DE GRUYTER OPEN

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 44 (57) 2016

DOI: 10.1515/slgr-2016-0002

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THEOLOGICAL METAPHORS IN MATHEMATICS

Abstract. Examples of possible theological influences upon the development of mathematics are indicated. The best known connection can be found in the realm of infinite sets treated by us as known or graspable, which constitutes a divine-like approach. Also the move to treat infinite processes as if they were one finished object that can be identified with its limits is routine in mathematicians, but refers to seemingly super-human power. For centuries this was seen as wrong and even today some philosophers, for example Brian Rotman, talk critically about "theological mathematics". Theological metaphors, like "God's view", are used even by contemporary mathematicians. While rarely appearing in official texts they are rather easily invoked in "the kitchen of mathematics". There exist theories developing without the assumption of actual infinity the tools of classical mathematics needed for applications (For instance, Mycielski's approach). Conclusion: mathematics could have developed in another way. Finally, several specific examples of historical situations are mentioned where, according to some authors, direct theological input into mathematics appeared: the possibility of the ritual genesis of arithmetic and geometry, the importance of the Indian religious background for the emergence of zero, the genesis of the theories of Cantor and Brouwer, the role of Name-worshipping for the research of the Moscow school of topology. Neither these examples nor the previous illustrations of theological metaphors provide a certain proof that religion or theology was directly influencing the development of mathematical ideas. They do suggest, however, common points and connections that merit further exploration.

As explained in the introduction to this volume, mathematical examples, analogies, models, methods, and even ideologies can be used or loosely invoked by theologians. In the present study we are interested in the reverse influence, or rather, the problem whether there exists an influence of theology on the development of mathematics. Theology is conceived here very broadly as any reflection of religion and religious matters, and not necessarily a rational, systematic analysis of religious concepts.

To begin the analysis of the problem one needs examples, both historical and contemporary. Below some examples are indicated. They seem

ISBN 978-83-7431-480-0 ISSN 0860-150X

to suggest the points where conceptual developments in mathematics may have been influenced by theology. This is suggested in a particularly strong manner by the terminology and metaphors used by mathematicians.[1]

My initial assumption is modest: we need to examine religious metaphors and theological terms used by mathematicians rather than begin by rejecting their relevance. My tentative conclusion is even more modest: these examples as such do not unquestionably prove by themselves that religion or theology directly influenced the development of mathematical ideas. They do suggest, however, connections that need to be explored further.

I. Managing Infinity

According to one estimate in our universe there are more than 10^{22} and perhaps more than 10^{23} stars. And on our Earth there are allegedly somewhat less than 10^{22} grains of sand. (It is easy to calculate that if the whole Earth was composed of grains of sand of the size 0.3 mm then there would be less than 10^{32} grains.) The numbers are so huge that in most contexts they are practically infinite even for us, despite the fact that we can give an estimation – as was understood already by Archimedes, and, what is even more remarkable, we can express it in a very concise way – which is great progress with respect to Archimedes. Obviously for the people whose consciousness is reflected in the Bible, the number of stars or grains of sand was completely beyond human capacity to describe, and in this sense it was infinite. That is, it was completely unmanageable, even if not necessarily infinite in the modern sense. In that meaning, indicating practical uncountability, the number of stars or grains of sand was used in the well-known images, illustrating the future of the Jewish people. Thus, in Gen. 22:17, it is said, "I will multiply thy seed as the stars of the heaven, and as the sand which is upon the sea shore". Concerning the stars, in Psalm 147 we read that God "counts (or sets) the number of stars, and gives a name to each."

מוֹנֶה מִסְפָּר לַכּוֹכָבִים˙ לְכֻלָּם שֵׁמוֹת יִקְרָא׃

The message is clear: God's mind is infinite in the sense that God can name each star, that is, treat each star as a separate individual. In other words, God can manage their multitude. For God the number is as finite or manageable as is for us the number of goats in our flock or, to make it more relevant to most of us, books in our possession. (My wife would surely object to the statement that I can manage the collection of books in my study, and she would be right, but we are attempting a purely theoretical analysis here.) We may think that this indicates God's infinite memory, either practically infinite or really infinite, or even absolutely infinite in Cantor's sense (beyond all alephs or ordinal numbers, the size of the proper class of all of them). What is more, the next verse of the Psalm mentions numbers directly again. It says *ein mispar*, there is no number to God's wisdom.

