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THEOLOGICAL UNDERPINNINGS OF THE MODERN PHILOSOPHY OF MATHEMATICS. PART II: THE QUEST FOR AUTONOMOUS FOUNDATIONS

Abstract. The study is focused on the relation between theology and mathematics in the situation of increasing secularization. My main concern in the second part of this paper is the early-twentieth-century foundational crisis of mathematics. The hypothesis that pure mathematics partially fulfilled the functions of theology at that time is tested on the views of the leading figures of the three main foundationalist programs: Russell, Hilbert and Brouwer.

> I believe that in my generation, the belief in a platonic mathematics has often been a substitute religion for people who have abandoned or even rejected traditional religions. Where can certainty be found in a chaotic universe that often seems meaningless? Mathematics has often been claimed to be the sole source of absolute certainty.
>
> Philip J. Davis (2004, p. 35).

1. Introduction

When mathematicians turned out to be in an unprecedented situation, (firstly) far enough from everyday empirical evidence due to the never-before-seen rise of abstraction in mathematics, (secondly) even further from divine support due to secular tendencies, and (thirdly) willing to preserve the prestigious popular philosophy of mathematics, they found themselves *anxious* and in a great need for *secure foundations* (Gray, 2004; 2008)[1]. Moreover, the needed foundations were to be *absolute*, in conformity with the popular philosophy of mathematics, and *autonomous*, in accordance with the separation of pure reason from both natural and supernatural worlds (see Part I of this paper).

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The most intensive period of foundational work in mathematics (the 1870s–1920s) witnessed the emergence of diverse programs for the foundations of mathematics. The three main foundationalist programs were: logicism, intuitionism, and formalism (Kleene, 1952, p. 43; Benacerraf & Putnam, 1983, pp. 1, 41). All three programs failed to provide pure mathematics with absolute autonomous foundations. Nevertheless, it would be unfair to call them fruitless. Those programs gave birth, on the one hand, to the contemporary philosophy of mathematics and, on the other, to a new research field within mathematics: *mathematical logic*. Imre Lakatos put it in a well-turned phrase: "'Certainty' is never achieved; 'foundations' are never found – but the 'cunning of reason'[2] turns each increase in rigour into an increase in content, in the scope of mathematics." (Lakatos, 1976, p. 56) Mathematical logic finally became just another branch of the branchy tree of mathematics (Mendelson, 1964/1979, p. 4; Rasiowa & Sikorski, 1963, pp. 6–9).

This paper proposes the hypothesis that mathematics, seen from the foundationalist perspective, served at the time as *a substitute for metaphysics* (and for theology as its part and parcel). According to this approach, the popular philosophy of mathematics substantially mediated the interplay between theology and mathematics in the very long 19$^{\text{th}}$ century (from the 1780s through the 1920s). In this context, I should pay special attention to the theological and quasi-theological ideas of the key figures of the three main foundationalist programs. I shall take Russell, Brouwer and Hilbert.

2. Quasi-theological perspective on mathematics in Russell

Sir Bertrand Russell is well-known for being an agnostic in theory and an atheist in practice. He rejected religion and in his quest for absolute certainty pinned most of his faith on mathematics. He eventually was deeply dissatisfied with mathematics, as mathematics failed to quench his thirst for something absolute.

His great expectations for mathematics are quite obvious and explicit in his essay *The Study of Mathematics*, written in 1902 and first published in 1907 (Russell, 1917, pp. 68–70). There, we have a brilliant example of the popular philosophy of mathematics at work. And we have even more than that: Russell speaks outright of mathematics in a quasi-religious language and with quasi-religious emotion. It is mathematics that maintains "the true dignity of reason" being "an investigation into the very heart and immutable essence of all things actual and possible". Mathematics takes us "into the region of absolute necessity [...] where our ideals are fully satisfied and our best hopes are not thwarted". Its shining beauty brings us salvation from the cruelty of the natural world, in which we drag out our miserable existence in "exile amid hostile forces". This world of pure mathematics is objective ("independent of us and our thoughts") but has autonomy ("a reciprocal liberty") from the natural world having a "purely ideal character"; in fact, it should be characterized as *quasi-supernatural*.

This interpretation can be corroborated by Russell's later confessions. Some of them can be found in his book *My Philosophical Development* (1959):

> My original interest in philosophy had two sources. On the one hand, I was anxious to discover whether philosophy would provide any defence for anything that could be called religious belief, however vague; on the other hand, I wished to persuade myself that something could be known, in pure mathematics if not elsewhere. (1959, p. 11)

This claim (which one can find on the very first page of the first chapter of Russell's book) from the very start connects *religion* (*theology*) and *mathematics*. When he gets to a more detailed account of the origins of his philosophy in the late 1880s (three years before getting to Cambridge) in chapter 3, he restates that connection once again:

> Most of my time was taken up by mathematics, and mathematics largely dominated my attempts at philosophical thinking, but the emotional drive which caused my thinking was mainly doubt as to the fundamental dogmas of religion. (1959, p. 28)

> I regretted my loss of religious belief. [...] It was not only as to theology that I had doubts, but also as to mathematics. [...] I hoped sooner or later to arrive at a perfected mathematics which should leave no room for doubts, and bit by bit to extend the sphere of certainty from mathematics to other sciences. Gradually during these three years my interest in theology grew less, and it was with a genuine sense of relief that I discarded the last vestiges of theological orthodoxy. (1959, pp. 35–36)

