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PHYSARUM POLYCEPHALUM SYLLOGISTIC L-SYSTEMS AND JUDAIC ROOTS OF UNCONVENTIONAL COMPUTING

Abstract. We show that in Kabbalah, the esoteric teaching of Judaism, there were developed ideas of unconventional automata in which operations over characters of the Hebrew alphabet can simulate all real processes producing appropriate strings in accordance with some algorithms. These ideas may be used now in a syllogistic extension of Lindenmayer systems (L-systems), where we deal also with strings in the Kabbalistic-Leibnizean meaning. This extension is illustrated by the behavior of *Physarum polycephalum* plasmodia which can implement, first, the Aristotelian syllogistic and, second, a Talmudic syllogistic by *qal wa-homer*.

I. Introduction

Among approaches to biological computing there is one represented by L-systems invented by Aristid Lindenmayer [4], which allow us to simulate the growth of plants by formal grammars [7], [14]. In the project [2], we consider *Physarum polycephalum* as the medium of computations to embody complex extensions of L-systems; about the computational power of *Physarum polycephalum* please see [1], [2], [8], [9], [10]. In this paper, we show that we can implement different syllogistics including Talmudic reasoning by *qal wa-homer* in the biological behavior of *Physarum polycephalum*: the Aristotelian syllogistic [5] and a Talmudic syllogistic formalized by Yisrael Ury [15]. The first is implementable within standard trees of appropriate L-systems considered as growing plasmodia. The second is implementable by growing appropriate L-systems only in one direction. Hence, the *Physarum polycephalum* plasmodium may be regarded as an L-system that is being spatially extended to implement different logics. This extension can be described as a *Physarum* L-system (Section 3). Within this system we can

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implement (i) Aristotelian syllogistic in the *Physarum* media (Sections 4, 5) and Talmudic reasoning formalized in [15] (Section 6).

In our project [2], we obtained the basis of a new object-oriented programming language for *Physarum polycephalum* computing [12]. Within this language we can check the possibilities of practical implementations of storage modification machines on plasmodia and their applications to behavioral science such as behavioral economics and game theory. The point is that experiments with plasmodia may show fundamental properties of any intelligent behavior. The language proposed by us can be used for developing programs for *Physarum polycephalum* by the spatial configuration of stationary nodes. Thus, in this language we can design the Aristotelian syllogistic [5] and a Talmudic syllogistic formalized by Yisrael Ury [15].

Let us notice that the ideas of unconventional computing, i.e. the ideas that any creating in the physical world such as L-systems is a kind of computation came to the minds of Kabbalists for the first time. According to the Kabbalists, each physical item is a string of an appropriate L–system. Each string of creature consists just of three characters of the Hebrew alphabet. Then Leibniz supposed that each string of creature consists of two characters called a subject and predicate with appropriate numerical values. The Judaic roots of syllogistic L-systems are examined in Section 2.

II. Judaic roots of unconventional computing within syllogistic extensions of L-systems

According to unconventional computing, any natural process can be considered a set of programmable strings. First the idea that the world is a class of strings (such as strings of L-systems) was thought up in Kabbalah, the esoteric teaching of Judaism. It was claimed that the world does not exist in itself and it is a combination of the characters of the sacred (Hebrew) language, in which the Holy Scripture is written. According to the *Sefer Yetzirah*, the best known Kabbalistic source, the entire Universe was created from the set of 22 Hebrew letters. Each character of the Hebrew alphabet is given an assigned number, beginning with one for **א**, two for **ב**, and so on. The tenth character, **י**, is numerically equivalent to 10, and successive characters equal 20, 30, 40, and so on. The letter **ק** equals 100, **ר** 200, **ש** 300 and the last letter, **ת**, equals 400. The final forms (sofit) of the Hebrew letters are counted as a continuation of the numerical sequence for the alphabet. These sofit letters are assigned from 500 to 900, respectively:

letter	numeric value	letter	numeric value	letter	numeric value
א	1	י	10	ק	100
ב	2	כ	20	ר	200
ג	3	ל	30	ש	300
ד	4	מ	40	ת	400
ה	5	נ	50	ך	500
ו	6	ס	60	ם	600
ז	7	ע	70	ן	700
ח	8	פ	80	ף	800
ט	9	צ	90	ץ	900

Kabbalah reconstructs the dynamics of natural phenomena from two points of view: on the one hand, Hebrew phrases of the Torah can be represented as numeric algorithms for describing natural phenomena, on the other hand, mathematical calculations over phenomena can be read as Hebrew phrases.

The roots of Kabbalah have grown from Gematria [**גימטריא**], the cryptogramme giving instead of an initial word its numerical value or replacing one word by another word whose letters correspond to the letters of the first word by a special combination of characters. The two forms of Gematria occuring in the Talmud and Midrash are as follows:

1) *Numeric values of words.* Two basic subcases are:

i) A number, hinting in the Holy Scripture at a subject. For example, in the Torah (*Gen.* 14:14) it is stated that Abraham, waging war against the four Eastern kings, armed and took 318 of his trained servants; meanwhile there exists a Midrash according to which Abraham took only one servant, Eliezer. Gematria solves this contradiction by referring that the sum of numerical values of letters of the name Eliezer is equal to 318 (**א** = 1; **ל** = 30; **י** = 10; **ע** = 70; **ז** = 7; **ר** = 200):

שמונה עשר ושלש מאות: רבותינו אמרו אליעזר לבדו היה, והוא מנין גימטריא של שמו:

three hundred and eighteen: Our Sages said (*Gen. Rabbah* 43:2, *Ned.* 32a): It was Eliezer alone, and it [the number 318] is the numerical value of his name.

ii) A word of the Holy Scripture hinting at a number, person or subject. For instance, a word can be accepted in its ordinary numerical value, for example the word הכסף, money, which King Ahasuerus promised to return to Haman, presaged העץ, the gallows-tree on which he was hanged, because both words = 165 (*Esther* 3:11). In this form Gematria is a simple arithmetic equation, for example צמח = מנחם = 138. Further, one of the members of equality or both members can be added up with a unit or two, an 'external number' for those words whose numerical value is regarded. For example: both תורה and יראת equal 611; by adding the number 2, (an 'external number' for both words), we sum up 613 (*Numb. Rabbah* 18:21).