גָּדוֹל אֲדוֹנֵינוּ וְרַב-כֹּחַ לִתְבוּנָתוֹ אֵין מִסְפָּר׃

This is usually translated as: "it is infinite" or "there is no measure to His wisdom." We may think that this indicates either God's infinite ("operational") memory, again either practically infinite or really infinite, or perhaps an infinite "CPU" (processor) behind the Creator's behavior.

God can handle the infinite multitude of stars, and, at least in later theological thought, God can grasp at once the infinity of time, of all past and future moments. Thus Boethius (6^{th} century) in The *Consolations of Philosophy* says that "God abides for ever in an eternal present, ... and, embracing the whole infinite sweep of the past and of the future, contemplates all that falls within its simple cognition as if it were now taking place." (Boethius, 1897, Book V, Song VI)

The same vision was applied to the geometrical line. For example, in the 14^{th} century, Walter Burley said that "God now sees all the points that are potentially on a line". According to Calvin Normore, for Burley "God sees all the points that are potentially on a line and all the intervals that separate them." (Shapiro 2011, 103)

How does this relate to mathematics? I think it does so in a very substantial way. Up to the 18^{th} century only potential infinity was considered meaningful. For example, Leibniz believed that "even God cannot finish an infinite calculation." (Breger, 2005, 490) Since the 19^{th} century we have been using actually infinite sets, and for more than a hundred years we have been handling them without reservations. Nowadays students are convinced that this is normal and self-evident as soon as they begin their study of modern mathematics. This constitutes the unbelievable triumph of Georg Cantor. There may have been precursors of Cantor, and as early as five centuries before him there had been ideas about completing infinite additions – as documented in the paper by Zbigniew Król in the present volume – but clearly it was Cantor who opened to us the realm of actually infinite structures.

As is well known, we handle, or at least we pretend we can handle, with complete ease the following infinite sets (and many other ones): the set of (all) natural numbers, real numbers etc.; the transfinite numbers – even though the totality of all of them seems harder to master; the set of (all) points in a given space, the sets of (all) functions, etc.

It is apparent that we behave in the way described by Boethius or Burley as being proper to God. Infinite structures are everyday stuff for mathematicians. What is more, we are used to handling infinite families of infinite structures. Thus the set (class) of all models of a set of axioms is routinely taken into account as is the category of topological spaces and many other categories approached as completed entities. In addition, in mathematical logic one unhesitantly considers such involved sets as the set of all sentences true in a specific set theoretical structure or in each member of an arbitrary family of structures.

Such behavior is so familiar that no mathematician sees it as remarkable. But the fact is that this is like being omniscient. We do play the role of God or, rather, the role not so long ago deemed appropriate only for God!

From where could the idea of actual infinity in mathematics have arisen? The only other examples of talk that remind of actual infinity are religious or theological, as the just mentioned verses from the psalms indicate. This fact is suggestive but it does not constitute a proof that post-Cantorial mathematics was derived from theology. Actually, we know that Cantor was stimulated by internal mathematical problems of iterating the operation of the forming of a set of limit points and performing the "transfinite" step in order to continue the iteration. This fact leads to a more general issue of infinite processes.

II. Reification of Infinite Processes

Basic examples of actually infinite processes have been considered at least since the time of Euler. Convergent sequences and finite sums of infinite series have since become completely familiar. One of the simplest examples is provided by the equation

$$0.999\ldots = 1.$$

William Byers in his highly interesting book (2007) tries to remind us of the ambiguity involved in the equation. On the left hand side, an infinite process is indicated; on the right hand side, a specific individual object is shown, the number one. The equality sign indicates that a process is identified with an object. We are all used to it, but according to Byers this is ambiguous and difficult to grasp by students. The identification of the dynamic and static aspects of the situation constitutes a move that has permeated contemporary mathematics dealing with limits, approximations, infinite operations. Actually, a simpler identification of this kind is needed even when we deal with numbers like 2/3; they can be seen as the result of a process of a finite division into parts. The infinite process introduces, however, something more: an unrealistically long operation. Doesn't that look like usurping infinite powers by finite human beings? Isn't it an appropriation of a divine power?