At the time, mathematics looked, from Russell's point of view, far more promising than Christian theology; and so the substitution took place. This is a core theme for his philosophical development. It is no wonder he devoted the whole chapter 17 of the very same book of his to what he saw in his later period as the illusion of Platonic heaven. The chapter is titled "The Retreat from Pythagoras". In the very first lines of it, he explicates why he needs Pythagoras to explain himself:

> My philosophical development, since the early years of the present century, may be broadly described as a gradual retreat from Pythagoras. The Pythagoreans had a peculiar form of mysticism which was bound up with mathematics. This form of mysticism greatly affected Plato and had, I think, more influence upon him than is generally acknowledged. I had, for a time, a very similar outlook and found in the nature of mathematical logic, as I then supposed its nature to be, something profoundly satisfying in some important emotional respects. (1959, p. 208)

"This kind of mathematical mysticism, which Plato derived from Pythagoras, appealed to me", he wrote elsewhere (1956, p. 18). Thus, Russell attributes to himself a sort of *mathematical mysticism* owing to mathematical logic. He also notes here an emotional satisfaction with mathematical logic that should be considered in comparison with the "emotional drive" linked to "the fundamental dogmas of religion" that he already mentioned in chapter 3. Russell's emotional drive was obviously *shifted* from theology to mathematics. Let us read on:

> My interest in the applications of mathematics was gradually replaced by an interest in the principles upon which mathematics is based. This change came about through a wish to refute mathematical scepticism. A great deal of the argumentation that I had been told to accept was obviously fallacious, and I read whatever books I could find that seemed to offer a firmer foundation for mathematical beliefs. This kind of research led me gradually further and further from applied mathematics into more and more abstract regions, and finally into mathematical logic. I came to think of mathematics, not primarily as a tool for understanding and manipulating the sensible world, but as an abstract edifice subsisting in a Platonic heaven and only reaching the world of sense in an impure and degraded form. My general outlook, in the early years of this century, was profoundly ascetic. I disliked the real world and sought refuge in a timeless world, without change or decay or the will-o'-the-wisp of progress. Although this outlook was very serious and sincere, I sometimes expressed it in a frivolous manner. My brother-in-law, Logan Pearsall Smith, had a set of questions that he used to ask people. One of them was, 'What do you particularly like?' I replied, 'Mathematics and the sea, and theology and heraldry, the two former because they are inhuman, the two latter because they are absurd'. (1959, pp. 209–210)

Initially, Russell tried to refute *religious* skepticism but failed. Now, his challenge was *mathematical* skepticism. He calls his general outlook of the period 'ascetic', apparently using a word from religious vocabulary. According to the context, it means here unearthly oriented, 'opposing Platonic heaven to the real world'. In the final joke, I would like you to take notice of the joint appearance of mathematics and theology.

Speaking of his reception of the set-theoretic paradoxes, Russell once again compares his own reaction to a religious one:

> I felt about the contradictions much as an earnest Catholic must feel about wicked Popes. And the splendid certainty which I had always hoped to find in mathematics was lost in a bewildering maze. (1959, p. 212)

In "Reflections on My Eightieth Birthday" (1952), Russell expressed this quasi-religious way of perceiving mathematics with the utmost clarity:

> I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere. But I discovered that many mathematical demonstrations, which my teachers expected me to accept, were full of fallacies, and that, if certainty were indeed discoverable in mathematics, it would be in a new kind of mathematics, with more solid foundations than those that had hitherto been thought secure. But as the work proceeded, I was continually reminded of the fable about the elephant and the tortoise. Having constructed an elephant upon which the mathematical world could rest, I found the elephant tottering, and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant, and after some twenty years of very arduous toil, I came to the conclusion that there was nothing more that *I* could do in the way of making mathematical knowledge indubitable. (1956, pp. 54–55)

> I set out with a more or less religious belief in a Platonic eternal world, in which mathematics shone with a beauty like that of the last Cantos of the [Dante's] *Paradiso*. I came to the conclusion that the eternal world is trivial, and that mathematics is only the art of saying the same thing in different words. [...] I may have conceived theoretical truth wrongly, but I was not wrong in thinking that there is such a thing, and that it deserves our allegiance. (1956, p. 58)

Here, Russell *explicitly* compares his attitude towards mathematics with religious faith. Should we take his claims seriously? If so, then the project of *Principia Mathematica* should be viewed as analogous to that of *Summa Theologiae* (only for the new religion of mathematics). Such a perspective on *Principia Mathematica* was taken, for example, by Hermann Weyl. He assessed 'the Russell universe' in such words:

> [I]n the resulting system mathematics is no longer founded on logic, but on a sort of logician's paradise, a universe endowed with an 'ultimate furniture' of rather complex structure and governed by quite a number of sweeping axioms of closure. The motives are clear, but belief in this transcendental world taxes the strength of our faith hardly less than the doctrines of the early Fathers of the Church or of the scholastic philosophers of the Middle Ages. (Weyl, 1946, p. 6)