2. *Cryptography.* All or some letters of a word can be replaced also by appropriate letters in accordance with the cryptographic alphabet. One of the methods of cryptography is *Atbash* [אתבש] that exchanges each letter in a word or a phrase by opposite letters. Opposite letters are determined by substituting the first letter of the Hebrew alphabet [א] with the last letter [ת], the second letter [ב] with the next to last [ש], etc. The result can be interpreted as a secret message or calculated by the standard Gematria methods. For example, Prophet Jeremiah (51:1) calls Babylon the name לב קמי for unclear reasons. By Atbash, לֵב קָמָי is a cryptography, having the value כַּשְׂדִּים (Chaldea). Another instance of Atbash is when the country שֵׁשַׁךְ is understood as בָּבֶל (*Jeremiah* 25:26 and 51:41).

Many Kabbalists had attempts to recreate some samples of the Divine speech creating the Universe by using different Gematria methods. They supposed that the Divine words may be generated by a purely mechanical process and might have no meaning in human language. In fact, as we see, the idea of automaton, a self-operating machine, flashed upon Kabbalists for the first time (for more details see [6]).

As an example of Kabbalistic automata let us consider one of the automata simulating the Divine speech proposed by Eleazar ben Judah ben Kalonymus of Worms (Rokeach), the Talmudist and Kabbalist, born probably at Mayence about 1176 and died at Worms in 1238 [6]. He was a descendant of the great Kalonymus family of Mayence.

The matter is that each Hebrew character has a name that is a combination of other Hebrew characters, e.g. the first letter of the Hebrew alphabet is אלף and its name is a combination of the following three letters: אלף, למד, פא. The first letter in the names of each Hebrew letter is this letter itself, e.g. the first letter of the name of אלף is אלף itself. This allowed Rokeach to define each letter as a production rule producing another letter. As a result, these calculations were very close to modern L-systems with coinductive definitions of production rules. In the following example of Rokeach's automaton, we can see, how the letters of the word 'Talmud' [תלמוד] emerge from אלף in an L-system:

Halting pattern: *t* [ת], *l* [ל], *m* [מ], *w* [ו], *d* [ד] (Talmud)

Input string: *'a* [א], *l* [ל], *f* [פ]

Production rules: *'a* → *lf*; *l* → *md*; *f* → *'a*; *m* → *m*; *d* → *lt*, *t* → *w*

Generation1: *lf* [לף], *md* [מד], *'a* [א]

Generation2: *md* [מד], *'a* [א], *m* [מ], *lt* [לת], *lf* [לף]

Generation3: *m*, *l**t***, ***lf***, ***m***, *md*, ***w***, *m**d***, *'a*

All five letters of the word 'Talmud' appear after three generations.

This automaton can be simulated in a *Physarum* L-system, where all the variables denote appropriate Hebrew characters and production rules correspond to the production rules of Rokeach's automaton.

Each root of a Hebrew word has only three characters. This fact allows Kabbalists to claim that each item denoted by suitable three Hebrew characters is a string expressing the symbolic meaning and the power of that item. In the *Sefer Yetzirah*, it is stated that each string of the length three is obtained by generations of L-systems from the three letters: *'alef* [א], *mem* [מ], *shin* [ש]:

(א) עשרים ושתים אותיות יסוד, שלש אמות ושבע כפולות וי״ב פשוטות: (ב) שלש אמות אמ״ש, תולדות השמים אש, תולדות הארץ מים, תולדות אויר רוח. אש למעלה ומים למטה ורוח חק מכריע בנתיים: (ג) שלש אמות אמ״ש, מ׳ דוממת, ש׳ שורקת, א׳ חק מכריע בנתיים: (ד) שלש אמות אמ״ש, יסודן כף זכות וכף חובה ולשון חק מכריע בנתיים:

There are twenty-two basic letters. Three of them are the first elements [water, air, fire], fundamentals or mothers, seven are double letters and twelve are simple letters. The three fundamental letters אמ״ש have as their basis the balance. In one scale is the merit and in the other criminality, which are placed in equilibrium by the tongue. The three fundamental letters אמ״ש signify, as מ is mute like the water and ש hissing like the fire, there is א among them, a breath of air which reconciles them (*Sefer Yetzirah*).

For example, man (in Hebrew *'ish*, i.e. a combination of *'alef*, *shin*) and woman (in Hebrew *'ishah*, i.e. a combination of *'alef*, *shin*) are considered obtained from fire (*shin*) and air (*'alef*). Each string of the length three contains different combinations of 22 letters. Different meanings of strings are implied by different combinations, because each character possesses a special symbolic value. In the following table, the basic symbolic values of Hebrew letters are considered:

letter	symbolic value	letter	symbolic value	letter	symbolic value
א	Air, Supremacy, Existence.	ט	Leo, Taste, Combination of *shin* and *kaf*. שך means 'thorn'.	פ	Mars, Power, Combination of *beth* and *daleth*. בד means 'cloth'.
ב	Mercury, Wisdom, Combination of *'alef* and *shin*. אש means 'fire'.	י	Virgo, Eros, Combination of *shin* and *peh*. שף means 'rub'.	צ	Aquarius, Imagine, Combination of *mem* and *resh*, Meditation. מר means 'bitter' or 'master'.
ג	Luna, Grace, Combination of *'alef* and *mem*. אמא means 'mother'.	כ	Jupiter, Life, Combination of *beth* and *gimel*. בג means 'food'.	ק	Pisces, Sleep, Combination of *mem* and *taw*. מת means 'dead'.
ד	Venus, Fertility, Combination of *shin* and *mem*. שם means 'name'.	ל	Libra, Work, Combination of *shin* and *resh*. שר means 'minister'.	ר	Sol, Peace, Combination of *gimel* and *daleth*. גד means 'fortune'.
ה	Aries, Sight, Combination of *'alef* and *kaf*. אך means 'indeed', 'but'.	מ	Water, Eternal matter without form.	ש	Fire, Energy, Power.
ו	Taurus, Hearing, Combination of *'alef* and *peh*. אף means 'although'.	נ	Scorpio, Movement, Combination of *shin* and *taw*. שת means 'be placed'.	ת	Saturn, Riches, Combination of *'alef*, *shin*, *mem*. אמש means 'past', 'last night'.
ז	Gemini. Smell, Combination of *'alef* and *resh*. אר means 'illuminate'.	ס	Sagitt, Anger, Combination of *mem* and *kaf*. מך means 'humble'.		
ח	Cancer, Speech, Combination of *'alef* and *taw*. את means an objectivization.	ע	Capricorn, Mirth, Combination of *mem* and *peh*. מף means 'index'.		

