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FROM RELIGION TO DIALECTICS AND MATHEMATICS
SCHLEIERMACHER'S THEOLOGICAL CONTRIBUTION TO THE DEVELOPMENT OF MODERN TENSOR CALCULUS IN GRASSMANN'S AUSDEHNUNGSLEHRE

Abstract. Hermann Grassmann is known to be the founder of modern vector and tensor calculus. Having as a theologian no formal education in mathematics at a university he got his basic ideas for this mathematical innovation at least to some extent from listening to Schleiermacher's lectures on Dialectics and, together with his brother Robert, reading its publication in 1839. The paper shows how the idea of unity and various levels of reality first formulated in Schleiermacher's talks about religion in 1799 were transformed by him into a philosophical system in his dialectics and then were picked up by Grassmann and operationalized in his philosophical-mathematical treatise on the extension theory (German: Ausdehnungslehre) in 1844.

1. Introduction

Hermann Grassmann (1809–1877) and Friedrich Schleiermacher (1768–1834) were towering figures in 19th century science in Germany. Schleiermacher can be called the church father of Protestant theology in the 19th century. Grassmann can be labelled as founding father of modern vector and tensor calculus. Both were theologians, and highly gifted self-taught lay-mathematicians. Neither of them ever listened to the lecture of a professional mathematician at a university. Nevertheless Grassmann worked out his extension theory (German: Ausdehnungslehre) from 1844 – in nearly complete isolation from the currents of contemporary mathematics – the foundations of modern vector and tensor calculus. Schleiermacher was professor in the newly founded University in Berlin for theology and actively engaged in the reorganisation of the Prussian University system, whereas Grassmann taught as a teacher of mathematics and other subjects at a Gymnasium in the city of Szczecin (German: Stettin) far from the German centres of professional mathematics.

When Grassmann published his extension theory in 1844 only very few copies were sold and it was completely ignored by his fellow mathematicians, due to the fact that it was written in a philosophical manner nearly without any mathematical theorems. A second edition in 1862 without a philosophical introduction and written in a rigorous formalistic manner proved to be as unsuccessful as the first edition. It was not earlier than around 1870, 30 years after his first edition, when he had already completely abandoned work in mathematical research that he gained increasing recognition by professional mathematicians. Alfred North Whitehead's *Treatise on universal Algebra* from 1898 was already based on Grassmann's extension theory. Whitehead wrote: "The greatness of my obligations in this volume to Grassmann will be understood by those who have mastered his two Ausdehnungslehres. The technical development of the subject is inspired chiefly by his work of 1862, but the underlying ideas follow the work of 1844."[1] And the German philosopher Ernst Cassirer contended, in his important book *Substanzbegriff und Funktionsbegriff* from 1910, about Grassmann's contribution to the foundation of modern mathematics, that it was Grassmann who changed the understanding of modern mathematics as a science of magnitudes to a science of structures, functions or relations.[2]

2. Schleiermacher and Grassmann

Now what was the relation between these two theologians and lay-mathematicians? First of all Grassmann was a student of Schleiermacher's and listened in Berlin to his lectures on dialectics, which inspired him greatly. In addition he read the publication of these lectures on dialectics from 1839 in the subsequent year 1840 together with his brother Robert just before he began to work on his extension-theory. In his autobiographical notes Grassmann testifies to the enormous influence of Schleiermacher's lectures on his thinking. This influence is echoed by his second biographer Friedrich Engel, a mathematician from Gießen. Grassmann wrote in 1833:

> Just as little will I claim that I had made Schleiermacher's views completely my own (since indeed much of them I did not understand); in the mean time he still exerted such a powerful influence on my development, I have so infinitely much to thank him for spiritually [...]. Yet only in the last year did Schleiermacher attract me completely; and although by that time I was more concerned with philology, still only then did I realize what one can learn from Schleiermacher

for every science, since he did not so much provide positive answers, as he made one skilled in attacking every investigation from the correct side and continuing independently, and thus in a position to find the positive answer oneself.[3]

Now in what way did Schleiermacher influence Grassmann? This question is far from being easy. On the one hand Grassmann did not mention Schleiermacher in his extension–theory, neither in his philosophical preface nor in the technical parts. On the other hand Schleiermacher did not deal with any mathematical problems in his dialectics although he frequently refers to mathematics and uses mathematical expressions such as approximation, for example, as metaphors.[4] And this unclearness is why it is a matter of dispute by historians of mathematics if Schleiermacher influenced Grassmann at all, and if so to what extent and in what way Schleiermacher exerted an influence on Grassmann.

Some have stated such influence whereas others have denied any influence of Schleiermacher on Grassmann.[5] However, this influence some scholars claim remains in their scholarly works rather vague. It is mostly restricted to the way in which mathematics is located in the framework of the sciences. For example Schleiermacher talks about real and ideal sciences, Grassmann about real and formal sciences.[6] But this influence is not directed to the mathematical content of the extension-theory and thus not very interesting.

Therefore I would like to take a different starting point sticking strictly to Grassmann's testimony on what he owes Schleiermacher. There is one phrase that gives us a hint: "I realize what one can learn from Schleiermacher for every science, since he did not so much provide positive answers, as he made one skilled in attacking every investigation from the correct side and continuing independently, and thus in a position to find the positive answer oneself." Two aspects can be identified which can help us further: the first is that Schleiermacher helped to develop the virtue of self-reliance, which explains why Grassmann did not mention his theological teacher. The other is important: Schleiermacher did not teach any particular content. This means already that it is in vain to look for any direct mathematical influence. Rather he taught a particular *method*; to be more precise he taught a *way of knowledge acquisition*.

In fact Schleiermacher's dialectics is meant as a methodological tool to generate new knowledge in contrast to his contemporaries Friedrich Hegel and Gottlieb Fichte and their speculative metaphysics to reach knowledge of the absolute. Thus he opposes their attempts to build a metaphysical

system with a metaphysical axiom from which reality could be derived by deductive reasoning.[7] Schleiermacher is much more humble in his dialectic. He is not claiming to attain understanding of the absolute. However he rather offers a method to proceed in the process of knowledge acquisition.[8] In this process the religious notions of unity and diversity play an important role. Now what does this mean more concretely?

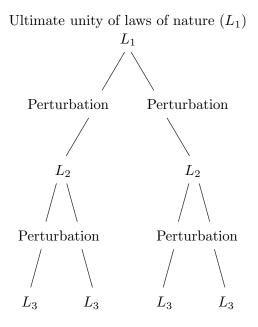### 3. Dialectic and extension theory – unity and diversity

Schleiermacher's dialectic focuses on knowledge acquisition in the context of two dialectically related key notions: unity and diversity. Unity and diversity form a conceptual space in which new knowledge and knowledge acquisition can be and must be located. And it is the very tension between unity and diversity – both ontologically and epistemologically – which lies at the core of Schleiermacher's dialectics and could have served as a driving force and as an intellectual tool for Grassmann's new mathematics.

