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ON LANGUAGE ADEQUACY[1]

Abstract. The paper concentrates on the problem of adequate reflection of fragments of reality *via* expressions of language and inter-subjective knowledge about these fragments, called here, in brief, ***language adequacy***. This problem is formulated in several aspects, the most general one being: *the compatibility of the language syntax with its bi-level semantics*: *intensional and extensional*. In this paper, various aspects of language adequacy find their logical explication on the ground of the formal-logical theory of syntax $\boldsymbol{T}$ of any categorial language L generated by the so-called classical categorial grammar, and also on the ground of its extension to the bi-level, *intensional* and *extensional* semantic-pragmatic theory $\boldsymbol{ST}$ for L. In $\boldsymbol{T}$, according to the *token-type* distinction of Ch. S. Peirce, L is characterized first as a language of well-formed expression-*tokens* (*wfe-tokens*) – material, concrete objects – and then as a language of *wfe-types* – abstract objects, classes of *wfe-tokens*. In $\boldsymbol{ST}$ the semantic-pragmatic notions of meaning and interpretation for *wfe-types* of L of *intensional semantics* and the notion of denotation of *extensional semantics* for *wfe-types* and constituents of knowledge are formalized. These notions allow formulating a postulate (an axiom of categorial adequacy) from which follow all the most important conditions of the language adequacy, including the above, and a structural one connected with three principles of compositionality.

Keywords: *token-type* distinction, categorial grammar, intensional semantics, meaning, interpretation, constituent of knowledge, extensional semantics, referring, ontological object, denotation, categorization, compatibility of syntax and semantics, algebraic models, truth, compositionality, communication.

1. Introduction

In the process of cognizing reality, we acquire knowledge about it, gathering knowledge in a certain system and representing it in some sign system, usually a language-based one (see Diagram 1). In the language system of representation, this knowledge is processed, leading to a new knowledge about the reality of interest to us, thus to a better cognition of it.

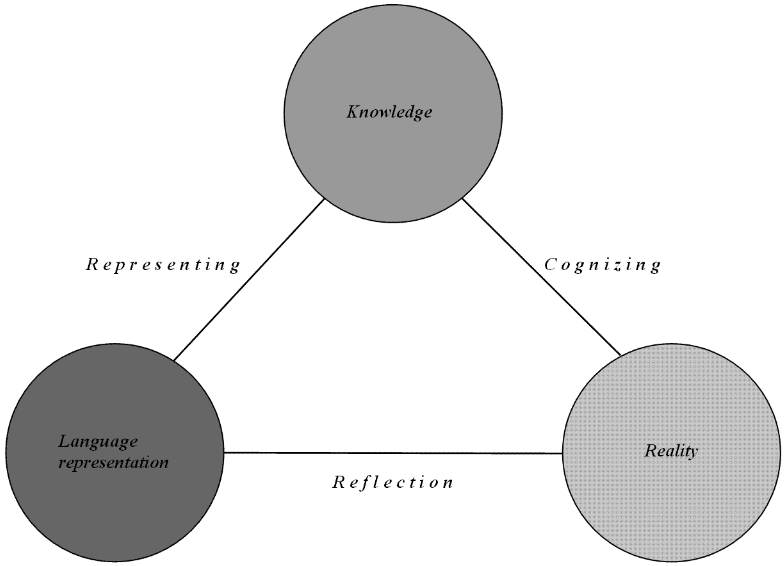

Diagram 1. Representation of knowledge

The effectiveness of cognition is dependent on mutual relations between the three elements of the triad:

Language – Knowledge – Reality.

This is obtained when the syntax of language reflects, in an adequate manner, its semantics, and thus the suitable fragment of the cognized reality, as well as the knowledge being the result of inter-subjective cognition.

2. The problem area of language adequacy

The problem of language adequacy in relation to cognition is, beside that of adequacy of cognition, one of the central, traditional philosophical problems. The question of adequate reflection of fragments of reality *via* expressions of language and inter-subjective knowledge about these fragments is called here, in brief, ***language adequacy***. This problem can be formulated in several aspects, the most general one being: *the compatibility of the language syntax with its bi-level semantics*:

intensional semantics,

in which to expressions of language correspond – as constituents of knowledge – their meanings (*intensions*), and

extensional semantics,

in which to these expressions correspond – as ontological objects of reality – their object references (*references*) and denotations (*extensions*).