Another historically important example of a reification of an infinite action is provided by the Axiom of Choice. Choosing one element from each set of an arbitrary family of (disjoint) sets must constitute a series of movements; if the family is infinite it must be an infinite series of operations. If there is a single rule according to which the choice is done then the resulting set of representatives can be defined and can be relatively safely assumed to exist. In the case of an arbitrary family of sets there is no such definition, and it is necessary to postulate the existence of the selection set. Its existence is not self-evident. The first uses of the Axiom of Choice were unconscious, but seemed natural to the advocates of unrestricted infinite mathematics. However, when the use of this axiom became understood, opposition against it arose. Among the opponents were important mathematicians, like the French "semi-intuitionists", who did handle infinite operations, but felt that some limitations were necessary. For example, in 1904 Emile Borel claimed that arbitrary long transfinite series of operations would be seen as invalid by every mathematician.[2] According to him the objection against the Axiom of Choice is justified since "every reasoning where one assumes an arbitrary choice made an uncountable number of times ... is outside the domain of mathematics".[3] Interestingly, against Borel, Hadamard saw no difference between uncountable and countable infinite series of choices. He rejected, however, an infinity of dependent choices when the choice made depends on the previous ones. (Borel 1972, 1253) All the just mentioned choice principles are considered obviously acceptable and innocent by contemporary mathematicians. The former opposition was clearly derived from the realization that an infinite number of operations is impossible. Or, it is impossible if our power is not divine.

Another familiar example of handling the result of an infinite process as if it was unproblematic is found in mathematical logic. Namely, we often consider the set of all logical consequences of a set of propositions. Of course, it is impossible to "know" all of them. It is also impossible to write down all of them – their number is infinite and most of these consequences are too long to be practically expressible – although when the initial set is recursive a program can produce the list (in a given language) if it runs infinitely long or infinitely fast. Thus, by assuming suitable idealizations we can assume that the set of all logical consequences can be seen as "given".

Many similar moves are routinely done in contemporary mathematical logic. An infinite process of deriving subsequent consequences is seen as one step. We behave as if we knew all the logical consequences. This is like being omniscient.

A theological language seems natural to express this behavior of mathematicians. The similarities are striking. Yet, as mentioned in the previous section, this fact does not prove that that religious or theological thinking directly led to the appearance of mathematical concepts. It is, however, an indication that the two areas are connected or, at least, may be connected. In each case a detailed historical study is needed to demonstrate dependence. We should not, however, ignore the possibility of religious thinking influencing mathematics.

Some recent attempts within the philosophy of mathematics indicate the need to react to the extraordinary ease with which we pretend we handle infinite processes. Among the most interesting is the work by Brian Rotman (1993) and (2000). According to him, mathematics is theological and this is wrong!

One of Rotmans's arguments recalls the turn of the 20$^{\text{th}}$ century discussions on actual infinity and the Axiom of Choice. A mathematical Subject, says Rotman in his 1988 paper "Toward a Semiotics of Mathematics" (cf. Rotman, 2000, 1–43), in order to execute an operation to which he is invited – e.g., consider a space, define a function, prove a formula – must imagine an Agent who performs an appropriate action. This imaginary agent can, unlike the Subject, perform infinite additions. Obviously, we, human beings, real physical Persons, in no way can execute an infinite number of operations in a finite time. Rotman fights Platonism, but this agenda is close to opposing theological and religious influences or traces in science. This becomes clear when the title of his (1993) book is considered: "The Ghost in Turing's Machine. Taking God Out of Mathematics and Putting the Body Back In." Rotman evokes the "disembodied Agent" again and remarks that the Agent is "as near to God as makes no difference" whether he is "a spirit, a ghost or angel." (Rotman, 1993, 10) The theological interpretation is imposed even when we accept Platonic assumptions: "the actual Platonic infinity seems to invoke some version of the Greek–Hebraic divinity" (Rotman, 1993, 49). Rotman reminds us that the greatest modern scholars, "Galileo, Descartes, Newton, Kant, Kronecker, Cantor, Husserl, Einstein, Brouwer, Gödel" spoke of God. They did not mean the same but they all referred to "the Western metaphysical dream of divine and timeless reason," and this culminated in the "unstated theism – implicit and unacknowledged – of twentieth-century mathematical infinitism..." (Rotman, 1993, 157). Notwithstanding Rotman's extreme criticism of scholars for being too religious, I believe that his basic insight is correct. Religious thinking is present in the approach of mathematicians to their subject matter. (Or rather, to follow Rotman, to their subject ghost?)