Now let us deal more concretely with unity and diversity in Schleiermacher's thinking. Unity and diversity have a history in Schleiermacher's theological thoughts which dates back as early as 1799, the year in which his epoch making book "Talks about religion to the educated among its dispraisers" appeared. In this influential book, unity and diversity appear in the context of religious mystical experience as well as features of nature. For Schleiermacher unity is in the tradition of mystic religious experience as well as in the tradition of Spinoza's pantheism and the Greek metaphysical thinking of the "Ἐν καὶ Πᾶν" a basic feature of religion. Thus he asserts in his second talk "About the essence of religion": "Religion lives its whole life also in nature, but in the infinite nature of the whole, the unity and totality."[9]

But what is the nature of this unity? In Schleiermacher's point of view, it is in the first place the religious mood, the religious feeling, the innermost mind to which he frequently refers the source of the idea of unity both in nature and religious experience.[10] Religious feelings in Schleiermacher's terminology, however, are not restricted to moods, or sentiments of awe and wonder, but include rationality and will. It is this combination of will and rationality, which lies at the very core of Schleiermacher's religious anthropology and it is this combination which he calls religious feeling (German: Gefühl) or mind (German: Gemüth).[11]
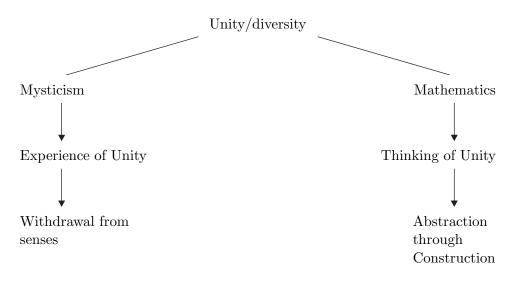
Starting from this source in the human mind in which unity is a more or less intensive religious feeling, the idea of unity is applied to the world and

nature outside. In this context Schleiermacher refers to the world outside as the "universe", which has different meanings in his talks. An essential one is that the universe as a representative of unity can be looked at as being governed by the laws of nature. The laws of nature therefore have a religious meaning and thus they must somehow be related to unity.[12]

But how is the unity of the laws of nature given to the religious mind as being related to unity? Now Schleiermacher offers some very interesting considerations concerning the relation between laws of nature and unity. In fact he holds that the unity of the laws of nature are not directly given to the scientist or religious person. Indeed, their unity is hidden. And it is only by the experience of disorder that in an act of intellectual effort the deeper underlying unity can be revealed and unveiled. Therefore disorder, chaos, and perturbations are essential steps towards the intellectual journey to acquire knowledge of the laws of nature, which are included in a system of ordered hierarchies of unity pointing towards an ultimate unity of the laws of nature.[13] The structure of these considerations concerning the hierarchy of levels of unity among the laws of nature $(L_1, L_2, \ldots, L_{n-1}, L_n)$ can be visualized as follows.

Ultimate unity of laws of nature (L_1)

$$L_1$$

Perturbation   Perturbation

$$L_2 \qquad\qquad L_2$$

Perturbation   Perturbation

$$L_3 \qquad L_3 \qquad L_3 \qquad L_3$$

Now it seems to me of utmost importance that these twin notions of unity and diversity reoccurred from the realm of religious mystical experience in his early "Talks on religion ..." from 1799 to purely intellectual notions of knowledge acquisition in his later elaborated dialectic published

in 1839. Obviously, Schleiermacher himself was aware of the unity-diversity correlation both operating in the field of science and mystical religious experience, because in his dialectic he writes: "The unity of thinking and being in this unity is the highest itself, the absolute. This relation of volition to thinking and vice versa the unity thereof is the divine in us. The religious person has it in life, the speculative in contemplation, but both have it only as something different."[14] The mutual connection of volition and rationality is an innovation in Schleiermacher's religious anthropology which avoids the one-sidedness of traditional theology either being focussed on the will or on rationality. Thus the concept of unity, which in former theology was only associated to feeling or will could migrate to the intellectual faculties of the mind.

And it is now in this context of intellectual knowledge that unity and diversity unfold their heuristic power. Let me depict this migration of concepts from religious experience to intellectual notions. We do have unity and diversity in mystical experience as well as in intellectual thinking. But they differ in the way they are realized.

Unity/diversity

Mysticism   Mathematics

Experience of Unity   Thinking of Unity

Withdrawal from   Abstraction
senses   through
                                                     Construction

In Schleiermacher's dialectic unity and diversity form the framework in which knowledge acquisition must be located. Unity is at one end and diversity at the other. Unity and diversity are understood both ontologically as well as epistemologically. With regard to epistemology the twin notions of unity and diversity are the boundaries of the continuous *process* of knowledge acquisition by abstraction through constructive thinking.[15] Schleiermacher considers in various ways how ontological and epistemolog-

ical aspects of unity and diversity are mutually inclusive: "In the absolute unity of being, diversity is dissolved, in the community of being, unity is dissolved; but both are always related to each other, because only by this is a transition from one to the other possible."[16] However the diversity in the community has to be overcome. There is a kind of epistemological imperative not to remain imprisoned in the diversity, but to attempt to reach by intellectual work higher degrees of unity and thus to reduce diversity.[17] He even claims that all knowledge should be condensed in a final unity by a method which he calls architectonic.[18] The aim of this architectonic method is to unify knowledge in a way that a law can be identified, which makes it possible to construct other knowledge at a lower level of abstraction but which is included in the architectonic unity.[19] There are two essential methodological elements for thinking in Schleiermacher's dialectic to generate knowledge: construction and connection (German: "Verknüpfung"). Both elements however must be related to an encompassing unity.[20] Construction means that adequate scientific notions or abstract terms must be invented or found.[21] Connection means more precisely that a manifold of knowledge in the form of notions must be connected by certain rules.[22] The aspect of connection and interrelation belongs to the worldly manifold of knowledge.[23] In addition, the constructed knowledge by connection mirrors the real world and is in this regard objective.[24] However, there are no preconceived rules for either the construction or the connection of knowledge. But the final aim of knowledge is clear. It is the unity of a correlated and constructed manifold.[25]

Unity as the centre of all knowledge is God, who can't be reached by intellectual reasoning but works as a driving force to reach higher degrees of intellectual abstraction. Abstraction thus is an intellectual tool for constructing higher knowledge. However, abstraction is not meant in the traditional philosophical sense as leaving out particular properties to reach the highest philosophical notion of "being" as completely devoid of any content. To put this approach of Schleiermacher into a nutshell: His method of knowledge acquisition is: New knowledge is found by constructive abstraction from diversity to the complexity of unity going through different levels of abstraction leaving behind perturbations at a particular level.

Mathematical knowledge was important for Schleiermacher and he even argues that the existence of mathematics in the thinking of a particular science is an indicator of its inherent value.[26] But it seems that Schleiermacher has not applied this mode of knowledge acquisition to mathematics itself. This is so because he still holds the traditional point of view that mathe-

matics is the science of magnitudes,[27] which are given by immediate sense perception.[28] If this is the case, it was Grassmann's ingenious achievement to transfer the concept of general knowledge acquisition in science to the particular science of mathematics.

4. Schleiermacher's method of knowledge acquisition operating in Grassmann's Ausdehnungslehre

Now what does Schleiermacher's method mean for concrete mathematical thinking? I would like to identify the following five examples, which are pertinent to Grassmann's extension theory, that illustrate this triple of unity – diversity – abstraction operating as a device of knowledge acquisition both in Schleiermacher's dialectics and Grassmann's extension theory.