From the First World War on, Bertrand Russell became dissatisfied with the results of his logicist project and turned his main attention away from mathematics. His quasi-religious Platonism about mathematics was finally displaced by a theory of analytic propositions (Russell, 1919, pp. 204–205) popularized later on by logical positivism:

> All this, though I still remember the pleasure of believing it, has come to seem to me largely nonsense, partly for technical reasons and partly from a change in my general outlook upon the world. Mathematics has ceased to seem to me non-human in its subject-matter. I have come to believe, though very reluctantly, that it consists of tautologies. I fear that, to a mind of sufficient intellectual power, the whole of mathematics would appear trivial, as trivial as the statement that a four-footed animal is an animal. I think that the timelessness of mathematics has none of the sublimity that it once seemed to me to have, but consists merely in the fact that the pure mathematician is not talking about time. I cannot any longer find any mystical satisfaction in the contemplation of mathematical truth. (Russell, 1959, pp. 211–212)

In the just-cited passage, I would like to stress Russell's words on *mystical satisfaction* pertaining to doing mathematics as well as his reluctance and regret in losing the pleasure.

3. Hilbert's axiomatic mathematics as quasi-theology

It is worth noting that the new axiomatic approach to mathematics, associated primarily with the name of David Hilbert, was also widely associated with theology.

The first great mathematical result by Hilbert was his 1888 finiteness theorem in the theory of algebraic forms whose proof was of the highest generality and of an existential (non-constructive) type. It was a perfect example of a new purely *conceptual* mathematics (begriffliche Mathematik) and a new working style, one which Hermann Minkowski called *the other Dirichlet principle*: "to overcome problems by a maximum of insightful thought and a minimum of blind calculation (mit einem Minimum an blinder Rechnung, einem Maximum an sehenden Gedanken die Probleme zu zwingen)" (Minkowski, 1905, S. 163; McLarty, 2012, p. 120). Hilbert's result is accompanied in the history of mathematics by a typical anecdote. According to it, Paul Gordan's reaction to Hilbert's theorem (and apparently to the new style of doing mathematics in general) was put into much-cited words: "Das ist nicht Mathematik, das ist Theologie" (McLarty, 2012). Eric Temple Bell even made a common noun out of them: "mathematical theologians – in Gordan's sense" (1945, p. 430).

In the same vein was Frege's reaction to the ideology of Hilbert's *Grundlagen der Geometrie* (1899). In his letter to Hilbert (Jan. 6, 1900), he mocked Hilbert's approach to mathematical existence as indistinguishable from a theological one.

> But what would you say about the following?:
> 'Explanation. We imagine objects we call Gods.
> 'Axiom 1. All Gods are omnipotent.
> 'Axiom 2. All Gods are omnipresent.
> 'Axiom 3. There is at least one God.' [...]
> Suppose we knew that the propositions
> (1) *A* is an intelligent being
> (2) *A* is omnipresent
> (3) *A* is omnipotent
> together with all their consequences did not contradict one another; could we infer from this that there was an omnipotent, omnipresent, intelligent being? This is not evident to me. (Frege, 1980, pp. 46–47; cf. Gray, 2008, p. 203)

Some of our contemporaries also take that point of view. For instance, Carlo Cellucci questions the thesis that "the axiomatic method expresses the real nature of mathematics" by comparing mathematics to theology in that respect.

> To the objection, 'Surely theological entities are not mathematical objects', one could answer: How do you know? If mathematics consists in the deduction of conclusions from given axioms, then mathematical objects are given by the axioms. So, if theological entities satisfy the axioms, why should not they be considered mathematical objects? (Cellucci, 2005, p. 137)

David Hilbert seemed to be agnostic and had nothing to do with theology proper or even religion. Constance Reid tells a story on the subject:

> The Hilberts had by this time [around 1902] left the Reformed Protestant Church in which they had been baptized and married. It was told in Göttingen that when [David Hilbert's son] Franz had started to school he could not answer the question, 'What religion are you?' (1970, p. 91)

In the 1927 Hamburg address, Hilbert asserted: "mathematics is presuppositionless science (die Mathematik ist eine voraussetzungslose Wissenschaft)" and "to found it I do not need a good God ([z]u ihrer Begründung brauche ich weder den lieben Gott)" (1928, S. 85; van Heijenoort, 1967, p. 479). However, from *Mathematische Probleme* (1900) to *Naturerkennen und Logik* (1930) he placed his quasi-religious faith in the human spirit and in the power of pure thought with its beloved child – mathematics. He was deeply convinced that every mathematical problem could be solved by pure reason: in both mathematics and any part of natural science (through mathematics) there was "no *ignorabimus*" (Hilbert, 1900, S. 262; 1930, S. 963; Ewald, 1996, pp. 1102, 1165). That is why finding an inner absolute grounding for mathematics turned into Hilbert's lifework. He never gave up this position, and it is symbolic that his words "wir müssen wissen, wir werden wissen" ("we must know, we shall know") from his 1930 Königsberg address were engraved on his tombstone. Here, we meet a ghost of departed theology (to modify George Berkeley's words), for to absolutize human cognition means to identify it tacitly with a divine one.