So far we have focused on the concept of infinity. When infinity is considered, theology is rather easily invoked. Or rather, Rotman style arguments aside, religious references were used when it was still far from obvious that we could act as if we could fully control infinite structures, perform infinite sequences of operations, and consider as given the results of an infinite number of acts. By now, such capacities are commonly seen as unproblematic. Even on a purely mathematical level, the opposition to "free"[4] infinitistic mathematics is virtually understandable to a contemporary university student.

Yet it would be wrong to conclude that the dominant attitudes show the objective truth about the subject matter of mathematics, and that the development toward the acceptance of actual infinities was somehow inevitable. There exist mathematical theories developing mathematics without actual infinity. The potential infinity of ever denser finite sets of reals is enough for mathematical analysis, as shown first by Jan Mycielski (1981). Most of traditional mathematics and some modern results, and above all the part of mathematical analysis "which is used or has potential applications in natural science," (Mycielski, 1981, 625) can be reproduced in this framework. The framework feels restricted from the classical point of view, but for applications appropriately dense finite approximations of the real line are sufficient. This approach recalls the ancient geometry in which only the potential extendibility of line segments was assumed. This was enough for the original Euclidean geometry. Only in the modern epoch has the existence of the whole line been assumed. This move, in which all the points of space are equally available, and their totality is grasped at once, has been called, not surprisingly, "the divine point of view." Early developments of this approach are presented in the paper by Zbigniew Król in the present volume.

The mention of Rotman's criticism and Mycielski's theory as well as the presence of the much better known story of Intuitionism support a position that can be expressed by the following sweeping thesis: Mathematics could have developed in another way, very different from what is known as our contemporary mathematics.

III. Some Other Theological Metaphors

Hilbert made the famous remark that we, mathematicians, will not agree to be purged from the "paradise" of infinite sets. "No one will drive us from the paradise which Cantor created for us." (Hilbert, 1926)[5]

The religious reference is obvious, but does it express something substantial? Most of us would say, No. The most probable guess is that he just wanted to use a nice metaphor to express the importance of the world of sets. Still, we can ask: What is the role of religious metaphors? This question is naïve, perhaps too naïve, but I believe that the ease with which mathematicians employ theological language must not be ignored.

The theological metaphors mentioned above are mostly about infinity. Let us note that when we talk about infinity we must talk in a metaphorical way. We cannot grasp it, we can only suggest, indicate, evoke, using devices like the inevitable three dots: "...". Yet we try to understand infinities, and as soon as we do this we feel that there is something paradoxical about it. It reaches beyond words and yet we talk about it; we form the concept and express infinity in finite terms. It is somewhat like the contradiction typical of mystics: their experience is ineffable and yet they talk so much about it.

God arithmetizes

There are other theological metaphors used by mathematicians. Sometimes they seem inessential, rather irrelevant to us. When people, from Pythagoras to Einstein, say something to the effect that God is a mathematician, we feel they want to say something about the nature of the physical world. On other occasions the reasons for references to God can be strictly mathematical, but a religious flavor remains. Gauss wrote that God does arithmetic, or rather "God arithmetizes," in order to oppose and replace the statement, attributed to Plato, that "God always geometrizes." (cf. Ferreirós, 2007, 235–7) The point was to show that broadly conceived arithmetic is more fundamental and absolute than the geometry favored by the ancients. Gauss began the era of the domination of arithmetic instead of geometry, of the arithmetization of mathematics, and, in our time, the digitalization of everything. (See Ferreirós (2007) and some other chapters in Goldstein, Schappacher, Schwermer (2007).) The reference to God seems to be made primarily in order to indicate the utmost importance and the absolute character of geometry for Plato and of arithmetic for Gauss. Yet the theological associations are detectable. According to Ferreirós, "the Gaussian motto implies that there is a theological explanation for the impressive applicability and effectiveness of mathematics: God's creation bears the mark of His thought, which we are able to grasp because we are of His lineage." (Ferreirós, 2007, 236)

Theological language is still used today. To be sure, nowadays theological metaphors appear only in informal speech. For example, when one wants to say why some topic is important or interesting, or what one's motivations or associations were, it can be perfectly natural to talk about God, on "God's standpoint", "God's mind", "God's language".