First example. As we have seen, abstraction belongs to the very core of Schleiermacher's dialectics. It is associated with various levels of abstraction. In Grassmann's extension theory abstraction reoccurs and is directly mentioned by the expression of various levels (German: "Stufe") of mathematical entities, which can be generated by certain changes at a particular level up to infinite realms.[29] Through abstraction, mathematical systems can be constructed up into infinite realms. By doing so mathematics as a science of structures is disassociated from immediate sense perception and its confinement to three-dimensional space and in addition it ceases to be a science of quantities.

Second example. This kind of abstraction through construction is already a new meta-mathematical way Grassmann elaborated upon. In fact all authors agree that Grassmann actually was the first mathematician preceding Brouwer's intuitionism who advocated a constructivist way of mathematical thinking. This constructivist approach can now be better understood within the framework of our twin notions of diversity and unity. Constructivism is the means to reach higher levels of abstraction containing lower levels of abstraction as special cases. The regulative principle of unity – to use a Kantian wording – is the driving force however behind the constructive activity of the human mind, which gives the constructivist approach a direction. This becomes already apparent in the preface of his extension theory in which he calls mathematics "pure thinking" liberated from being imprisoned by three dimensional geometry.[30]

In his subsequent introduction he makes the distinction between real science and formal science. Formal sciences are philosophy and mathematics. Whereas he defines real sciences in a traditional way as the concordance

of being and thinking, he understands mathematics as a science which generates its objects by pure thinking and is concerned only with logical consistency.[31]

In addition he distinguishes the general and particular. Philosophy deals with the general, seeking the unity in all thinking, and mathematics with the particular, but from the perspective of a general rule.[32] As a result mathematics as a formal science dealing with particular entities from the perspective of a general rule generated by pure thinking allows him to understand mathematics as a science of forms which are created constructively by the human mind.

"Pure mathematics is therefore the science of the particular being as created by thinking."[33] However the particular is only the starting point from which the constructive abstraction reaches, through the interrelation or connection (German: "Verknüpfung") of mathematical notions, a more generalized form.[34]

There is one aspect of this constructive understanding of mathematics which is important with regard to the aspect of unity-diversity as found in Schleiermacher. Grassmann's understanding of mathematics as being created by pure thinking opens a new conceptual space for mathematical novelties. And these novelties can be found only if one is able to interrelate mathematical notions in a new way. Interrelation or connection (German: "Verknüpfung") is directed towards unity.[35] However there is no general method for finding such mathematical novelties as were sought by Leibniz in his *ars combinatoria*. Instead Grassmann refers to this in terms of the romantic notion of intuitiveness (German: "Ahnung").[36]

As a result of this constructive understanding of mathematics he excludes traditional geometry from mathematics because it is empirically given through sense perception and thus not a matter of pure thinking.[37] This leads us to the next example.

Third example. This way of abstraction through construction can also be identified in his extension of geometry to more than the usual three dimensions. Grassmann claims that geometry needs to be completely reframed by his new approach because it lacks the necessary scientific foundation, which he can offer.[38] Geometry with more than three dimensions is usually ascribed to Bernhard Riemann (1826–1866) who discovered it again about 10 years later without knowing the work of Grassmann. In fact this was already a revolution in mathematics. The extension of geometry to more than three dimensions has two important aspects belonging to this tool of abstraction by construction. The first is the liberation of mathematics from being based in the immediate sense perception of a three dimensional

space. Already in his preface he explains the relation of his new abstract science of extension to geometry: "[...] so as a result I realized soon that I had embarked here on the area of a new science, from which geometry is only a special application."[39] And he continues: "My extension theory is the abstract foundation of geometry. This means it is a mathematical science which is disentangled from all immediate space-perception. Its application to space is geometry."[40] Or in a different way: "However, the theorems of the extension theory are not applications of theorems of geometry in an abstract language, but they do have a more general meaning; because, whereas geometry remains restricted to *three dimensions of space*, abstract science is free from this barrier."[41]

Essential is that abstraction through construction has to be done in a meaningful way by using the right building block in order to get the next level of abstraction right. Having acquired thus a higher perspective the lower level of abstraction can be understood as a special case.

Thus Grassmann opened the prison of the three dimensions in which Kant wanted to confine mathematics. In association with that, mathematics became a science of pure thinking – ideally thinking in terms of abstraction through construction – no longer depending on sense perception but solely on the logical relations of notions and concepts. In this sense the category of constructive relatedness becomes essential for mathematics.

Fourth example. The fourth example is of a more technical character and applies directly to his new approach of constructive relatedness. It is his non-commutative algebra. In normal algebra the law of <u>commutativity</u> is valid. However if one changes the way of relations and gets to a higher degree of abstraction then the law of commutativity becomes invalid. Grassmann elaborates this in §§ 5–12 in his extension theory.[42] He writes:

"We will get to know in our science kinds of multiplication to which the commutativity of the factors does not apply, but for which all theorems so far formulated are valid."[43] Thus we get the equivalent of the modern vector product:

$$a \times b = -b \times a \text{[44]}$$

Again we see the essential building block of constructive abstraction with a particular form of connection and interrelation (German: "Verknüpfung") as formulated already by Schleiermacher as an essential mode of knowledge acquisition. These kinds of interrelated mathematical notions or abstract terms are now our fifth and last example.

Fifth example. As we have already seen in Schleiermacher's dialectic, the interrelatedness of notions is an essential part of knowledge. However,

in mathematics in general and in algebra in particular the human mind is free to construct such connections between abstract terms. The vector product just mentioned is an example of how a relation between mathematical entities can be changed by abstraction and by changing the kind of interrelation. For Grassmann, interrelation (German: "Verknüpfung") is an essential part of his mathematical epistemology. This becomes already apparent in his introduction to his extension theory. In § 2 he introduces a very abstract notion of interrelation devoid of any concrete content or specification, which must be applicable in any system of mathematics.[45] He even defines a particular notation for this abstract term of interrelatedness. It looks like this: If there is any mathematical term A and another B and C then the interrelation is $A \frown B \frown C$. This he calls an analytic interrelation, whereas $A \smile B \smile C$ is a synthetic interrelation. Addition and subtraction are only special cases of such interrelation, such of the first order, multiplication of the second order.[46] Grassmann does not exclude the possibility that there are other more general forms of addition, which might entail more abstract forms of interrelation.[47] In fact he develops such an addition in § 47 of his extension theory, from which normal addition is only a special case.[48]

Summary

Let me summarize: Schleiermacher's influence on Grassmann can be understood in various phases:

1. The migration of unity and diversity from mystical experience to an intellectual space.
2. The creative tension between unity and diversity as a driving force for abstraction at various levels
3. The construction of various levels of abstraction in the context of unity and diversity.
4. Invention of non-Euclidean geometry by abstraction from the limitation of the three dimensional space
5. Invention of new forms of interrelations between mathematical magnitudes like the non-commutativity of the vector product: $a \times b = -b \times a$.
6. This tension between complex unity and diversity and the levels of abstraction by far exceeds this first influence on Grassmann. It can be seen as a pattern operating in the subsequent history of mathematics and physics.