In the *Sefer Yetzirah* all the possible pairs of Hebrew letters are called the two hundred and thirty-one 'gates of knowledge' (*sha'arim*). They are calculated as follows: $(22 \cdot 21)/2! = 231$, i.e., we take one of the 22 Hebrew letters and match it with each of the 21 remaining letters which results in the number 462 of all the possible permutations of two-letter strings including the same two letters (2!). Hence, we obtain 231 unique 'gates of knowledge', i.e., 231 unique two-letter strings. These strings occur in a square array, in which each column and each line corresponds to one of the eleven supernal *sefirot* with an inner dimension and an outer dimension. In this way we can generate a complete array of $22 \cdot 11 = 242$ possible pairs:

(יא) אל בת גש דר הק וצ זפ חע טס ינ כמ
אב גת דש הר וק זצ חפ טע יס כנ למ
אג דת הש ור זק חצ טפ יע כס לנ במ
אד בג הת וש זר חק טצ יפ כע לס מנ
אה בד ות זש חר טק יצ כפ לע מס גנ

או בה גד זת חש טר יק כצ לפ מע נס
אז בו גה חת טש יר כק לצ מפ נע דס
אח בז גו דה טת יש כר לק מצ נפ סע
אט בח גז דו ית כש לר מק נצ ספ הע
אי בט גח דז הו כת לש מר נק סצ עפ
אכ בי גט דח הז לת מש נר סק עצ ופ
אל בכ גי דט הח וז מת נש סר עק פצ
אמ בל גכ די הט וח נת סש ער פק זצ
אנ במ גל דכ הי וט זח סת עש פר צק
אס בנ גמ דל הכ וי זט עת פש צר חק
אע בס גנ דמ הל וכ זי חט פת צש קר
אפ בע גס דנ המ ול זכ חי צת קש טר
אצ בפ גע דס הנ ומ זל חכ טי קת רש
אק בצ גפ דע הס ונ זמ חל טכ רת יש
אר בק גצ דפ הע וס זנ חמ טל יכ שת
אש בר גק דצ הפ וע זס חנ טמ יל כת
את בש גר דק הצ ופ זע חס טנ ימ כל:
(*Sefer Yetzirah*).

All the possible triples of Hebrew letters are called the 1,540 roots (*shorashim*). We can calculate the number of all three-letter strings in the following manner: $(22 \cdot 21 \cdot 20)/3! = 1,540$.

The idea of Kabbalistic automata in which operations over characters of the Hebrew alphabet should simulate physical and biological processes producing appropriate strings, suggested to Leibniz an idea to work at a *characteristica universalis* or '*universal characteristic*', built on an alphabet of human thought in which each fundamental concept would be represented by a unique character. In universal characteristic complex thoughts would be represented by combining characters for atomic thoughts. Characters of the Hebrew alphabet, which in Kabbalists' intention are atoms of any information, were the prototypes of characters in Leibniz' meaning. For Leibniz such atoms should be presented by a system of the coding of all elementary concepts by using prime numbers due to the uniqueness of prime factorization (this idea was also used in Gödel numbering).

Universal characteristic should have become an automaton simulating all physical processes, an automaton which knows the "Answer to the Ultimate Question of Life, the Universe, and Everything". This automaton should have become a universal checker which can check and verify any thought mechanically. Due to such possibilities, universal characteristic may be regarded as sacral language, i.e. a unique language in which it is possible to think unmistakably, language that objectively reflects any physical and biological process, language which already coincides with reality. It is obvious that sacred language in Kabbalistic meaning was a prototype of universal language in the Leibnizian meaning.

Leibniz' intention to construct universal characteristic was extremely grandiose. In his days people trusted in unlimited powers of reason. It seemed to them, that a bit later and the whole world would be completely investigated. In this paper, I realize some of the ideas developed by Leibniz; in particular I show that we can completely define a universal characteristic for Aristotelian syllogistics and implement all syllogistic reasoning within *Physarum* L-systems assuming Leibnizean-style characters (numbers) for all concepts. Then I also implement Talmudic reasoning by the inference rule *qal wa-homer* [15] within *Physarum* L-systems. The main difference between Aristotelian syllogistics and Talmudic reasoning is that we are concentrating on fusions of plasmodium in the case of the implementation of Aristotelian syllogistics in *Physarum* L-systems and on multiplications of plasmodium in the case of the implementation of Talmudic reasoning in *Physarum* L-systems.

Thus, Kabbalists considered strings in the Hebrew alphabet, and all the strings have a length not exceeding 3 characters. Leibniz proposed to consider strings built up by real characters and these strings do not exceed the length 2: they are constructed from a subject symbol and a predicate symbol. So, these strings are syllogistic. In this paper, I am going to develop some Kabbalistic-Leibnizian presuppositions within *Physarum* L-systems to build up syllogistic systems simulating the creation of plasmodia.

III. Physarum L-system

The behavior of *Physarum polycephalum* plasmodia can be stimulated by attractants and repellents. We have the following entities which can be used in programming plasmodia:

- The set of *active zones* of *Physarum* $\{A_1, A_2, \ldots\}$, from which any behavior begins to carry out.
- The set of *attractants* $\{N_1, N_2, \ldots\}$; they are sources of the nutrients on which the plasmodium feeds, or pheromones which chemically attract the plasmodium. Any attractant is characterized by its position and intensity.
- The set of *repellents* $\{R_1, R_2, \ldots\}$. Plasmodium of *Physarum* avoids light and some thermo- and salt-based conditions. Thus, domains of high illumination (or a high level of salt) are repellents such that each repellent is characterized by its position and intensity, or force of repelling.