It is also worth mentioning that Hilbert used religious vocabulary to speak of the realm of pure mathematics. He famously said in his 1925 Münster address that "No one shall be able to expel us from the paradise that Cantor created for us (Aus dem Paradies, das Cantor uns geschaffen, soll uns niemand vertreiben können)" (Hilbert, 1926, S. 170; van Heijenoort, 1967, p. 376). In a lecture version (1924/25), the image of mathematical paradise is even richer. If we accept Brouwer and Weyl's innovations, said Hilbert, "then we expel from the paradise not only the devil but also the angels, and then we lastly change Cantor's paradise into a wasteland (aber damit nicht nur die Teufel, sondern auch alle Engel aus dem Paradiese vertreiben, und damit tatsächlich das Cantorsche Paradies in eine Einöde verwandeln wollen)" (Hilbert, 2013, p. 742; Kragh, 2014, p. 9). Surely, his language is metaphorical. Nevertheless, I will not add '*just* metaphorical', for usually, there is no coincidence in choice of metaphors. In this very context, it should be remembered that Cantor's theory of transfinite numbers, which was held in high esteem by Hilbert and was praised by him as "the most admirable flower of the mathematical intellect and in general one of the highest achievements of purely rational human activity (die bewundernswerteste Blüte mathematischen Geistes und überhaupt eine der höchsten Leistungen rein verstandesmäßiger menschlicher Tätigkeit)" (Hilbert, 1926, S. 167; van Heijenoort, 1967, p. 373), was intensively underpinned by theological and religious considerations, at least in the case of its creator. On the one hand, the theory of transfinite numbers is a characteristic example of new mathematics, but on the other, for Georg Cantor, his mathematics remained closely tied both to theology and natural philosophy (Ferreirós, 2004).

Moreover, the opposition of classic (set-theoretic and axiomatic) mathematics vs. constructive mathematics is often associated with an opposition of divine vs. human mathematics. For example, according to an American mathematician, Kenneth Kunen, *platonism*, *finitism*, and *formalism* in the philosophy of mathematics "are roughly analogous to *religion*, *atheism*, and *agnosticism*" (Kunen, 2009, p. 186). This was eloquently put by constructivist Errett Bishop:

> In the words of Kronecker, the positive integers were created by God. Kronecker would have expressed it even better if he had said that the positive integers were created by God for the benefit of man (and other finite beings). Mathematics belongs to man, not to God. We are not interested in properties of the positive integers that have no descriptive meaning for finite man. When a man proves a positive integer to exist, he should show how to find it. If God has mathematics of his own that needs to be done, let him do it himself. (1967, p. 2)

In such a context, it is natural to expect that Hilbert's main opponent – a Dutch mathematician, L.E.J. Brouwer – should be associated with atheism and solely human mathematics. Amazingly, this is not the case. There is a considerable difference between constructivism (or finitism) and Brouwer's intuitionism.

4. The case of Brouwer: A mystical prelude

Brouwer's initial motivation for his revolution in mathematics ("Brouwer – das ist die Revolution!" Weyl, 1921, S. 56.) was religious or, to put it more correctly, *mystical*. At the age of seventeen, already as a student at the University of Amsterdam, he consciously joined the Remonstrant Church, which "had a reputation for open-mindedness and tolerance", while his parents were members of the Dutch Reformed Church (van Dalen, 2013, pp. 16, 19). In *My Profession of Faith* (*mijn Geloofsbelijdenis*, 1898) (van Stigt, 1990, pp. 387–393; van Dalen, 2013, pp. 16–19), written for the occasion, he put into words a sort of solipsistic theology, which makes a perfect introduction to his thoughts in general.

Russell failed to find foundations for the existence of God, and consequently failed to ground pure mathematics as something more than a bunch of tautologies. In contrast to Russell, Brouwer thought he had found a satisfactory foundation for his faith in God, though outside the intellect. He operates in his 1898 profession with three interconnected categories: "God", "ego" and "representations". Ego is something that has representations: "to me the only truth is my own ego of this moment, surrounded by a wealth of representations in which the ego *believes*, and that makes it *live*". He discovers God as the origin of his ego, which gives him his representations. Ego is the center. On the one hand, this ego *believes* in its representations, on the other, it *senses* God and *trusts* in God, and the last two are not representations. God is something independent of him and his representations; God is outside, under and above his world of representations. "The belief in God is a *direct spontaneous emotion* in me. [...] Only the *sensing of my God* belongs to my proper religion." An intellectual deduction of the existence of God is impossible, but "this belief in God is the bedrock, from which [something] can be deduced, but that itself is not deduced" (van Dalen, 2013, pp. 16–18). Could mathematics be founded on this belief in God in Brouwer's view? The answer was still to come.

As we see from his 1898 profession, Brouwer's mystical religion was of a strictly individualistic nature ("only me and my God"), which tended to stand above the standard divisions between denominations and even religions. He had a clear tendency to universal mysticism easily uniting, for instance, Christianity and Hinduism: Brouwer quoted with equal respect both Christian mystics, such as Meister Eckhart and Jakob Böhme, and the *Bhagavad Gita* (Brouwer, 1905/1996, pp. 392–393, 400–401, 420–421, 423–424; Brouwer, 1975, pp. 486–487). It is no wonder that, in his later life, he was rather indifferent to concrete religious affiliations (van Dalen, 2013, p. 16).