Conclusion

Schleiermacher's twin notions of unity and diversity opened via Grassmann's extension theory the gate for a new field in mathematics, vectors and tensors and non-Euclidean geometry.

In the later application to physics we again see this principle, of abstraction through construction and thus seeking unification, operating. In Einstein's Special Relativity Theory and General Relativity Theory we can identify it in the invariance of the metric. There is always one physical magnitude helping to bridge the gap to the next higher level of abstraction. In the case of the leap from Newtonian Mechanics to SRT, it is the velocity of light "c", thus changing the metric:

$$[\text{NM: } \Delta x^2 = (x_1)^2 + (x_2)^2 + (x_3)^2$$
$$\rightarrow \text{SRT: } ds^2 = (x_1)^2 + (x_2)^2 + (x_3)^2 - (ct)^2].$$

In the case of the leap from SRT to GRT, it is the inclusion of mass, again changing the metric to a higher level of abstraction:

$$[\text{SRT: } ds^2 = (x_1)^2 + (x_2)^2 + (x_3)^2 - (ct)^2 \rightarrow \text{GRT: } ds^2 = \Sigma g_{\mu,\nu} dx_\mu dx_\nu]$$

In current research on super string theory, theoretical physicists seek to push the frontiers of unification even further by combining GRT and quantum mechanics.[49] The inverse of the velocity of light, the Newtonian constant of gravity, and the Planck quantum of action are interconnected to proceed to the next level of unification.

Thus the level of abstraction again has to be raised in order to include all of these components. As seen in the way from NM to SRT and from SRT to GRT, there must be a component which helps to elevate from GRT to string theory in a way that GRT emerges as a special case if one or more of the mentioned magnitudes (e.g. inverse of the velocity of light, the Newtonian constant of gravity, or Planck's quantum of action) are eliminated, formally if they equal zero. At least the Planck action principle has to be included for the leap to the next layer of abstraction and unification. Thus in the current string theories a metric with 10 dimensions, one of time and nine of space are required to make the new layer of unification mathematically coherent.

$$[\text{ART: } ds^2 = \Sigma g_{\mu,\nu} dx_\mu dx_\nu \rightarrow \text{SST: ?}]$$

Theory	Relations	Invariance	Relativity of...	Mediate abstraction
Classical Mechanics	time	$\Delta x_2 = (x_1)^2 + (x_2)^2 + (x_3)^2$	space	velocity of light: "c"
SRT	space-time	$ds^2 = (x_1)^2 + (x_2)^2 + (x_3)^2 - (ct)^2$	space-time	matter, gravitation: "m"
ART	space-time-matter	$ds^2 = \Sigma g_{\mu,\nu} dx_\mu dx_\nu$	space-time-matter	Planck quantum of action: "h"
String Theory	Space-time-matter and quantum mechanics	?	?	?

Thus we see that the mystical longing for unity with the divine has entered, in a very fruitful way, the sciences and brought them to a high level of technical sophistication. However without this driving force mathematics and physics remain a fruitless game with symbols. This religious driving force for unity was also vivid in Einstein's longing for a unified field theory as well as in his philosophical commitment to Spinoza.

In this sense he wrote in 1939: "It is true that science is directed towards linking rules which connect facts and thus lead to prediction. But this is not all it is longing for. It seeks also to reduce the connections to a low number of independent notions. By this striving towards rational unification of diversity it celebrates its greatest success, even though it is running the risk of falling prey to this longing for illusions. However he who has successfully worked in this manner and experienced it intensely will be seized by a deep reverence towards the wisdom which is manifested in being."[50]

N O T E S

[1] Alfred North Whitehead, A treatise on universal algebra: with applications, The University Press, Cambridge: 1898 p. X.

[2] "Hier ist in der Tat das erste und markanteste Beispiel der allgemeinen Betrachtungsweise gegeben, die vom Größenbegriff zum Funktionsbegriff von der Quantität zur Qualität als dem eigentlichen Fundament zurückleitet." (Translation: "Here is indeed the first and distinctive example for the general point of view, which leads from the notion of magnitude to the notion of function, from quantity to quality as the real fundament."). Ernst Cassirer, Substanzbegriff und Funktionsbegriff. Untersuchungen über die Grundfragen der Erkenntniskritik, Wissenschaftliche Buchgesellschaft, Darmstadt [7] 1994, p. 131.

[3] Hans-Joachim Petsche, "Ernst Abbe's reception of Grassmann in the light of Grassmann's reception of Schleiermacher", in: Hans–Joachim Petsche et al. (ed.), Hermann Graßmann. From Past to Future: Graßmann's Work in Context. Graßmann Bicentennial Conference, September 2009, Birkhäuser, Springer, Basel 2011, p. 169.

⁴ Friedrich Schleiermacher, Dialektik, Friedrich Schleiermacher's sämtliche Werke, Dritte Abteilung zur Philosophie, Vierten Bandes zweiter Teil, Friedrich Schleiermacher's literarischer Nachlass, Zur Philosophie, zweiten Bandes zweite Abtheilung, Justus Jonas (ed.) Verlag G. Reimer, Berlin 1839, p. 15, 54, 55, 62, 164, 192, 196, 242.

⁵ Albert C. Lewis, Hans Joachim Petsche, Inken Mädler und Johannes Dittmer have argued in different ways for the influence of Schleiermacher on Grassmann whereas Gert Schubring argues against such an influence.

⁶ "Die oberste Teilung aller Wissenschaften ist die in reale und formale, [...]." Hermann Grassmann, Bd. I, 1, 1894, p. 22. (Translation: "The supreme division of all sciences is the one in real and formal, [...]") "Das allgemeine Theilungsprinzip ist der doppelte Gegensatz des idealen und realen in Vernunft und Natur." Friedrich Schleiermacher, Dialektik, Justus Jonas, (ed.) 1839, § 341, p. 308. (Translation: "The general principle of division is the double opposite of the ideal and real in reason and nature.")

⁷ "Zweitens aber sagen wir uns von dem Verfahren aller derer los, welche, indem sie einen Inbegriff von Sätzen aufstellen, der das wesentliche des Wissens so enthalten soll, dass das weitere sich daraus entwickeln läßt, mögen sie ihn nun Wissenschaftslehre nennen, Logik oder Metaphysik oder Naturphilosophie oder wie sonst immer, hiebei einen sogenannten Grundsatz an die Spitze stellen als denjenigen, mit dem das Wissen notwendig anfange, [...]." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.), 1839, p. 594. (Translation: "Secondly, however, we dissociate ourselves from the procedure of all those, who, by positioning an epitome, which is to include the essence of all knowledge, so that everything further can be deduced, may it be called Wissenschaftslehre, logic or metaphysics or natural philosophy or however else, pose an axiom at the top as one, with which knowledge necessarily starts, [...]".)

⁸ This focus on the process of knowledge acquisition is highlighted in Johannes Dittmers excellent book on Schleiermacher's dialectics. Johannes M. Dittmer, Schleiermachers Wissenschaftslehre als Entwurf einer prozessualen Metaphysik in semiotischer Perspektive. Triadizität im Werden. DeGruyter, Berlin 2001.