Plasmodia grow from active zones. At these active zones, according to Adamatzky's experiments [2], [3], the following three basic operations stimulated by nutrients (attractants) and some other conditions can be observed: fusion, multiplication, and direction operations (see Fig. 1):

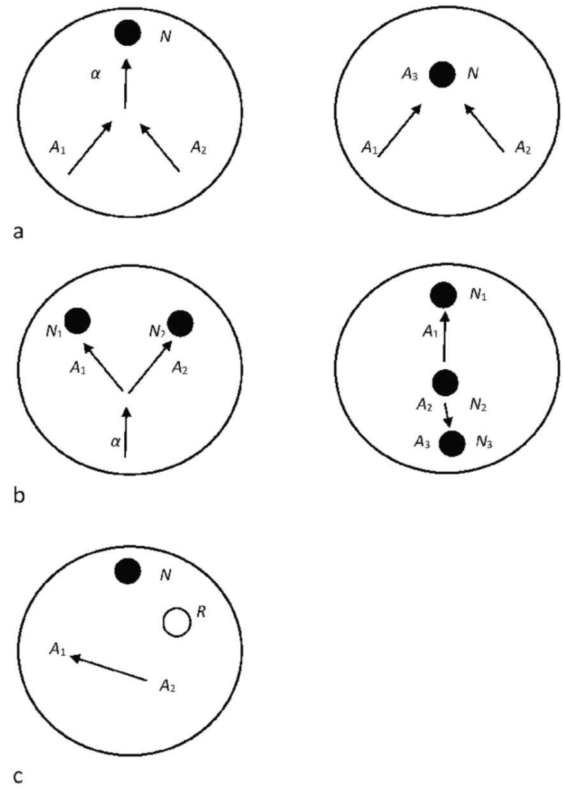

Figure 1. The stimulation of the following operations in *Physarum* automata: (a) fusion, (b) multiplication, and (c) direction, where A_1, A_2, A_3 are active zones, N, N_1, N_2, N_3 are attractants, α is a protoplasmic tube, R is a repellent.

(1) The *fusion*, denoted *Fuse*, means that two active zones A_1 and A_2 produce new active zone A_3: $Fuse(A_1, A_2) = A_3$.
(2) The *multiplication*, *Mult*, means that the active zone A_1 splits into two independent active zones A_2 and A_3, propagating along their own trajectories: $Mult(A_1) = \{A_2, A_3\}$.
(3) The *direction*, *Direct*, means that the active zone A is translated to a source of nutrients with certain initial velocity vector v: $Direct(A, v)$.

These operations, *Fuse*, *Mult*, *Direct*, can be determined by the attractants $\{N_1, N_2, \ldots\}$ and repellents $\{R_1, R_2, \ldots\}$.

On the basis of active zones $\{A_1, A_2, \ldots\}$, attractants $\{N_1, N_2, \ldots\}$, and repellents $\{R_1, R_2, \ldots\}$, we can define a *Physarum* L-system. Let us remember that an L-system consists of (i) an alphabet of symbols that can be used to make strings, (ii) a collection of production rules that expand each symbol into some larger or shorter string of symbols, and (iii) an initial string from which we move. These systems were introduced by Lindenmayer [4], [7], [14] to describe and simulate the behavior of plant cells.

The *Physarum* L-system is defined as follows: $\mathbf{G} = \langle G, \omega, Q \rangle$, where (i) G (the *alphabet*) is a set of symbols containing elements that can be replaced (*variables*); namely they are active zones $\{A_1, A_2, \ldots\}$ which can be propagated towards attractants $\{N_1, N_2, \ldots\}$ by protoplasmic tubes and avoid repellents $\{R_1, R_2, \ldots\}$, i.e., $G = \{A_1, A_2, \ldots\} \cup \{N_1, N_2, \ldots\} \cup \{R_1, R_2, \ldots\}$; (ii) ω (*start*, *axiom* or *initiator*) is a string of symbols from G defining the initial state of the system, i.e., ω always belongs to $\{A_1, A_2, \ldots\}$; (iii) Q is a set of *production rules* or *productions* defining the way variables can be replaced with combinations of constants and other variables, i.e., production rules show a propagation of active zones by protoplasmic tubes towards attractants with avoiding repellents.

Let A, B, C be called primary strings, their meanings run over symbols $A_1, A_2, \ldots, N_1, N_2, \ldots$ Production rules allow us to build composite strings from primary strings. So, a production $A \to_Q B$ consists of two strings, the *predecessor* A and the *successor* B. Some basic cases of productions are as follows: (i) the *fusion*, denoted $AB \to_Q C$, means that two active zones A and B produce new active zone C at the place of an attractant denoted by C; (ii) the *multiplication*, $A \to_Q BC$, means that the active zone A splits into two independent active zones B and C propagating along their own trajectories towards two different attractants denoted then by B and C; (iii) the *direction*, $A \to_Q B$, means that the active zone A is translated to a source of nutrients B.

L-systems can generate infinite data structure. Therefore it is better to define some production rules, denoted by $A \to B$, recursively like is: $A \to BA$, producing an infinite sequence $BABABABA\ldots$ from A, or $A \to BCA$, producing an infinite sequence $BCABCABCABCA\ldots$ from A. In the *Physarum* L-system, the rule $A \to BA$ means that we will fulfill the *direction*, $A \to_Q B$, infinitely many times; the rule $A \to BCA$ means that we will fulfill the *multiplication*, $A \to_Q BC$, infinitely many times. Let us consider an example of recursive production rules. Let $G = \{A, B\}$ and let us start with the string A. Assume $(A \to BA)$ and $(B \to B)$. Thus, we obtain the following strings:

Generation $n = 0$: A
Generation $n = 1$: BA
Generation $n = 2$: BBA
Generation $n = 3$: $BBBA$
Generation $n = 4$: $BBBBBA$
Generation $n = 5$: $BBBBBBA$

In an appropriate *Physarum* L-system, these generations are represented as an infinite tree by permanent additions of new attractants before plasmodium propagation. In other words, we obtain the binary tree labeled with s and t, and whose interior nodes are either one unary node labeled with B or one binary node labeled with A (Fig. 2).

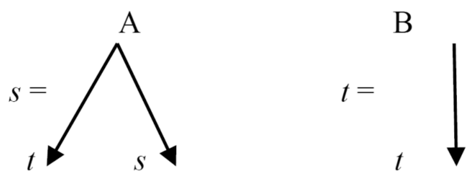

Figure 2. Example of labels for binary trees.