Occasionally traces of this way of talking can be retained in an "official" text. Thus, as mentioned before, we can talk about performing infinitely many acts (or even a huge finite number of steps that is practically inaccessible) as if we had an unlimited, "divine" mind; we can refer to a complete knowledge (for instance, taking the set of all sentences true in a given interpretation) as if we were actually omniscient. We can also refer to paradise in Hilbert's sense. This paradise was challenged by Wittgenstein who built upon the metaphor saying that rather than fear expulsion we should leave the place. "I would do something quite different: I would try to show you that it is not a paradise–so that you'll leave of your own accord." (Wittgenstein, 1976, 103)

The kitchen of mathematics

One could say that all such figurative utterances using, directly or indirectly, theological terms are irrelevant and should be ignored in reflections about the nature of mathematics; they are mere chatting, present around mathematics, but not part of it.

Yet this loose conversation does constitute a part of real mathematics, says Reuben Hersh in (1991). His argument is ingenious: let us consider seriously the fact that mathematics, like any other area of human activity, has a front and a back, a chamber and a kitchen.[6] The back is of no less importance since the product is made there. The guests or customers enter the front door but the professionals use the back door. Cooks do not show the patrons of their restaurant how the meals are prepared. The same can be said about mathematics, and for this reason its mythology reigns supreme. It includes, says Hersh, such "myths" as the unity of mathematics, its objectivity, universality, certainty (due to mathematical proofs). Hersh is not claiming that those features are false. He reminds, however, that each one has been questioned by someone who knows mathematics from the perspective of its kitchen. Real mathematics is fragmented; it relies on esthetic criteria, which are subjective; proofs can be highly incomplete, and some of them have been understood in their entirety by nobody. And it is here where the ancient or primitive references can be retained. It is deep at "the back" that we could say that only God knows the entire decimal representation of the number π. If we were to say that "at the front", we would stress it was just a joke.

In the kitchen, mathematicians borrow liberally from religious language. One telling example is the saying of Paul Erdös, the famous author of some 1500 mathematical papers (more than anyone else), according to which there exists the Book in which God has written the most elegant proofs of mathematical theorems. Erdös was very far from standard religiosity, but he reportedly said in 1985, "You don't have to believe in God, but you should believe in The Book." (Aigner & Ziegler, 2009)

Probably the most famous example of direct use of theology in mathematics can be found in the reaction, in 1888, of Paul Gordan to Hilbert's non-constructive proof of the theorem on the existence of finite bases in some spaces. Gordan said, "Das ist nicht Mathematik. Das ist Theologie." It is worth adding that later, having witnessed further accomplishments of Hilbert, he would admit that even "theology" could be useful (Reid, 1996, 34, 37).

One can easily dismiss such examples. Almost everyone would say that while the criticism of a non-constructive approach to mathematics is a serious matter, the use of theological language is just a rhetorical device and has no deeper significance. The same would be said about Hilbert's mention of "the paradise" in his lecture presenting "Hilbert's Program". However, in another classic exposition of a foundational program, Rudolf Carnap, in 1930, while talking about logicism, used the phrase "theological mathematics." According to him, Ramsey's assumption of the existence of the totality of all properties should be called "theological mathematics" in contradistinction to the "anthropological mathematics" of intuitionists; in the latter, all operations, definitions, and demonstrations must be finite. When Ramsey "speaks of the totality of properties he elevates himself above the actually knowable and definable and in certain respects reasons from the standpoint of an infinite mind which is not bound by the wretched necessity of building every structure step by step." (Benacerraf & Putnam, 1983, 50)

Carnap's statement brings us back to the issue of being omniscient, considered above in Section II. There are other examples of religious references which do not deal directly with infinity. In the 19th century, the trend arose to provide foundations for mathematics, and it turned out to be very fruitful. The very idea of the foundations of mathematics assumes the presence of an absolute solid rock on which the building of mathematics is securely built. This image has been challenged, and the vision of mathematics without foundations is now favored by many philosophers of mathematics. The question that can be asked in our context is, Whence did the idea of foundations come from? It could have come from everyday experience. However, the idea of absolute certainty has a theological flavor. In our world, in our lives, foundations are hardly absolute, unchanging, unquestionable. As soon as we hope for absolutely secure foundations we invoke a religious dimension. The metaphor of the rock on which we can firmly stand is as much common human experience as it is a Biblical image: God is called the Rock, truth means absolute reliability, etc. For instance, in chapter 32 of the Book of Deuteronomy alone God is referred to as the Rock several times, e.g.:

הַצּוּר תָּמִים (Deut 32:4), **צוּר יְשֻׁעָתוֹ** (Deut 32:15),
כִּי לֹא כְצוּרֵנוּ צוּרָם (Deut 32:31).