Did Brouwer's religious credo affect his way into mathematics? Many years after, in 1946, he recollected his doubts about becoming a professional mathematician. Then (in 1899–1900), after two or three years of studying mathematics, he was still of two minds "whether to stay or go", because he "still could see the figure of the mathematician only as a servant of natural science or as a collector of truths: – truths fascinating by their immovability, but horrifying by their lifelessness, like stones from barren mountains of disconsolate infinity". He also added: "as far as I could see there was room in the mathematical field for talent and devotion, but not for vocation and inspiration" (Brouwer, 1975, p. 474). Brouwer especially praised Gerrit Mannoury (Bergmans, 2005; van Dalen, 2013, pp. 41–44) for showing him different mathematics whose harmonies could give aesthetic pleasure (Brouwer, 1975, pp. 474–475).

Apparently Brouwer faced inner conflict between his mystical inclinations and traditional ways of doing mathematics. Mannoury was one who practiced a philosophically informed perspective on mathematics and exhibited a strong interest in its foundations and in the philosophy of mathematics (Heijerman, 1990). He gave Brouwer no ready-to-use settlement of the conflict but inspired his hope that such settlement was worth seeking. It is quite probable that Brouwer owed his determination to write his dissertation on the foundations of mathematics primarily to Mannoury. Brouwer's own solution to the problem of mathematics and mysticism through differentiation of 'good' mathematics from 'bad' ones, which gradually crystallized through his first decade in the University of Amsterdam (1897–1907), despite all its originality, was by no means independent of a diverse range of intellectual influences. Mannoury's lectures on the philosophy of mathematics (since 1903) in the University of Amsterdam evidently played a special role in the formation of Brouwer's views.

So Brouwer aspired after different mathematics, mathematics that would not distract him from "the cultivation of [his] power and the development of [his] clairvoyance in the service of God". That is why he planned to make his dissertation on 'The value of Mathematics' with the motto 'Οὐδεὶς ἀγεωμετρικὸς εἰσίτω [Let no one unskilled in geometry enter]' (A letter from Brouwer to C.S. Adama van Scheltema, July 4, 1904, van Dalen, 2011, pp. 20–21). Brouwer's motto loosely reproduces the words "ἀγεωμέτρητος μηδεὶς εἰσίτω" ascribed to Plato by Neoplatonic commentators of Aristotle in the 6th century (namely Joannes Philoponus and Elias) as well as by Joannes Tzetzes, a Byzantine author of the 12th century. It means that *the only true way to divine reality leads through mathematics* (cf. *Republic* VII). I would like to stress Brouwer's implicit reference to Plato here, for reasons to be revealed later on.

In 1905, his mystical-philosophical manifesto *Life, Art and Mysticism* (*Leven, Kunst en Mystiek*) (Brouwer, 1905/1996) appeared. It was based on Brouwer's lectures delivered at the Technical University at Delft. At the time, he was already working on his thesis on the foundations of mathematics. Brouwer felt he was acting concurrently as a prophet and as a mathematician. In his Delft lectures, he showed his worth as a charismatic figure and a teacher of life. To understand Brouwer's account of mathematics, we should review the ideas of this little book in some detail. It was intended by the author as "a philosophical confession" and the prologue of his dissertation on the value of mathematics (A letter from Brouwer to C.S. Adama van Scheltema, July 4, 1904, van Dalen, 2011, pp. 20–21).

In Brouwer's solipsistic version of paradise, a person maintained a subtle balance of his life in solitude, guided by "the old instinct of separation and isolation"; human beings did not interfere with one another as well as with other living creatures. The original sin is in the lust for power, in the ambition to control inanimate nature, animals, other people and, finally, the future; it turned our world into a battleground where "everyone has power but at the same time suffers oppression", into "the sad world". The salvation, according to Brouwer, is in "turning into oneself" from "this perceptional world", that is to say, from *representations* to *ego*. If you succeed in this turning, "you will feel dead to the old world of perception, of time and space, and all other forms of plurality; and your eyes, no longer blindfolded, will be opened to a scene of joyful quiescence" (Brouwer, 1905/1996, pp. 391–393).

The crucial point in this turning is not connected with the *intellect*, reasoning, and words of reasoning (the language of science) but with the *will*. There are two main orientations of our will: it can be oriented *outwards*, to the perceptional world of plurality in a search for gaining power over others, or *inwards*, to our self in its unity with God. About the latter *unio mystica*, Brouwer makes Meister Eckhart and Jakob Böhme speak for him. In this very state, he quotes Böhme: "God will hear and see in you" (Brouwer, 1905/1996, p. 393). In this new state, you obtain true *freedom*, integral *understanding* surpassing intellect, and a clear vision of your own *destiny* and *mission* in this sad world. The way to obtaining the true look at the *outer* world leads through turning to the *inner* self.