To sum up, we obtain the infinite binary tree of Fig. 3.

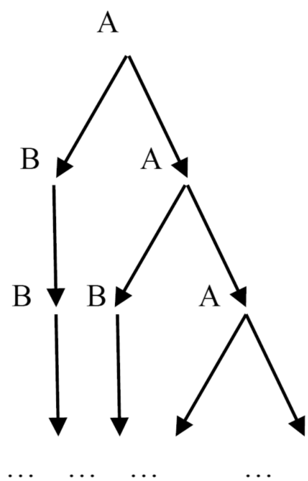

Figure 3. Example of an infinite binary tree.

If we are limited just by the *multiplication*, $A \to_Q BC$, and the *direction*, $A \to_Q B$, we can build up binary trees in *Physarum* L-systems using the following definition of binary trees labeled with $x, y, \ldots$, whose interior nodes are either unary nodes labeled with $u_1, u_2, \ldots$ or binary nodes labeled with $b_1, b_2, \ldots$:

1. the variables $x, y, \ldots$ are trees;
2. if t is a tree, then adding a single node labeled with one of $u_1, u_2, \ldots$ as a new root with t as its only subtree gives a tree;
3. if s and t are trees, then adding a single node labeled with one of $b_1, b_2, \ldots$ as a new root with s as the left subtree and t as the right subtree again gives a tree;
4. trees may go on forever.

Let Tr be the set of trees that we have defined. Then our definition introduces a coalgebra [16]:

$$Tr = \{x, y, \ldots\} \cup (\{u_1, u_2, \ldots\} \times Tr) \cup (\{b_1, b_2, \ldots\} \times Tr \times Tr).$$

Thus, within L-systems, we can obtain complex structures including infinite structures defined coalgebraically. In some cases, it is better to deal with infinite structures (infinite trees), assuming that all strings are finite.

IV. Aristotelian trees

Let us consider Aristotelian syllogistic trees, which can be large, but their strings are only of the length 1 or 2. An *Aristotelian syllogistic tree* is labeled with $x, y, \ldots$, its interior nodes are n-ary nodes labeled with $b_1, b_2, \ldots$, and it is defined as follows: (1) the variables $x, y, \ldots$ are Aristotelian syllogistic trees whose single descendants are underlying things (*hypokeimenon*, ὑποκείμενον) such that for each $x, y, \ldots$, parents are supremums of descendants (notice that all underlying things are mutually disjoint); (2) if $t_1, t_2, \ldots, t_n$ are Aristotelian syllogistic trees such that their tops are concepts which are mutually disjoint and their supremum is $b_x \in \{b_1, b_2, \ldots\}$, then adding a single node labeled with b_x as a new root with $t_1, t_2, \ldots, t_n$ as its only subtrees gives an Aristotelian syllogistic tree; (3) an Aristotelian syllogistic tree is finite.

The idea of *hypokeimenon* allowed Aristotle to build up finite trees. He starts with underlying things as primary descendants of trees in constructing syllogistic databases. Now, let us define syllogistic strings of the length 1 or 2 by means of a *Physarum* L-system. Let each $b_x \in \{b_1, b_2, \ldots\}$ be presented by an appropriate attractant and underlying things by initial active zones of *Physarum*. So, first trees $x, y, \ldots$, whose single descendants are underlying things, are obtained by fusion or direction. Their supremums are denoted by attractants which were occupied by the first plasmodium propagation.

These trees are considered subtrees for the next plasmodia propagation by fusion or direction. At the end, we can obtain just one supremum combining all subtrees. Let $a_1, a_2, a_3, \ldots$ be underlying things. Then they are initial strings, i.e., they can be identified with active zones of plasmodia. Their meanings are as follows: "there exists a_1", "there exists a_2", "there exists a_3", ... Assume that in the tree structure the supremum of a_1 and a_2 is b_1, the supremum of a_2 and a_3 is b_2, ... These supremums are fusions of plasmodia. Then, we have the strings $a_1b_1, a_2b_1, a_2b_2, a_3b_2, \ldots$ Their meanings are as follows: "a_1 is b_1", "a_2 is b_1", "a_2 is b_2", "a_3 is b_2", ... Further, let b_n be a supremum for b_1 and b_2. It denotes an attractant that was occupied by the plasmodium at the third step of the propagation. Our new strings are as follows: "b_1 is b_n", "b_2 is b_n", etc. Now we can appeal also to the following new production rule: if "x is y" and "y is z", then "x is z". Thus, we have the strings: $a_1b_n, a_2b_n, a_2b_n, a_3b_n, \ldots$

V. Aristotelian syllogistic

The symbolic system of Aristotelian syllogistic can be implemented in the behavior of *Physarum polycehalum* plasmodium. Let us design cells of *Physarum* syllogistic which will designate classes of terms. We can suppose that cells can possess different topological properties. This depends on the intensity of chemo-attractants and chemo-repellents. The intensity entails the natural or geographical neighborhood of the set's elements in accordance with the spreading of attractants or repellents. As a result, we obtain Voronoi cells [3], [12]. Let us define what they are mathematically. Let $\mathbf{P}$ be a nonempty finite set of planar points and $|\mathbf{P}| = n$. For points $p = (p_1, p_2)$ and $x = (x_1, x_2)$, let

$$d(p, x) = \sqrt{(p_1 - x_1)^2 + (p_2 - x_2)^2}$$

denote their Euclidean distance. A planar Voronoi diagram of the set $\mathbf{P}$ is a partition of the plane into cells, such that for any element of $\mathbf{P}$, a cell corresponding to a unique point p contains all those points of the plane which are closer to p in respect to the distance d than to any other node of $\mathbf{P}$. A unique region

$$vor(p) = \bigcap_{m \in \mathbf{P}, m \neq p} \{z \in \mathbf{R}^2 : d(p, z) < d(m, z)\}$$

assigned to the point p is called a *Voronoi cell* of the point p. Within one Voronoi cell, a reagent has a full power to attract or repel the plasmodium.