The parallel between the theological thinking and the foundationalist approach to mathematics as the domain of absolute certainty is richly illustrated in the papers by Vladislav Shaposhnikov in the present volume. He also suggests the possibility of a genetic link between theology and the perception of mathematics as an immutable, or “divine”, domain.

Byers (2007, 303–4) indicates the paradoxical character of the mathematical study of randomness. It is defined by the absence of a pattern while mathematics can be defined as the science of patterns. So is it a legitimate object of mathematics? This problem reminds us of that of infinity. The religious dimension can be seen, says Byers, in the fact that rituals were supposed to ensure randomness, to avoid human manipulation, so that the gods could have a channel to convey information. Of course, even if that was a correct picture, one can wonder whether religious rituals actually influenced mathematical developments.

It must be admitted that all the examples mentioned above do not offer a convincing argument for the essential presence of theological terms in mathematics. They are used as a part of vague metaphors. And, after all, all sorts of metaphors are employed by mathematicians in their “kitchen”. They evoke the cultural background, but to state that the role of the background was essential, more examples from the actual history of mathematics are needed.

IV. Some Specific Historical Examples

A certain very broad statement by Einstein concerns science in general rather than only mathematics: to create science we need “the aspiration toward truth and understanding [which] springs from the sphere of religion” (quoted by Koetsier & Bergmans, 2005, 41). This does provide a useful framework for our study, but it seems too general to be of much help in analyzing specific situations.

Examples illustrating the problem of the purported role of theology in mathematics can play their roles well only when studied in depth and when the role of broad cultural context is properly taken into account. Below, a few items are only briefly indicated.

Ritual origin of counting and geometry

The most far-reaching thesis was formulated by Abraham Seidenberg. According to him arithmetic and geometry originated from religious rituals, in which there existed such needs as counting the participants or using some regular forms like moving in a circle imitating the behavior of stars. (See (1962a), (1962b), (1978); there are more papers by him.) In addition, Seidenberg advocated the theory that mathematics was introduced in one place and then spread to other lands. He indicated the religious rituals described in the Sulva (or Sulba) Sutras in India as the probable source of Indian and hence Babylonian and Greek mathematics.

Seidenberg's claims were adopted by Van der Waerden (1983). They give the most direct link between religion and mathematics. They refer to the most basic mathematical ideas and refer to the ancient epochs. That is why it is unclear how well justified they are and if they will remain unchallenged.

Zero

The story of zero is much less old than the origins studied by Seidenberg. Zero appeared in the 5th century CE in India. It was a great idea that made possible decimal notation and then later developments in mathematics and recently in computer science. It is hard to disagree with John Barrow that "the Indian system of counting is probably the most successful intellectual innovation ever devised by human beings. ... It is the nearest thing we have to a universal language." (Barrow, 2000, 69) The main problem with the appearance of zero is, why in India? Why didn't the great Greek mathematicians invent zero? One answer is that it happened by chance, with no special reason. This explanation may be true, but is highly unsatisfactory as it leaves no possibility of understanding. Despite Kahneman's (2011) well founded warning that attempts to find causes behind chance can be misleading, it is much better to look for possible causes (while retaining a healthy criticism). The answer proposed by various authors, including Barrow (2000) and Byers (2007), refers to cultural background. And the cultural background is very strongly linked to its religious aspects.