There is no doubt that Brouwer should be counted as a representative of the *voluntaristic* tradition in theology and philosophy. A comparison with Schopenhauer, though he is mentioned in Brouwer's book only once and in passing (1905/1996, p. 413), inevitably comes to mind. Brouwer also mentioned Schopenhauer in some private letters and notebooks (van Dalen, 2011, pp. 44, 310; van Dalen, 2008, pp. 4, 17; cf. Koetsier, 2005). Schopenhauer's criticism of Euclid's geometrical proofs (*Über die vierfache Wurzel des Satzes vom zureichen den Grunde*, § 39; *Die Welt als Wille und Vorstellung*, Erster Band, § 15, Zweiter Band, Kapitel 13) is worth comparing with Brouwer's intuitionism. While Kant proposed only an account of mathematics, Schopenhauer outlined a reform of mathematics on the basis of Kant's account. Schopenhauer criticized the overuse of logical reasoning in geometry and suggested substituting it with direct *a priori* intuition of the reciprocal determination of space and time divisions, which he called "the reason of being [der Grund des Seins]" (*Über die vierfache Wurzel...*, § 36–38). From a mathematical point of view, Schopenhauer's criticism looked rather naïve (Klein, 1909, S. 503–508). In a sense, it was Brouwer who developed Schopenhauer's reform into something mathematically serious.

The enlightened state of mind is pictured by Brouwer as a sort of *double* or single but polarized vision (Brouwer, 1905/1996, pp. 393–394, 412–413):

> Freely staying outside, you live at the same time your imprisoned bodily life in this human world, live with your shackles but you are fully aware that you have accepted them in freedom and that they bind you only as long as you wish. (Brouwer, 1905/1996, p. 394)

Brouwer opposes *self-reflection* to *intellect* as good to evil. His position is not only *voluntaristic*, but *anti-intellectualistic* as well. Our intellect creates this sad world of time, space, and total causality. All our industry and almost all culture in general are accused by Brouwer from this point of view. It is important to stress that he also accuses *science* as an offspring of intellect. Science is a manifestation of the Fall: "Science [...] even expresses its resignation in God's will, while it is all the result of rebellion against his will!" (Brouwer, 1905/1996, pp. 391–392) It seems that there is almost no place for *true* science in Brouwer's Weltanschauung. Brouwer especially mocks *pure* science with its anxiety and the endless quest for secure, autonomous *foundations*. His account sounds like a severe caricature, and he really means it (1905/1996, pp. 396–398):

> Science places whatever is perceived, outside the self, in a world of perception independent of the self; the bond with the self, its only source and guide, is lost. It then constructs a mathematical-logical substratum which is completely alien to life, an illusion, one which acts in life as a Tower of Babel with its confusion of tongues. (1905/1996, p. 412)

Science is abandoned as an end in itself, but as a route within the double vision, it is tolerable. It seems that, despite their mystical radicalism, Brouwer's Delft lectures leave some room for true art, true religion, and perhaps even true science, though the point is left rather obscure by the author.

It is worth noting that Brouwer said next to nothing about *mathematics* in his 1905 manifesto. At the time, he was already working on his dissertation devoted to the *value* of mathematics. In his manifesto, he said nothing to reveal this value. The only mathematics mentioned was the one united with logic and doomed to rejection as a part of 'bad' science. Why did he pass over in silence this issue so vital for him? Perhaps the issue was not clear for him yet. He still needed some *alternative* mathematics as something valuable against the background of his mystical Weltanschauung to excuse his mathematical career.

5. The case of Brouwer: Different mathematics

It seems to me impossible to count the high level of Brouwer's engagement with mathematics as having nothing to do with his main goals or even leading him astray (van Dalen, 2013, p. 81). His 1903/1904 private correspondence (despite its high style) leaves no doubt of him being a self-confessed prophet living within and under the direct grace of God and having a certain mission in this sad world. For instance, he wrote to his friend, the Dutch poet Carel Adama van Scheltema:

> If I were looking for kingship on earth, it might be good to wall myself in mathematics, and have myself crowned like a pope in the Vatican, a prisoner on his throne. But I covet a Kingship in better regions, where not the goal but the motive of the heart is of primary importance. We are not on earth for our pleasure, but with a mission that we have to render account for. And a small kingdom by the Grace of God is better than a large one by the will of the people. (A letter from July 4, 1904, van Dalen, 2011, p. 21)

So Brouwer, as a "Christian King" (A letter from Brouwer to C.S. Adama van Scheltema, Nov. 15, 1903, van Dalen, 2011, p. 18) did not want "to wall himself in mathematics", at least in 1904. Nevertheless, he was persuaded to start his career as a *privaatdocent* (unpaid lecturer) at the University of Amsterdam in 1909. Did Brouwer just compromise with the Devil, or did he find some form of doing mathematics appropriate for a mystic and a prophet? To answer the question, we should address his 1907 dissertation, *On the Foundations of Mathematics* (*Over de Grondslagen der Wiskunde*) (Brouwer, 1975, pp. 11–101) and accompanying materials: Brouwer's notebooks (van Stigt, 1990, pp. 394–403; van Dalen, 2013, pp. 77–81) and rejected parts of his dissertation (van Stigt, 1979; 1990, pp. 405–415).

His work on the problem of the foundations of mathematics led him to recognize *two* different types of mathematics. The first one is mathematics as a *science* and within science. The other one is mathematics as a true *art*, practiced for its own sake (for God's sake?).