The distance d is defined by the intensity of reagent spreading like in other chemical reactions simulated by Voronoi diagrams. A reagent attracts or repels the plasmodium and the distance in that it is possible corresponds to the elements of a given planar set $\mathbf{P}$. When two spreading wave fronts of two reagents meet, this means that on the board of meeting the plasmodium cannot choose its one further direction and splits (see Fig. 4). Within the same Voronoi cell, two active zones will fuse.

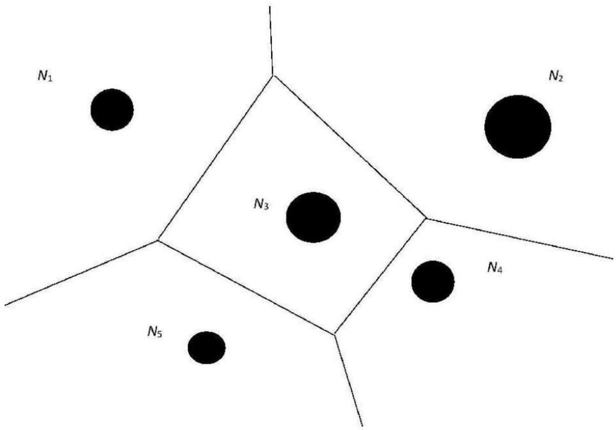

Figure 4. The Voronoi diagram for Physarum, where different attractants have different intensity and power.

Now, we can obtain coordinates $(x, y) \in \mathbf{Z}^2$ for each Voronoi center. The number (x, y) can be assigned to each concept as its character. If a Voronoi center with the coordinates (x_a, y_a) is presented by an attractant that is activated and occupied by the plasmodium, this means that in an appropriate *Physarum* syllogistic model there exists a string a with the coordinates (x_a, y_a). This string has the meaning "a exists". If a Voronoi center with the coordinates (x_a, y_a) is presented by a repellent that is activated and avoided by the plasmodium, this means that in an appropriate *Physarum* syllogistic model there exists a string $[a]$ with the coordinates (x_a, y_a). This string has the meaning "a does not exist". If two neighbor Voronoi cells with the coordinates (x_a, y_a) and (x_b, y_b) of centers contain activated attractants which are occupied by the plasmodium and between both centers there are protoplasmic tubes, then in an appropriate *Physarum* syllogistic model there exists a string ab and a string ba where a has the coordinates (x_a, y_a) and b has the coordinates (x_b, y_b). The meaning of those strings is the same and it is as follows: "ab exists", "ba exists", "some a is b", "some b is a".

If one neighbor Voronoi cell with the coordinates (x_a, y_a) of its center contains an activated attractant which is occupied by the plasmodium and another neighbor Voronoi cell with the coordinates (x_b, y_b) of its centre contains an activated repellent which is avoided by the plasmodium, then in an appropriate *Physarum* L-system there exists a string $a[b]$ and a string $[b]a$ where a has the character (x_a, y_a) and $[b]$ has the character (x_b, y_b). The meaning of those strings is the same and is as follows: "ab do not exist, but a exists without b", "there exists a and no a is b", "no b is a and there exists a", "a exists and b does not exist".

If two neighbor Voronoi cells with the coordinates (x_a, y_a) and (x_b, y_b) of their centers contain activated repellents which are avoided by the plasmodium, then in an appropriate *Physarum* L-system there exists a string $[ab]$ and a string $[ba]$ where $[a]$ has the character (x_a, y_a) and $[b]$ has the character (x_b, y_b). The meaning of those strings is the same and is as follows: "ab do not exist together", "there are no a and there are no b", "no b is a", "no a is b". Hence, existence propositions of Aristotelian syllogistic are spatially implemented in *Physarum* L-systems.

Let y' denote all neighbor Voronoi cells for x which differ from y. Now, let us consider a complex string $xy\&x[y']$. The sign & means that we have strings xy and $x[y']$ simultaneously and they are considered one complex string. The meaning of the string $xy\&x[y']$ is a universal affirmative proposition "all x are y".

As a consequence, each *Physarum* L-system is considered a discourse universe verifying some propositions of Aristotelian syllogistic.

VI. Talmudic reasoning by qal wa-homer

One of the most important rules in Talmudic reasoning is called *qal wa-homer*. It is a parallel concurrent deduction which partly corresponds to the scholastic proof a fortiori ('a minori ad majus' or 'a majori ad minus'); according to the latter what applies in less important cases will apply in more important ones too, i.e. this rule allows one to entail from the simple to the complex or vice versa. However, there are important distinctions from the scholastic proof a fortiori. The process of deduction in the *qal wa-homer* proceeds under the assumption that the inferred statement conclusion may contain nothing more than is found in the premise. This limitation is called the *dayo* principle. A syllogism implicitly drawn from a minor case upon a more important one: "If X is true of Y and Z is of greater weight than Y, then how much more X must be true of Z (but not more than of Y)." Example: using the following passage as premise "If thou meet thy enemy's ox or his ass going astray, thou shalt surely bring it back to him again" (Ex. 23:4), we can conclude that if that be one's conduct toward an enemy, how much more should one be considerate toward a friend. In *qal wa-homer* two or more parallel deductions concur under the following conditions: (i) they have joint premises (ii) one deduction of the set of concurrent deductions is much more certain. As a result, a certainty of that deduction is expanded to cases of other concurrent deductions. Notice that *qal wa-homer* does not hold in Judaic criminal procedure, i.e. by using this rule nobody can be sentenced to an execution.

Now let us consider how syllogistic strings in a *Physarum* L-system can verify the *qal wa-homer* reasoning as well. For the first time, Yisrael Ury [15] has proposed using a spatial interpretation to model conclusions by *qal wa-homer*. Let us concentrate on strings of the form xy', where y' denotes all neighbor Voronoi cells for x which differ from y. So, for each pair x and y we have the following four possible strings: $xy,\ xy',\ x'y,\ x'y'$.

Let us take the diagram

xy'	xy
$x'y'$	$x'y$

that plays the role of the 'universe of discourse' for Talmudic reasoning over cells $x,\ y,\ x',\ y'$, because we can deal in this case just with strings $xy, xy', x'y, x'y'$. Let us suppose that we have black counters and if a black counter is placed within a cell XY, this means that "this cell XY is occupied by the *Physarum polycephalum* plasmodium" (i.e. the syllogistic meaning: "there is a non-empty syllogistic string XY"; the Talmudic legal meaning: "an appropriate Talmudic rule XY should be obeyed"). So, the cell XY that does not contain a black counter indicates a situation in which the obligation XY is not fulfilled, whereas the cell XY containing a black counter indicates a situation in which the obligation XY is fulfilled.