The main idea in the explanation referring to Indian background is that their religious culture treated Nothing as Something (unlike the Greeks for whom there was no way to talk about non-being). This was expressed in the concept of *sunya*. "The sunya included such a wealth of concepts. Its literal meaning was 'empty' or 'void' but it embraced the notions of space, vacuousness, insignificance and non–being as well as worthlessness and absence." (Barrow, 2000, 63) At the same time, "the logic of the Greeks prevents them having the idea at all and it is to the Indian cultures that we must look to find thinkers who are comfortable with the idea that Nothing might be something." (Barrow, 2000, 29) And "the assimilation of Indian mathematics by the Arab world led to the literal translation of sunya into Arabic as assifr, which also means 'empty' or the 'absence of anything'." (Barrow, 2000, 72)

The religious component of both the Indian and the Greek views was essential. Western religious traditions sought to flee from nothingness, but in India, "Nothing as a state from which one might have come and to which one might return – indeed these transitions might occur many times, without beginning and without end." (Barrow, 2000, 65) If this does not seem convincing enough because, one would argue, in Greece it was philosophy that formed the general background of the development of mathematics, let us take into account the opinion of Robert Jenson: "We usually refer to the work of Greece's theologians with their own name for it, 'philosophy.' ... But this is a historical illusion; Greek philosophy was simply the theology of the historically particular Olympian-Parmenidean religion, later shared with the wider Mediterranean cultic world." (Jenson 1997, 9–10)

Georg Cantor

As already mentioned, Cantor was motivated by mathematical considerations. He had, however, or sought for, deeper reasons to believe in the reality of his infinite numbers. Deeper means theological. In an 1895 letter to Hermite, Cantor referred to St. Augustine to state that natural numbers "exist at the highest level of reality as eternal ideas in the Divine Intellect" (after Dauben, 1977, 94). Having adopted this view, it is easy to accept the existence of arbitrary infinite objects; after all, there is no reason to impose any limitations on the "divine intellect".

Cantor faced an opposition of mathematicians led by the influential Leopold Kronecker. Trying to defend his concepts Cantor wrote to one of his few allies, Magnus G. Mittag-Leffler, that his ideas about infinity were inspired by God. He also wrote that by his work he wanted to serve the Catholic Church. In correspondence with Constantin Gutberlet, Cantor claimed that his infinite numbers make God greater, not smaller. Cantor was able to gain the acceptance of Cardinal Franzelin, who was satisfied by the Cantorian distinction between absolute infinity and actual infinity.

It is apparent that Cantor not only used theological arguments to back his theory, but he also found consolation among Catholic theologians. Granting this, one can still ask whether theologians and mathematicians speak about the same object. More generally, the problem is whether Cantor, in addition to using theological notions in order to defend his pioneering work, used them in his mathematical considerations (or whether he applied mathematical concepts in theology).

The Cantorian distinction between the "usual" and the "absolute" infinity was mathematically right. It gave rise to the mathematical distinction between sets and proper classes that may not be elements of other classes. However, Cantor's theological-philosophical justification of the distinction did not stay. For him, a set, also an infinite set, could be treated as a unity (*Einheit*), or one completed whole, whereas those absolutely infinite collections could not be treated as completed unities. Yet von Neumann, Bernays, Gödel and others formulated an axiomatic theory of classes in which both sets and proper classes are treated as objects in the same way; they both can equally be "treated as unities without contradiction." Cf. the paper by Thomas-Bolduc, in the present volume, where the issue of the status of the absolutely infinite is discussed in detail.

Mysticism as a possible source of Brouwer's Intuitionism

Brouwer created mathematical intuitionism and was a mystic. The relationship between the two must not be excluded even though Brouwer seemed to deny any connection. In 1915, he wrote that neither "practical nor theoretical geometry can have anything to do with mysticism." (after van Dalen, 1999, 287) On the other hand, in a 1948 lecture *Consciousness, Philosophy, and Mathematics*, he summed up his famous picture of the mental – or, indeed, is it mystical? – origins of arithmetic, and eventually of the whole of mathematics:

> Mathematics comes into being, when the two-ity created by a move of time is divested of all quality by the subject, and when the remaining empty form of the common substratum of all two-ities, as a basic intuition of mathematics is left to an unlimited unfolding, creating new mathematical entities ... (Brouwer, 1949, 1237; or 1975, 482)

For more data see Brouwer (1975), van Dalen (1999).

Name worshipping – Imiaslavie

The story of a group of Russian mathematicians of the early 20th century is told in detail by Loren Graham and Jean-Michel Kantor (2009). Summaries can be found in Graham (2011) and Kantor (2011). The mathematicians who established the Moscow school of mathematics, Dimitri Egorov, Nikolai Luzin, and Pavel Florensky (who was also a priest), unlike their French colleagues, were not afraid of infinities and contributed in a decisive way to the creation of descriptive set theory. The historians' claim is that they were motivated by their religious beliefs and practice. Specifically, they were involved in a heretical (to the Russian Orthodox Church) religious movement called "Imiaslavie" or "Name Worshipping." Name Worshippers believed in the divine power contained in the *name* of God. They repeated prayers, especially the so-called Jesus Prayer, believing that the repetition of Jesus' name brings divine presence, makes God, in some sense, present. The connection of this practice to mathematics is supposedly to be seen in the fact that objects like transfinite numbers exist "just from being named." Naming a certain infinite set using appropriate logical formula makes sure that the set exists.