The first one should be cursed and rejected (van Dalen, 2013, p. 80); the second one is treated as a positive alternative:

> Science [...] makes sense only when man in his struggle against nature and his fellow men, uses the calculations of counting and measuring; in other words, physical science has value only as a *weapon*, it does not concern life – indeed it is a disturbing and distracting factor like *everything* in any way connected with struggle. But mathematics practised for its own sake can achieve all the harmony (i.e., an overwhelming multiplicity of different visible, simple

> structures within one and the same all-embracing edifice) such as can be found in architecture and music, and also yield all the illicit pleasures which ensue from the free and full development of one's faculties without external force. (van Stigt, 1979, p. 399)

Let us keep in mind that Brouwer's original theme for his doctoral thesis was *The Value of Mathematics* and the original plan for it had six chapters (the final variant retained only three chapters); the last two (never written) were to be on the value of mathematics "for society" and "for the individual" respectively (A letter from Brouwer to D.J. Korteweg, Oct. 16, 1906, van Dalen, 2011, p. 24). Moreover, he associated that quest for value with *Plato*, as we have already seen. There are some striking similarities between Brouwer and Plato. Plato, in his *Republic* VII (525cd), opposes two perspectives on mathematics. Mathematics can be studied "for the sake of buying and selling" and seen in a *dealer's perspective* (Plato speaks of ἔμποροι καὶ κάπηλοι that is "tradesmen and retailers" or "merchants and hucksters") or "for ease in turning the soul around, away from becoming and towards truth and being" and seen in a *philosopher's perspective* (Plato, 1997, p. 1142).

Classical mathematics is the mathematics seen in the dealer's perspective, according to Brouwer's preliminary notes towards his dissertation, in which he repeatedly associates mathematics with merchandise and marketing (van Dalen, 2013, pp. 79, 81). Here are some notes on mathematics from the philosopher's perspective: "Nothing in art or science that is true has value (i.e. commercial value)" (van Stigt, 1990, p. 400). "Let the motivation behind mathematics be the craving for the good, not passion or brains" (van Dalen, 2013, p. 79).

Finally, let us have a look at Brouwer's notes on the problem of foundations in the light of the opposition of the two perspectives.

> The role of foundational research must be: *given* the temptations of the devil, who is the world and its categories, to appreciate the true value of the world, and to relate it constantly to God.
> Not worrying about the 'foundations', and just doing mathematics is the same as: not worrying about economy and economic morals, and just doing business and earning money and making a career. In both one can be very clever, and yet a zero. [...]
> Mathematics justifies itself, needs no deeper grounds than moral mysticism. (van Dalen, 2013, pp. 79, 81)

In other words, Brouwer was drawn by moral and religious motivations and saw his task in foundational research as a shift from the dealer's to the philosopher's perspective on mathematics put in the context of his turning-into-oneself preaching. The result is well known: in his 1907 dissertation, Brouwer formulated the main principles of his intuitionism, which was in 1911–1912 opposed to formalism in mathematics (Brouwer, 1975, pp. 121–138). Among the advocates of formalism, Brouwer counted Dedekind, Peano, Russell, Hilbert and Zermelo. Walter van Stigt (1979, p. 389) was the one to assert and document the straightforward connection between Brouwer's mystical Weltanschauung and his intuitionism in the foundations of mathematics. Is it possible to treat mathematical intuitionism as the successful resolution of the conflict between mysticism and mathematics?

The answer is not that obvious. Brouwer so clearly and persistently opposed mysticism and mathematics (Brouwer, 1905/1996, pp. 419–420; van Stigt, 1979, p. 398; van Dalen, 2013, p. 282) that some scholars have concluded that the bond between the two is only a negative one in the case of Brouwer (van Atten & Tragesser, 2003; van Atten, 2015, pp. 181–182). Nevertheless, I do not think this conclusion is the right one. Firstly, "mathematical systems" and "geometry" in the texts in question refer to classical rather than intuitionistic mathematics. Secondly, the whole context suggests another conclusion. To reach it, let us address one of Brouwer's later works.

In his 1948 Amsterdam address, *Consciousness, Philosophy, and Mathematics* (Brouwer, 1975, pp. 480–494), Brouwer pretended to show that "research in foundations of mathematics is inner inquiry with revealing and liberating consequences, also in non-mathematical domains of thought" (1975, p. 494). Though Brouwer said nothing about God in this address, in the core of his work, we find the same mythologem of our initial home lost and regained as in the 1905 Delft lectures. The point of departure is the "deepest home" of our consciousness: "*Consciousness* in its deepest home seems to oscillate slowly, will-lessly, and reversibly between stillness and sensation" (1975, p. 480). Here, we have consciousness in the initial point of its birth. This state of consciousness precedes the subject-object dichotomy. We constantly lose the deepest home and proceed through several phases of exodus while reaching the familiar world of representation and total causality. Brouwer seems to be especially interested in reverse gradual ascent. In contrast to the maximalism of his 1905 manifesto, he stresses the *gradualness* of the process: "there may be wisdom in a patient tending towards reversible liberation [...] perhaps at the end of the journey the deepest home vaguely beckons" (1975, p. 487).

Intutionistic mathematics is "inner architecture" (1975, p. 494), "deducing theorems exclusively by means of introspective construction" (1975,

p. 488). It is not the ultimate goal; nonetheless, it plays a vital role on our way back to the roots of consciousness (and thus to God, we may add, while Brouwer kept silence on the subject) (1975, pp. 480, 482). In the "mathematical deeds" of an intuitionist, according to Brouwer, there are *truth* (1975, p. 488) and *beauty*.

