nevertheless, as also indicated, the real meaning of that sentence refers to  $\neg p$ , and Spanish native speakers almost always tend to interpret sentences such as that as  $\neg p$ . So, all of this requires an explanation.

However, the mental logic theory does not appear to be able to give that explanation. A wider framework is necessary, a framework that assumes that human reasoning is not basically syntactic and that factors such as semantics can have a very important influence on it as well. Really we have a framework of this kind. That is the one of the mental models theory and, as it will be explained in the next section, that approach does not seem to have the difficulties of the mental logic theory to explain phenomena such as that of the double negations in Spanish.

### The double negation and the mental models theory

From the mental models theory perspective, human reasoning is not a syntactic process, but a semantic one. Given a sentence, individuals tend to identify the possibilities to which that sentence refers, and reasoning is basically to review and combine such possibilities. Thus, for example, this theory explains why individuals often make inferences with the structure of *Modus Ponendo Ponens* without resorting to any rule or formal schema. In principle, the first premise of *Modus Ponendo Ponens*, which is a conditional, refers to these three semantic possibilities:

- |       |                |                |
|-------|----------------|----------------|
| (I)   | ANTECEDENT     | CONSEQUENT     |
| (II)  | NOT-ANTECEDENT | CONSEQUENT     |
| (III) | NOT-ANTECEDENT | NOT-CONSEQUENT |

And this is so because the remaining possibility (ANTECEDENT & NOT-CONSEQUENT) is the only possibility in which, a priori, a conditional would be false (think, e.g., about the truth tables of standard logic).

But the case is that, given that the second premise of *Modus Ponendo Ponens* provides that the antecedent is true, the possibilities (II) and (III)

are removed, and the result is that only one scenario can be taken into account. That scenario is obviously (I), i.e., a scenario in which the consequent is also true, which leads one to accept that, when the antecedent of a conditional is true, it is only possible that its consequent is true too.

However, this alone, without other details, can be considered to be a simplification of the theory. Strictly speaking, it has nothing to do with standard logic or truth tables. Firstly, (I), (II), and (III) are the 'Fully Explicit Models' of the conditional, and people do not always identify these three scenarios. (I) is easy to note, but the other two possibilities require further effort. Secondly, semantics and pragmatics are very important in this theory. They are so important that they cause modulation processes in which some of the possibilities are removed or modified. A clear example of this can be this:

"If Pat is in Italy then she is not in Rome" (Orenes & Johnson-Laird, 2012, p. 361).

As it can be noted, the combinations that can be attributed to this sentence are only (I) and (II), and this is so because (III) would imply that Pat is not in Italy and she is in Rome, which is impossible. In this way, the theory can explain why, despite the fact that 'if Pat is in Italy then she is not in Rome' is a conditional, people can draw that Pat is not in Rome from that conditional and the information that she is not in Italy. That derivation is not correct in standard calculus, but individuals can make it because the only possibilities are (I) and (II), the information that Pat is not in Italy removes (I), and in (II), the only possible option, Pat is not in Rome.

So, the approach of the mental models theory seems to be more appropriate to account for phenomena such as that of the double negation in Spanish. The sentence indicated in the previous section really includes two negations, but semantics and pragmatics reveal that the actual possibility to which it refers is not  $\neg\neg p$ , but  $\neg p$  ('nobody comes'). As said, this is a phenomenon that a purely syntactic theory cannot explain and, for this reason, it can be thought that the mental models theory has the potential to better describe human reasoning, regardless of the language spoken by the particular individuals.

It is true that the mental logic theory also assigns a role to pragmatics (see, e.g., Braine & O'Brien, 1998b, p. 46), but that role is very limited. What pragmatics does in the mental logic theory is to add pragmatic premises that are not explicitly said. For example, given the information that Pat is in Rome, pragmatics can add another premise: 'if Pat is in Rome then she is in Italy.' And this is so because individuals have the information that

Rome is in Italy by virtue of their general knowledge. Hence it is obvious that these pragmatic mechanisms cannot solve the problem of the previous Spanish sentence mentioned above, since no pragmatic premise can help us understand that, even though the sentence denies twice, it has to be interpreted as if it only denied once. To understand that, it is necessary to consider something else than syntax (which only reveals that there are two terms denying in the sentence) and pragmatic premises. It is also necessary a global view of the real sense of the sentence, which only appears to be possible from a wider framework such as that of the mental models theory. This last framework can ignore the terms in the sentences that can be related or referred to particular logical forms, and give priority to semantics and the semantic possibilities.

And all of this can be combined with other facts. I have only described here an example linked to the conditional of how the mental models theory works. Nonetheless, this later theory, as the literature on cognitive science shows, can also account for the results achieved in other conditional reasoning tasks and other exercises using other traditional connectives of logic (e.g., conjunction or disjunction). So, it appears to be obvious which of the two theories is the most appropriate to describe the real way in which the human mind works, no matter what the language is spoken.

## Conclusions

It is well known that the mental logic theory and the mental models theory have been rival approaches for a long time. In this way, it is also known that there are many cognitive phenomena that can be explained by both of them (see, e.g., López-Astorga, 2015b, 2015c). For this reason, it appears to be very important to find points or facts that only are consistent or coherent with one of these frameworks, since that can help us make a decision with regard to the theory that should be assumed.

It is true that very interesting discussions and debates between the proponents of these two approaches are to be found in the literature (and in particular in the works corresponding to both frameworks cited in this paper). Of course, the arguments presented in that literature can lead one to be more inclined towards one of these theories. However, I think that the phenomenon reviewed here is crucial. If something has been revealed with great clarity by this study, it is that the fact that, in some languages such as Spanish, there are double negations meaning just one negation seems absolutely incompatible with the basic assumptions of the mental logic the-



ory, which gives high importance to the words or terms appearing in the sentences, since they lead to logical forms.

Maybe it can be thought, and this is a possible objection against my arguments here, that the mental models theory has a problem in this way too. It does not explain in detail which exact mental process leads one to understand correctly the sentences with two negations in certain languages is. As shown, the mental models theory is consistent with this fact, but it neither has explicitly addressed it nor has provided a well-defined account in this regard (at least, as far as I know). Nevertheless, in my view, the key point is that the particular use of the double negation in some languages causes relevant problems to the mental logic theory, and not to the mental models theory. Thus, the fact that that use is not a difficulty for the latter can be already an indication that the mental models theory is the theory that must be followed and the one that will probably give a clear explanation of that phenomenon in the future. In any case, for now, it can be said that the mental models theory seems to be compatible with the different grammatical particularities of every language, and that, at least, has the potential to explain them. This, as argued, is hard to say with respect to the mental logic theory.

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