Although to a modern skeptic there is hardly a special connection between those theological views and mathematics, the fact is that Luzin, Egorov, and some others saw the connection. In addition, a somewhat similar view was later expressed by another mathematical genius, Alexander Grothendieck; he stressed the importance of naming things in order to isolate the right entities from the complex scene of mathematical objects and "keep them in mind". "Grothendieck, like Luzin, placed a heavy emphasis on 'naming,' seeing it as a way to grasp objects even before they have been understood." (Graham & Kantor, 2009, 200)

The emergence of the cumulative hierarchy of sets

A well-known foundational approach to mathematics uncovers the role of theological categories: the void and infinite power.

In standard set theory zero is identified with the empty set, and then 1 is defined as $\{0\}$, 2 as $\{0, \{0\}\}$, and, in general, $n + 1$ as $\{0, 1, 2, \ldots, n\}$. This construction, introduced by John von Neumann, is the most convenient one, but not the only way to define natural numbers as sets. Other numbers – integers, rationals, reals, complex numbers – can be easily defined. Actually, in a similar way all mathematical entities investigated in traditional mathematics – functions, structures, spaces, operators, etc. – can be defined as "pure" sets, that is, sets constructed from the empty set. The construction must be performed in a transfinite way. Note that the universe of pure sets arises via a transfinite induction, indexed by ordinal numbers.

In other words, from zero we can create "everything," or rather the universe of sets sufficient for the foundations of mathematics. The construction assumes the reality of the infinity of ordinal numbers, which means that in order to create from zero we need infinite power. Nothing, emptiness, is combined with infinite power and a kind of unrestricted will to continue the construction *ad infinitum*. Together they give rise to the realm of sets where mathematics can be developed. This is a rather normal way of describing the situation. Mathematicians would reject suggestions that this has something to do with theology. Yet terms like "infinite power," "all-powerful will" are unmistakably theological. If Leibniz had known modern set theory, he would have rejoiced, both as a theologian and as a mathematician. He claimed that "all creatures derive from God and nothing." (Breger, 2005, 491) When he introduced the binary notation, he gave theological significance to zero and one: "It is true that as the empty voids and the dismal wilderness belong to zero, so the spirit of God and His light belong to the all-powerful One." (after Barrow, 2000, 33)

All the examples, suggestions, and speculations mentioned in this paper are hardly indisputable proof that theology gave rise to some mathematical developments. They provide, however, a field where one can explore connections between the two domains, and the theological influences and stimulae extending to living mathematics.

NOTES

[1] Some considerations contained in this paper were included in my Polish book (2011).

[2] In his note in Mathematische Annalen 59, 1904, 514–516, he said, "aucun mathematician ne regardera comme valable ce dernier raisonnement." See Borel, 1972, 1252.

[3] "en dehors du domaine des mathematiques." (Borel, 1972, 1252).

[4] Mathematics is a tool and backbone of physics, but on the other hand mathematics is autonomous, detached from physics and other applications, indeed it is close, in some sense, to art. It is developing freely. Georg Cantor emphasized that the essence of pure mathematics is its freedom, "Das *Wesen* der *Mathematik* Liege gerade in ihrer *Freiheit*." (Uber unendliche, lineare Punktmannigfaltigkeiten, *Math. Annalen* 21 (1883), 564) Cantor, it seems, meant here an additional point – the central role of consistency. The formula about the "freedom" of mathematics was to defend the significance of his pioneering work on actual infinity which before his work had been seen by mathematicians as devoid of sense. He was right at least in the sense that his theory of infinite sets is consistent and compatible with the existing mathematics.

[5] "Aus dem Paradies, das Cantor uns geschaffen, soll uns niemand vertreiben können." (Hilbert, 1926, 170)

[6] Hersh employs concepts used by Erving Goffman in *The Presentation of Self in Everyday Life*.

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