⁹ "Die Religion lebt ihr ganzes Leben auch in der Natur, aber in der unendlichen Natur des Ganzen, des Einen und Allen." Friedrich Schleiermacher, Reden über die Religion an die Gebildeten unter ihren Verächtern, Reclam Verlag, Stuttgart 2010, p. 35.

¹⁰ "[...] was ist Individualität und Einheit? Diese Begriffe [...] habt ihr sie aus der Natur? Stammen sie nicht ursprünglich aus dem Inneren des Gemüts her und sind erst von da auf jenes gedeutet? Darum ist es auch das Gemüt eigentlich, worauf die Religion hinsieht und woher sie Anschauungen von Welt nimmt." Friedrich Schleiermacher, Reden über die Religion an die Gebildeten unter ihren Verächtern, Reclam Verlag, Stuttgart 2010, p. 59. (Translation: "[...] what is individuality, what is unity? These notions [...] did you get them from nature? Do they not stem originally from your innermost mind (= Gemüt) and are interpreted from here to there? Therefore actually it is the innermost mind, to which religion looks and from where it gains the perception (= Anschauung) of the world.")

¹¹ "Demgemäß nun haben wir auch den transzendentalen Grund nur in der relativen Identität des Denkens und Wollens, nämlich im Gefühl." (Translation: "Accordingly we now have the transcendental ground in the relative identity of rationality and will, namely in feeling."), Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.), § 215, p. 151–152.

¹² "Was in der Tat den religiösen Sinn anspricht in der äußeren Welt, das sind nicht ihre Massen, sondern ihre Gesetze. Erhebt euch zu dem Blick, [...], und dann sagt, ob ihr nicht anschaut die göttliche Einheit und die ewige Unwandelbarkeit der Welt." (Translation: "What appeals indeed to the religious mind in the outer world are not their masses but their laws. Elate yourself to this point of view [...] and then say if you are not apprehending the divine unity and the eternal immutability of the world." Friedrich Schleiermacher, Re-

den über die Religion an die Gebildeten unter ihren Verächtern, Reclam Verlag, Stuttgart 2010, p. 56.

[13] In the poetic language of Schleiermacher this reads as follows: "Nur niedere Gottheiten, dienende Jungfrauen hatten die Aufsicht in der Religion der Alten über das gleichförmig Wiederkehrende, dessen Ordnung schon gefunden war, aber die Abweichungen, die man nicht begriff, die Revolutionen, für die es keine Gesetze gab, diese waren eben das Werk des Vaters der Götter. Die Perturbationen in dem Laufe der Gestirne deuten auf eine höhere Einheit, auf eine kühnere Verbindung als die, welche wir schon aus der Regelmäßigkeit ihrer Bahnen gewahr werden, und die Anomalien, die müßigen Spiele der plastischen Natur, zwingen uns zu sehen, daß sie ihre bestimmtesten Formen mit einer Willkür, mit einer Phantasie gleichsam, behandelt, deren Regel wir nur aus einem höheren Standpunkte entdecken können." Friedrich Schleiermacher, Reden über die Religion an die Gebildeten unter ihren Verächtern, Reclam Verlag, Stuttgart 2010, p. 57. (Translation: "Only lower goddesses, serving virgins had the surveillance in the religion of the ancients over the uniformly reoccurring, for which the order was already discovered, but the perturbations, which one did not understand, the revolutions, for which no laws existed, these were the work of the father of the Gods. The perturbations in the course of the stars point to a higher unity, to a keen connection as the one, which we became aware of by the regularity of their orbits, and the anomalies, the otiose games of plastic nature, force us to see that it deals with its determined forms with capriciousness, with a fantasy so to speak, of which we can discover the rule only from a higher point of view.")

[14] "Die Einheit des Denkens und Seins in dieser Einheit ist das höchste selbst, das absolute. Diese Beziehung des Wollens auf das Denken und umgekehrt und die Einheit davon ist das göttliche in uns. Der religiöse hat es im Leben, der spekulative in der Betrachtung, aber beide haben es nur an etwas anderem." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.) 1839, § 216 p. 155.

[15] Inken Mädler has rightly pointed out that Schleiermacher focuses indeed on knowledge acquisition as an continuous *process*. He was doing so because as a highly gifted lay mathematician he knew the concept of a function, which includes both movement and change and a rule according to which the change is processed. Inken Mädler, Kirche und bildende Kunst der Moderne. Ein an F. D. E. Schleiermacher orientierter Beitrag zur theologischen Urteilsbildung, Verlag Mohr Siebeck, Tübingen 1997, S. 225–295.

[16] "Also in der absoluten Einheit des Seins ist die Vielheit aufgehoben, in der Gemeinschaftlichkeit des Seins die Einheit; aber beides ist immer in Beziehung auf einander, weil nur dadurch ein Uebergang von der einen in die andere möglich ist. Beziehen wir das auf ein Wissen im Werden: so erhalten wir die Formel Die absolute Einheit des Seins setzen wir voraus, indem wir die Gemeinschaftlichkeit des Seins hervorbringen im Denken und die Gemeinschaftlichkeit des Seins setzen wir voraus, indem wir die absolute Einheit des Seins hervorbringen im Denken." Friedrich Schleiermacher, Dialektik, Justus Jonas, (ed.), 1839, Beilage E, p. 508. Further important discussions on this subject: Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.), 1839, § 92 p. 92, § 118–119 p. 3–64, § 145–149 p. 84–86, § 166 p. 93, § 219 p. 162, § 247–256 p. 195–201, 170, 238, 289, 525.

[17] "Wenn alle Einzelheiten auf dem Gebiete des Wissens gegeben sind, aber wegen der Getheiltheit als atomistische Mannigfaltigkeit: so kann die Aufgabe keine geringere sein, als diese Mannigfaltigkeit zu einer in sich verbundenen Einheit zusammen zu fassen." Friedrich Schleiermacher, Dialektik, Justus Jonas, (ed.) 1839, § 335, p. 301. (Translation: "If all particulars are given in the field of knowledge, but because of their fraction as an atomistic manifold: so must the question be to integrate this manifold to a unity connected in itself.").

[18] "Die Vollendung des Wissens ist die erfüllte Idee der Welt, die Zusammenordnung aller fragmentarisch entstandenen Erkenntnisse in Eins. Eben diese Zusammenordnung ist die Aufgabe des architektonischen Verfahrens, [...]." Friedrich Schleiermacher, Dialektik,

Justus Jonas (ed.), § 340, p. 303. (Translation: "The completion of knowledge is the fulfilled idea of the world, the configuration of all fragmentary cognition in one. This configuration is the task of the architectonic method [...].").