> [T]he fullest constructional beauty is the introspective beauty of mathematics, where [...] the basic intuition of mathematics is left to free unfolding. This unfolding is not bound to the exterior world, and thereby to finiteness and responsibility; consequently its introspective harmonies can attain any degree of richness and clearness. (Brouwer, 1975, p. 484)

I would like to point out that the word *harmony*, used in this passage, is a weighty and positive one for Brouwer (cf. two other examples of its use already indicated above: van Stigt, 1979, p. 399; Brouwer, 1975, p. 475). He wrote in 1915 that the emotion of beauty "awakens in our diseased bodies the frozen consciousness of God" (van Dalen, 2013, p. 282).

According to Dennis Hesseling, "Heyerman [1981, p. 40][3] conjectured that Brouwer's mathematical experience has its place in the process of turning into oneself, between the experience of the multitude of the outer world and the mystical experience of unity" (Hesseling, 2003, p. 45). My independent analysis has given the same result. It means that Brouwer's intuitionistic mathematics plays a role similar to the one of mathematics in Plato's *Republic*.

L.E.J. Brouwer was a rebel who tried to oppose mystically oriented mathematics to Hilbert's formalist account. He also opposed the popular philosophy of mathematics in general. There is no need for the prefix "quasi-" in the religious preconditions of his intuitionism. Nevertheless, they were mainly hidden, according to the standards of the time (van Dalen, 2013, p. 283). His skeptical attitude towards language as well as logic, and the solipsistic tendencies of his intuitionism in general, which emphasized serious obstacles to opening the mathematical mind *outwards* (to other human beings), make sense only in the light of the mystical opening of our mind *inwards* (to the divine realm). Being devoid of its divine roots, intuitionist mathematics would turn into a mere individual fantasy.

The main point of the Hilbert-Brouwer controversy was articulated by Hermann Weyl in a discussion with György Pólya in Zürich in 1919: "Separating mathematics as formal from spiritual life [Geistesleben] kills it, turns it into a shell" (Hesseling, 2003, p. 129). This way of opposing formalism and intuitionism as agnosticism (or atheism) vs. religious outlook seems to me rather superficial, and it is no wonder it can be easily reversed in opposing classical mathematics and constructivism. At a deeper level, the Hilbert-Brouwer controversy was a conflict between two *theological* traditions: intellectualist and voluntarist.

6. Conclusion

One can find many additional illustrations to the central ideas of this paper in Daniel Cohen's *Equations from God* (2007). His work shows that, in the Anglo-American world, even through the 19th century, the alliance between mathematics and theology continually resisted the final secular rapture up to the 1860s–1870s. Many famous mathematical names were involved in the process: Benjamin Peirce, George Boole, Augustus De Morgan, etc.

I paid special attention here to the later period with Russell, Hilbert, and Brouwer as main characters. All three have been found to have been engaged with theology, but in diverse ways. Bertrand Russell developed his version of logicism trying to make mathematics take the part of the absolute-certainty-giver in the place of failed theology. David Hilbert built his axiomatic mathematics as a quasi-theology. Egbertus Brouwer stays all by himself in the picture. He recognized his own solipsistic modification of the TCA-triangle (see Part I of this paper): 'God – ego – representations' or 'God – self – perceptual world (God – zichzelf – aanschouwingswereld)'. Brouwer's TCA-triangle was the basis for his intuitionism. He suggested a voluntarist alternative to the intellectualist theology and the popular philosophy of mathematics based on it. This replacement of the theological base resulted in no less than an intuitionistic revolution in mathematics. Mathematics was also substituted for theology in Brouwer, though only in a biographical aspect: he became a professional mathematician, not a professional theologian or public prophet. Moreover, he was by no means inclined to parade his underlying theological reasons for doing mathematics; it made his intuitionism look for the general public to be one more attempt to provide mathematics with autonomous foundations. It had been taught to him already by his 'promotor' (supervisor) Professor Diederik Korteweg to separate mathematics from metaphysics and theology for public presentations, according to the norms of the time (a letter from D.J. Korteweg to Brouwer, Nov. 11, 1906, van Dalen, 2011, p. 32).

Theological and quasi-theological ideas played a decisive role in modern mathematics. In my view, those ideas functioned as a sort of *theological underpinnings* of the modern philosophy of mathematics. That is, they served as something inconspicuous but supporting and strengthening of our understanding and acceptance of highly abstract mathematics, and so helped mathematicians to continue their research with a comfortable sense of security. Perhaps it is no mere coincidence that the most intensive historical period of foundational work in mathematics (the 1870s–1920s) is fringed by two manifestly theologically friendly figures – Georg Cantor and Kurt Gödel.

NOTES

[1] Jeremy Gray declares the arrival of *modernism* in mathematics around 1890–1930. He especially stresses the autonomy of new mathematics from natural science and the world of nature, as well as its "anxious character", but says nothing about the crucial role of separation from theology and the supernatural.

[2] I have no intention to express any solidarity with this Hegelian idea (die List der Vernunft) or to discuss it here. It is the very fact that I am interested in.

[3] Unfortunately, this work has been inaccessible to me.

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