Thus, if we have two rows and two columns (i.e. only four strings), there are sixteen possible ways to cover such a diagram by means of black counters, but Yisrael Ury notes that only six of them satisfy *qal wa-homer* reasoning (Fig. 5).

Let x mean legal proposition 1 and y mean legal proposition 2. Then they in the case of accepting diagrams of Fig. 5 have the following sense: (a) It is necessary and sufficient to obey x; (b) It is necessary and sufficient to obey y; (c) It is sufficient to obey either x or y; (d) It is not sufficient to obey x and/or y; (e) It is necessary and sufficient to obey both x and y; (f) It is not necessary to obey either x or y.

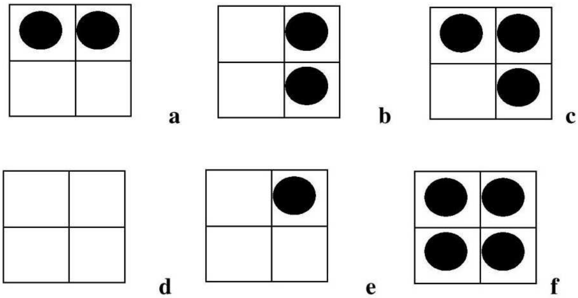

Figure 5. Ury's diagrams for conclusions by *qal wa-homer*.

Let us consider a Talmudic example of *qal wa-homer*:

אמרת קל וחומר ומה שומר שכר שפטור משבורה ומתה חייב בגניבה ואבידה
שואל שחייב בשבורה ומתה אינו דין שחייב בגניבה ואבידה וזה הוא קל וחומר
שאין עליו תשובה מאי אין עליו תשובה וכ"ת איכא למיפרך מה לשומר שכר
שכן משלם תשלומי כפל בטוען טענת לסטים מזויין אפ"ה קרנא דשואל עדיפא
איבעית אימא קסבר לסטים מזויין גזלן

You can reason a minori: if a paid bailee, who is not responsible for injury and death, is nevertheless liable for theft and loss, then a borrower, who is liable for the former, is surely liable for the latter too! And this is an a minori argument which cannot be refuted. Why state that it 'cannot be refuted?'—For should you object, It may be refuted thus: as for a paid bailee, [he is responsible for theft and loss] because he must make restitution of twice the principal [if discovered] in a [false] plea of [loss through] an armed robber, [I would reply,] notwithstanding the fact that the borrower is responsible for the principal is a greater severity. Alternatively, he maintains that an armed robber is a gazlan (*Baba Mezia* 95a).

The point is that in *Exodus* 22, there is a statement that if a man borrows an animal or a thing, and the animal dies or the object is destroyed, the borrower is responsible for the loss. But there is no information whether the borrower is also responsible when the borrowed animal or thing is stolen. Nevertheless, we can apply inference rule *qal wa-homer* by using the depositary as minor and the borrower as major. Then the conclusion is that the borrower, who is responsible for damage and death, is liable also to restore the thing stolen from him. Let x be 'borrower', x' 'depositary', y' 'destroyed', y 'stolen'. Then we obtain the following diagram:

xy'	xy
$x'y'$	$x'y$

Assume that a black counter means an obligation to pay 100% the cost of damage as compensation and an absense of the black counter means an obligation to pay 0% the cost of damage as compensation. Then we cover this diagram by counters in Fig. 6.

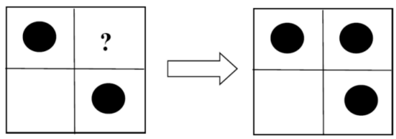

Figure 6. Ury's diagrams for the conclusions whether the borrower is responsible for the stolen.

Our rule for inferring by *qal wa-homer* is formulated thus: *if a cell contains a black counter, all cells above it and to its right also contain a black counter; if a cell does not contain any counter, all cells below it and to its left are also without counters.*

In the *Physarum* topology, Talmudic diagrams are built on the basis of syllogistic strings of the form $xy, yx, x'y, yx', xy', y'x, x'y', y'x'$, where x and y in xy are interpreted as two neighbor attractants connected by protoplasmic tubes, x' is understood as all attractants which differ from x, but they are neighbors for y, and y' is understood as all attractants which differ from y and are neighbors for x. We can simplify the Talmudic diagrams for the *Physarum* simulation as follows:

x	y
y'	x'

where x' is a non-empty class of neighbor attractants for y and y' is a non-empty class of neighbor attractants for x. Then *qal wa-homer* tells us if we had a multiplication in the plasmodium's propagation at points x and/or y. In Fig. 7, all the possible conclusions inferred by *qal wa-homer* in relation to x and y are considered and it is defined if we have a multiplication at those points.

Hence, the main difference between the Aristotelian syllogistic and Talmudic reasoning is that, on the one hand, we are concentrating on fusions of plasmodium in the case of implementation of the Aristotelian syllogistic in the *Physarum* topology and, on the other hand, we deal with multiplications of plasmodium in the case of implementation of Talmudic reasoning. Talmudic reasoning can describe only fragments of plasmodium behaviors