[19] "Wenn ich sage, die unter die architektonische Einheit zusammengefassten Erkenntnisse sollen die übrigen repräsentieren, so heißt das, es soll in ihnen das Gesetz liegen, wonach sich jeder in die verschwiegenen aber unter ihnen mitbegriffenen Erkenntnisse mit konstruieren wird. Je vollkommener dies erreicht wird, um desto vollkommener ist das architektonische Verfahren." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.), 1839, § 340, p. 304. (Translation: "If I say the cognitions which are subsumed under the architectonic unity should represent the others, then this means, in themselves must lie a law, according to which everybody can construct the concealed but resonating cognitions. The more perfectly this is accomplished, the more perfect is the architectonic method."),

[20] "Wir haben zu suchen die Theorie der Construction und die der Combination, d.h. wir haben zu fragen, wenn ich im Denken von Einem begriffen bin wie habe ich es zu machen, daß es ein Wissen werde? Wie habe ich es zu machen, wenn ich Eins habe, um von diesem zu einem anderen zu kommen, und zwar so, daß der Übergang, die Verknüpfung, den Charakter eines Wissens annehme, und das neue und das alte Ein Wissen, eine Einheit werde." Friedrich Schleiermacher, Dialektik, Justus Jonas, (ed.), 1839, § 233, p. 178. (Translation: "We have to seek the theory of construction and the one for combination, viz. we have to ask, if I am grasped in thinking by unity, how have I to deal that it becomes knowledge? How do I have to make it, if I have one, to get from this one to another one in a way, that the Passover, the interrelation, gets the character of knowledge and that the old one knowledge, a unity.").

[21] „Die Theorie der Construction ist auf der einen Seite Theorie der Begriffsbildung und auf der anderen Theorie der Urteilsbildung, denn Begriff und Urtheil sind die Formen, unter denen allein ein Wissen möglich ist. Ich frage also, Wie muß ich es machen, ein Wissen zu produzieren, wenn ich in der Begriffsbildung, und wie, wenn ich in der Urteilsbildung mich befinde? Hier ist nun gleich zu unterscheiden das Wissen, das sich mehr darbietet, und das, welches mehr gewollt wird. [...]. Die Theorie der Construction zerfällt uns als in die der Begriffsbildung und die der Urtheilsbildung, und jede von diesen in die der Bildung auf dem Wege der Erfahrung und in die der Bildung auf dem Wege des Suchens." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.), 1839, § 233, p. 179. (Translation: "The theory of construction is on the one side a theory of concept formation and on the other a theory of conclusion, because notion and conclusion are forms under which solely knowledge is possible. I ask therefore, how have I to do it, produce a knowledge, if I am in the process of concept formation and if I am in the process of conclusion formation? Here one has to distinguish knowledge which presents itself and the one which is more pursued. [...]. The theory of construction is divided in the theory of concept formation and the theory of conclusion, and each of them in the one of construction in the form of experience and in the one of the way of seeking.").

[22] "Dies ist nur richtig, wenn jedes gegebene Wissen in sich ein verknüpftes, d.h. ein mannigfaltiges ist." Friedrich Schleiermacher, Dialektik, Justus Jonas, § 80, p. 36. (Translation: "This is only true, if every given knowledge is in itself connected, e.g. manifold.").

[23] "Wenn nun das transcendentale und formale nicht getrennt sind, sondern dasselbe: so muss auch in beiden Ideen der formale Gehalt sich verhalten wie der transcendentale, und also ist die Idee der Gottheit die Form jedes Wissens, an und für sich, die Idee der Welt aber die Verknüpfung des Wissens." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.) 1839, § 226, p. 170. (Translation: "Now if the transcendental and formal are not segregated, but the same: so must in both ideas the formal content be like the transcendental and so the idea of the divine is the form of all knowledge, for itself, the idea of the world is the interrelation of knowledge.").

[24] "Die Einsicht in die Natur des Wissens als auf die Gegenstände sich beziehend kann sich in nichts anderem aussprechen und verkörpern als in den Regeln der Verknüpfung. Denn Sein und Wissen kommen nur vor in einer Reihe von verknüpften Erscheinungen." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.) 1839, § 15, p. 7. (Translation: "The insight in the nature of knowledge as being related to objects cannot express itself other than and embody itself as in the rules of interrelation. Because being and knowledge appear only in a sequence of interrelated appearances.").

[25] "Was die systematische Form betrifft, so soll in dieser die Einheit, wie sie eine strengere ist, der Inbegriff der verknüpften Mannigfaltigkeit, ein Begriff werden, und die Vollkommenheit besteht darin, wie vollkommen der Begriff sei." Friedrich Schleiermacher, Dialektik, Justus Jonas, (ed.), 1839, § 340, p. 307. (Translation: "As far as the systematic form is concerned in this unity should, if it is severe, the connected manifold become a notion [...].").

[26] "In jedem realen Denken ist daher soviel Wissenschaft als darin ist Dialektik und Mathematik." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.), § 346, p. 309 (Translation: "In all real thinking is therefore so much science as dialectics and mathematics are pertinent to it.").

[27] "Die Idee des Wissens unter der isolierten Form des besonderen ist die Mathematik." Friedrich Schleiermacher, Dialektik, Justus Jonas, (ed.), 1839 § 344, p. 309. (Translation: "The idea of knowledge under the isolated form of the particular is mathematics.") see also: Friedrich Schleiermacher, Dialektik, Justus Jonas, (ed.), 1839, § 346, p. 310.

[28] "Die rechten Mathematiker geben auch nichts auf den Syllogismus, sondern führen alles auf die Anschauung zurück." Friedrich Schleiermacher, Dialektik, Justus Jonas, (ed.), 1839, § 329, p. 288. (Translation: "The real mathematicians disregard syllogisms, but they trace back everything to perception [German: Anschauung]).".

[29] "§ 16 Systeme höherer Stufen. Nehme ich nun, um zu den Verknüpfungen verschiedenartiger Strecken zu gelangen, zumindest zwei verschiedenartige Grundänderungen an, und lasse ein Element die erste Grundänderung (oder deren entgegengesetzte) beliebig fortsetzen und dann das so geänderte Element in der zweiten Aenderungsweise gleichfalls beliebig fortschreiten, so werde ich dadurch aus einem Element eine unendliche Menge neuer Elemente erzeugen können, und die Gesammtheit der so erzeugbaren Elemente nenne ich ein System zweiter Stufe. [...] Und da dieser Erzeugungsweise dem Begriffe nach keine Schranke gesetzt ist, so werde ich auf diese Weise zu Systemen beliebig hoher Stufen fortschreiten können." Hermann Grassmann, Bd. I, 1, 1894, p. 51–52. (Translation: "§ 16 Systems of higher levels. If I make, in order to get to the interrelations of different lines, at least two different basic changes, and let one element the first basic change (or their opposite) in any order continue, and then the thus changed element in the second mode of change likewise in any order continue, then I will be able to create an infinite amount of new elements out of one element, and the totality of the thus created elements I call a system of the second level. [...] And because this mode of creation is according to its notion limitless I will be able to proceed to systems of any high level.") Grassmann gives an analogy of what he means here with his "Stufen" by comparing it with a line, a plane and a three dimensional space. But then the analogy stops because geometry is confined to three dimensions whereas his science goes beyond that. "Und weiter kann die Geometrie nicht fortschreiten, während die abstrakte Wissenschaft keine Gränze kennt." Hermann Grassmann, Bd. I, 1, 1894, p. 53. (Translation: "And further geometry can not proceed, whereas the abstract science has no limit.").

[30] "[...] und dass es daher einen Zweig der Mathematik geben müsse, der in rein abstrakter Weise ähnliche Gesetze aus sich erzeuge, wie sie in der Geometrie an den Raum gebunden erscheinen." Hermann Grassmann, Bd. I, 1, 1894, p. 10. (Translation: ("[...] and that there must be a branch of mathematics, which in a purely abstract way generates similar theorems, how they appear in geometry to be related to space.").

[31] "Die oberste Teilung aller Wissenschaften ist die in reale und formale, von denen die ersteren das Sein, als das dem Denken selbständig gegenübertretende, im Denken abbilden, und ihre Wahrheit haben in der Uebereinstimmung des Denkens mit jenem Sein; die letzteren hingegen das durch das Denken selbst gesetzte zum Gegenstande haben, und ihre Wahrheit haben in der Uebereinstimmung der Denkprozesses unter sich." Hermann Grassmann, Bd. I, 1, 1894, p. 22. (Translation: "The most general division is the one in real and formal, from which the first map being – as the one which independently confronts to thinking – in thinking and have their truth in correspondence of thinking and being; the latter however have by thinking created entities as objects which have their truth in the consistency of the processes of thinking."). In addition: Hermann Grassmann, Bd. I, 1, 1894, p. 22.

[32] "Der Gegensatz zwischen Allgemeinem und Besonderem bedingt also die Theilung der formalen Wissenschaften in Dialektik und Mathematik. Die erstere ist eine philosophische Wissenschaft, indem sie die Einheit in allem Denken aufsucht, die Mathematik hingegen hat die entgegengesetzte Richtung, indem sie jedes Gedachte einzeln als ein Besonderes auffasst." Hermann Grassmann, Bd. I, 1, 1894, p. 23. (Translation: "The contrast between the general and the particular entails the division of the formal sciences in dialectic and mathematics. The first one is a philosophical science, by seeking unity in all of thinking; mathematics, however, has the opposite direction, insofar as it apprehends each thought as something particular.") This combination between dialectic and mathematics is also found in Schleiermacher's dialectic: "Außerdem giebt es nur noch zweierlei, was sich hiervon trennen läßt, sofern es die Idee des Wissens selbst zur Anschauung bringt, nämlich die Dialektik und Mathematik, welche beide das reale Wissen umschließen und kritisieren, [...]." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.), 1839, p. 311. (Translation: "In addition there is only a bifocal perspective, what can be separated, insofar as it brings the idea of knowledge itself to perception, namely dialectic and mathematics, which both encompass real knowledge and criticise it."). Grassmann relies even in the combination of the general with dialectic and the particular with mathematics on Schleiermacher's dialectic. Schleiermacher writes: "§ 344 Die Idee des Wissens unter der isolierten Form des allgemeinen ist die Dialektik. "§ 345 Die Idee des Wissens unter der isolierten Form des besonderen ist die Mathematik. § 346 In jedem realen Denken ist daher soviel Wissenschaft als darin ist Dialektik und Mathematik." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.), 1839, p. 309. (Translation: "§ 344 The idea of knowledge under the isolated form of the general is dialectic. "§ 345 The idea of knowledge under the isolated form of the particular is mathematics. § 346 In every real thinking is therefore so much science as there is dialectic and mathematics in it.").

[33] "Die reine Mathematik ist daher die Wissenschaft des besonderen Seins als eines durch Denken gewordenen." Hermann Grassmann, Bd. I, 1, 1894, p. 23.

[34] "Das Eigenthümliche der philosophischen Methode ist, dass sie in Gegensätzen fortschreitet, und so vom Allgemeinen zum Besonderen gelangt; die mathematische Methode hingegen schreitet von den einfachsten Begriffen zu den zusammengesetzten fort, und gewinnt so durch Verknüpfung des Besonderen neue und allgemeinere Begriffe." Hermann Grassmann, Bd. I, 1, 1894, p. 30. "[...] denn in der Philosophie ist eben die Einheit der Idee das ursprüngliche, die Besonderheit das abgeleitete, in der Mathematik hingegen ist die Besonderheit das ursprüngliche, hingegen die Idee das letzte, angestrebte; wodurch die entgegengesetzte Fortschreitung bedingt ist." Hermann Grassmann, Bd. I, 1, 1894, p. 30. Also Schleiermacher uses the word "Verknüpfung" in the context of construction of new knowledge: "§ 14 Die Regeln der Verknüpfung, wenn man sie wissenschaftlich besitzen will, sind nicht von den innersten Gründen des Wissens zu trennen. Denn um richtig zu verknüpfen als die Dinge verknüpft sind, wofür wir keine andere Bürgschaft haben als den Zusammenhang unseres Wissens mit den Dingen." Jusus Jonas 1839, p. 7. See also § 13, p. 6, § 14–15, p. 7, § 226, p. 169, p. 241, 429, p. 447f., 473, 477,

[35] Hermann Grassmann, Bd. I, 1, 1894, p. 24–25.

[36] "Die (= Ahnung) scheint dem Gebiet der reinen Wissenschaft fremd zu sein und am allermeisten dem mathematischen. Allein ohne sie ist es unmöglich, irgend eine neue Wahrheit aufzufinden; durch blinde Kombination der gewonnenen Resultate gelangt man nicht dazu; sondern, was man zu kombinieren hat und auf welche Weise, muss durch die leitende Idee bestimmt sein, und diese Idee wiederum kann, ehe sie sich durch die Wissenschaft selbst verwirklicht hat, nur in der Form der Ahnung erscheinen. Es ist daher diese Ahnung auf dem wissenschaftlichen Gebiet etwas Unentbehrliches. "Hermann Grassmann, Bd. I, 1, 1894, p. 31. (Translation: "It (= intuition) seems alien to the area of pure science and most of all in mathematics. However without it, it is impossible, to detect new truth; by blind combination of results one will not attain it; but only what one has to combine and in what way must be guided by an overarching idea, and this idea in turn can, before it is realized through science itself, appear only in the form of intuition. Therefore intuition is something indispensible.") Schleiermacher uses the same idea, even the same word (in the old fashioned form of "Ahndung"): "Es gehört also zu diesem Verfahren ein eigenes divinatorisches Talent, was aber dabei zum Grunde liegt, ist nichts anderes als die Ahndung von der Zusammengehörigkeit aller Begriffe, nur auf das Verhältnis bestimmter, gegebener Begriffe angewandt." Friedrich Schleiermacher, Dialektik, Justus Jonas (ed.), 1839, § 333 p. 298. See also p. 247. (Translation: "To this procedure belongs a certain talent of divination. But what is at the bottom of it, is nothing else than the intuition of the connectivity of all notions, with regard to the relation of particular given notions.").