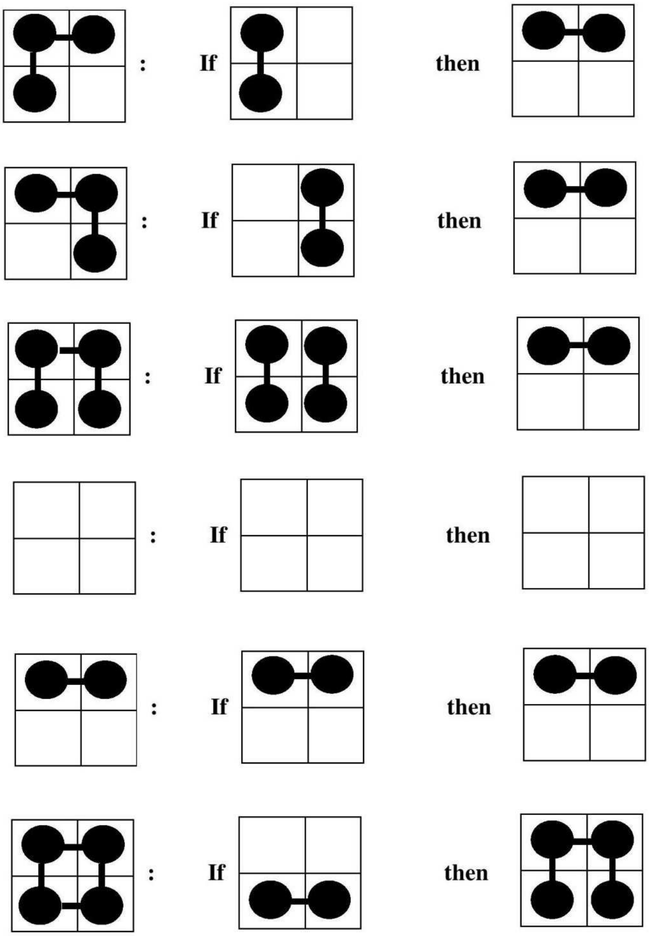

Figure 7. *The Physarum diagrams for qal wa-homer syllogisms*: (a) If the string xy' is verified, then the string xy is verified, too (i.e. if xy' is verified, then x has a multiplication of plasmodium). (b) If the string yx' is verified, then the string xy is verified, too (i.e. if yx' is verified, then y has a multiplication of plasmodium). (c) If the strings xy' and yx' are verified, then the string xy is verified, too (i.e. if both xy' and yx' are verified, then both x and y have multiplications of plasmodium). (d) If no string is verified, then there is no multiplication of plasmodium. (e) If the string xy is verified, then there is no multiplication of plasmodium. (f) If the strings $x'y'$ is verified, then the strings xy', yx' and xy are verified, too (i.e. if $x'y'$ is verified, then both x and y have multiplications of plasmodium).

as well as the Aristotelian syllogistic describes some fragments of plasmodium propagations. Only the pragmatic syllogistic is sound and complete on plasmodium interactions. For more details on Talmudic reasoning and its modern formalizations see [15].

VII. Conclusion

We have shown the Judaic roots of unconventional computing ideas and constructed two syllogistic versions of a storage modification machine in *Physarum polycephalum*: Aristotelian syllogistic and Talmudic reasoning by *qal wa-homer*. While Aristotelian syllogistic may describe the concrete directions of *Physarum* spatial expansions, *qal wa-homer* describes propagations in more than one direction. Therefore, while for the implementation of Aristotelian syllogistic we need repellents to avoid some possibilities in the *Physarum* propagations, for the implementation of *qal wa-homer* we do not need them. Hence, the second syllogistic can simulate massive-parallel behaviors, including different forms of propagations such as processes of public opinion formation.

In our opinion, the general purpose of *Physarum* computing covers many behavioural sciences, because the slime mould's behaviour can be considered the simplest natural intelligent behaviour. Thus, our results may have an impact on computational models in behavioural sciences in general.

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REFERENCES

[1] A. Adamatzky, V. Erokhin, M. Grube, Th. Schubert, and A. Schumann, "Physarum Chip Project: Growing Computers From Slime Mould," International Journal of Unconventional Computing, 8(4), 2012, pp. 319–323.

[2] A. Adamatzky, "Physarum machine: implementation of a Kolmogorov-Uspensky machine on a biological substrate," Parallel Processing Letters, vol. 17, no. 04, 2007, pp. 455–467.

[3] A. Adamatzky, Physarum Machines: Computers from Slime Mould (World Scientific Series on Nonlinear Science, Series A). World Scientific Publishing Company, 2010.

[4] A. Lindenmayer, "Mathematical models for cellular interaction in development. parts I and II," Journal of Theoretical Biology, 18, 1968, pp. 280–299; 300–315.

[5] J. Łukasiewicz, Aristotle's Syllogistic From the Standpoint of Modern Formal Logic. Oxford Clarendon Press, 2nd edition, 1957.

[6] Y. Matveyev, "Symbolic computation and digital philosophy in early Ashkenazic Kabbalah," [in:] Schumann, A. (ed.), *Judaic Logic*. Gorgias Press, 2010, 245–256.

[7] K. Niklas, Computer Simulated Plant Evolution. Scientific American, 1985.

[8] A. Schumann and A. Adamatzky, "Logical Modelling of Physarum Polycephalum," Analele Universitatii Din Timisoara, seria Matemtica-Informatica 48 (3), 2010, pp. 175–190.

[9] A. Schumann and A. Adamatzky, "Physarum Spatial Logic," New Mathematics and Natural Computation 7 (3), 2011, pp. 483–498.

[10] A. Schumann and L. Akimova, "Simulating of Schistosomatidae (Trematoda: Digenea) Behaviour by Physarum Spatial Logic," Annals of Computer Science and Information Systems, Volume 1. Proceedings of the 2013 Federated Conference on Computer Science and Information Systems. IEEE Xplore, 2013, pp. 225–230.

[11] A. Schumann, "On Two Squares of Opposition: the Leśniewski's Style Formalization of Synthetic Propositions," Acta Analytica 28, 2013, pp. 71–93.

[12] A. Schumann and K. Pancerz, "Towards an Object–Oriented Programming Language for Physarum Polycephalum Computing," in M. Szczuka, L. Czaja, M. Kacprzak (eds.), Proceedings of the Workshop on Concurrency, Specification and Programming (CS&P'2013), Warsaw, Poland, September 25–27, 2013, pp. 389–397.

[13] A. Schumann, "Two Squares of Opposition: for Analytic and Synthetic Propositions," Bulletin of the Section of Logic 40 (3/4), 2011, pp. 165–178.

[14] P. Prusinkiewicz and A. Lindenmayer, The Algorithmic Beauty of Plants. Springer-Verlag, 1990.

[15] Yisrael Ury, Charting the Sea of Talmud: A Visual Method for Understanding the Talmud. 2012.

[16] J. J. M. M. Rutten, "Universal coalgebra: a theory of systems," Theor. Comput. Sci., 249 (1), 2000, pp. 3–80.