[37] "Ehe wir zur Theilung der Formenlehre übergehen haben wir einen Zweig auszusondern, den man bisher mit Unrecht ihr zugerechnet hat, nämlich die Geometrie. Schon aus dem oben aufgestellten Begriffe leuchtet ein, dass die Geometrie, eben wie die Mechanik, auf ein reales Sein zurückgeht; nämlich dies ist für die Geometrie der Raum; und es ist klar, wie der Begriff des Raumes keineswegs durch das Denken erzeugt werden kann." Hermann Grassmann, Bd. I, 1, 1894, p. 23. (Translation: "Before we proceed to the division of the morphology we have to single out a branch, which was until now wrongly attributed to it, namely geometry. Already the notion proposed above elucidates that geometry likewise mechanics goes back to a real being; namely this is for geometry space; and it is evident, how the notion of space can not be constructed by through thinking.") It is interesting to note Grassman's argument against geometry as belonging to mathematics. If someone claimed geometry belonging to constructive mathematics one would have to deduce by pure thinking the necessity of three-dimensional space. "Wer das Gegentheil behaupten wollte, müsste sich der Aufgabe unterziehen, die Nothwendigkeit der drei Dimensionen des Raumes aus den reinen Denkgesetzen abzuleiten, eine Aufgabe, deren Lösung sich sogleich als unmöglich darstellt." Hermann Grassmann, Bd. I, 1, 1894, p. 23. (Translation: "He who claimed the opposite would have to undergo the task to deduce the necessity of three-dimensional space by pure thinking, a task of which the solution presents itself as impossible.") "Die Stellung der Geometrie zur Formenlehre hängt von dem Verhältnis ab, in welchem die Anschauung des Raumes zum reinen Denken steht." Hermann Grassmann, Bd. I, 1, 1894, p. 24. (Translation: "The relation of geometry to the morphology depends on the relation between the perception of space to pure thinking.").

[38] "Ich behaupte nämlich, dass die Geometrie noch immer eines wissenschaftlichen Anfangs entbehre, und dass die Grundlage für das ganze Gebäude der Geometrie bisher an einem Gebrechen leide, welches einen gänzlichen Umbau desselben nothwendig mache." Hermann Grassmann, Bd. I, 1, 1894, p. 63. (Translation: "I contend namely that geometry still lacks a scientific foundation, and that the foundation for the whole edifice of geometry suffers from an infirmity, which entails the necessity of a complete reconstruction.").

[39] "[...] sondern ich gewahrte bald, dass ich hier auf das Gebiet einer neuen Wissenschaft gelangt sei, von der die Geometrie selbst nur eine specielle Anwendung sei." Hermann Grassmann, Bd. I, 1, 1894, p. 10.

⁴⁰ Hermann Grassmann, Bd. I, 1, 1894, p. 297. See also: „Die relative Beschränktheit des Raumes wird dargestellt durch den Grundsatz: Der Raum ist ein System dritter Stufe." Hermann Grassmann, Bd. I, 1, 1894, p. 66. (Translation: "The relative restriction of space is displayed through the principle: space is a system of the third level.")

⁴¹ "Der wesentliche Vorteil, welcher durch diese Auffassung erreicht wurde, war der Form nach der, dass nun alle Grundsätze, welche Raumanschauungen ausdrückten, gänzlich wegfielen, und somit der Anfang ein ebenso unmittelbarer wurde, wie in der Arithmetik, dem Inhalte nach aber der, dass die Beschränkung auf drei Dimensionen wegfiel. Erst hierdurch traten die Gesetze in ihrer Unmittelbarkeit und Allgemeinheit ans Licht und stellten sich in ihrem wesentlichen Zusammenhange dar, und manche Gesetzmäßigkeit, die bei drei Dimensionen entweder noch gar nicht, oder nur verdeckt vorhanden war, entfaltete sich nun bei dieser Verallgemeinerung in ihrer ganzen Klarheit." Hermann Grassmann, Bd. I, 1, 1894, p. 10. Hermann Grassmann, Bd. I, 1, 1894, p. 10, also p. 297. (Translation: "The essential advantage, which was accomplished by this outlook, was that now all axioms, which convey the perception of space, ceased completely to exist. And thus the beginning became an immediate like in arithmetic, according to the content however, that the limitation to three dimensions fell away. Only hereby could the laws appear in their immediacy and generality and expose themselves in their essential coherency and some principles, which in three dimensions existed either not at all or only in a concealed way, unfolded now in this generalisation themselves in their complete lucidity.").

⁴² Hermann Grassmann, Gesammelte Mathematische und Physikalische Werke, Bd. I, 1. Friedrich Engel (ed.), Teubner, Leipzig 1894, p. 36–44.

⁴³ Hermann Grassmann, 1894, p. 44.

⁴⁴ "Hierin liegt schon das Gesetz, dass $a \times b = -b \times a$ ist. Denn wenn b von a aus betrachtet nach links liegt, so muss a von b aus betrachtet nach rechts hin liegen und umgekehrt." Hermann Grassmann, Bd. I, 1, 1894, p. 90. (Translation: "Here lies the law that $a \times b = -b \times a$. Because if b seen from a lies left, so must a seen from b lie right and vice versa.").

⁴⁵ "Wenn zwei Größen oder Formen [...] unter sich verknüpft sind, so heißen sie Glieder der Verknüpfung, die Form, welche durch die Verknüpfung beider dargestellt wird, das Ergebnis der Verknüpfung." Hermann Grassmann, Die Wissenschaft der extensiven Grösse oder Ausdehnungslehre, eine neue mathematische Disziplin, dargestellt und durch Beispiele erläutert, Verlag Otto Wigand, Leipzig 1844, § 2, p. 2. (Translation: "If two entities or forms [...] are interrelated then they are called elements of interrelation, the form which is depicted by interrelation, is the result of the interrelation.")

⁴⁶ Hermann Grassmann, Ausdehnungslehre, 1844, § 8–10, p. 8–12.

⁴⁷ "Sollte sich späterhin ein erweiterter Begriff von Addition ergeben, so würde eine solche Verknüpfung nicht eher als Addition festgestellt sein, als bis auch ihre additive Beziehung zu der bisher dargelegten Multiplikation nachgewiesen ist." Hermann Grassmann, Ausdehnungslehre, 1844, § 36, p. 60. (Translation: "If later an enlarged notion of addition is about to result, then such an interrelation would not be identified as an addition unless its additive relation to the so far exposed multiplication is proved.")

⁴⁸ "§ 47 Verknüpfung der Ausdehnungsgrössen höherer Stufen", Hermann Grassmann, Ausdehnungslehre, 1844, § 47, p. 73. (Translation: „§ 47 Interrelation of extensive entities of higher level"). "Um diese Beschränkung aufzuheben, werden wir daher den Begriff der Addition für höhere Ausdehnungsgrößen erweitern müssen. Der so erweiterte Begriff muss von der Art sein, dass er erstens bei gleichartigen Ausdehnungsgrößen in den gewöhnlichen umschlägt, und dass für ihn die Grundbeziehung der Addition zur Multiplikation gilt." Hermann Grassmann, Ausdehnungslehre, 1844, § 36, p. 60–61. (Translation: "In order to abrogate these limitations, we will have to expand the notion of addition for higher extension entities. The thus enlarged notion must be of the kind, that it in the first place

it changes into the ordinary for similar extension entities and that for it the basic relation of addition to multiplication is valid.")

[49] M.B. Green, J.H. Schwarz, E. Witten, Superstring theory, Vol., 1 Introduction, Cambridge Monographs on mathematical physics, Cambridge University Press 2002.

[50] Albert Einstein, Aus meinen späteren Jahren, Ullstein, Frankfurt [4]1993, p. 46–47.

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