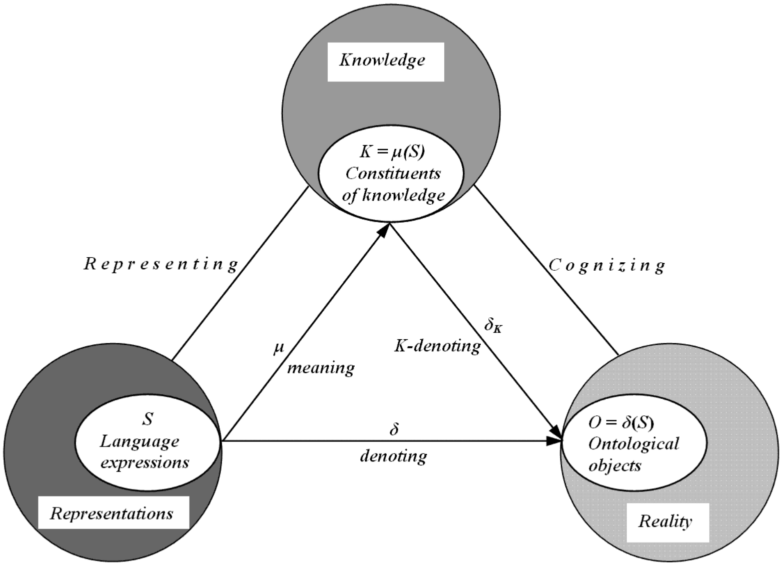

Diagram 2. Semantic adequacy

The problem area of *language adequacy* (discussed in Section 4 of this paper) will be considered formally on the ground of the logical theory of syntax ***T*** (outlined in Section 3.1 of this paper) and its extension to the semantic theory ***ST*** (characterized in Section 3.2 of this paper), describing the bi-level semantics of categorial language. The theories ***T*** and ***ST*** are presented in the author's papers (1985, 1989, 1991, 1998, 2005–2009) and are built in the spirit of Leśniewski's (1929, 1930) and Ajdukiewicz's (1935, 1960) theories of syntactic (semantic) categories, with simultaneous retention of Frege's ontological canons (1879).[2]

In the theory of syntax ***T***, the notion of a *well-formed expression* (meaningful) and that of the *syntactic category* are defined. In the semantic theory ***ST*** – with reference to Frege's (1892) distinction: *Sinn–Bedeutung*, or Carnap's (1947): *intension–extension* – such notions as: *meaning* (*intension*) of a meaningful expression, its *interpretation*, its *object reference* (*reference*), as well as *denotation* (*extension*) are defined, and also two notions of *semantic category*: the notion of *intensional category* and that of *extensional category* are introduced.[3]

The *meanings* (*intensions*) of rational expressions are treated as certain constituents of inter-subjective knowledge: logical notions, logical judgments, operations on such judgments or on such notions, on the former and the latter, on other operations.

Object references (*references*) of language expressions, and also constituents of knowledge, are objects of the cognized reality: individuals, states of things, operations on the indicated objects, and the like. *Denotations* (*extensions*) of meaningful expressions of language and constituents of knowledge are sets of such objects. ***Semantic adequacy*** – the agreement of these denotations – is illustrated in Diagram 2.

Semantic adequacy is one of the aspects of language adequacy, taking into account the bi-level semantic.

3. An outline of the theory of categorial language

In this paper, various aspects of language adequacy find their logical explication on the ground of the formal-logical theory ***T*** of any categorial language, describing its syntax, and also on the ground of its extension to the theory ***ST***, describing the bi-level semantics (*intensional* and *extensional*) for such a language. The theories ***S*** and ***ST*** are based on first order predicate logic and set theory.

Let *L* be any, yet – in our consideration – an established language characterized *categorially*. The language *L* is defined when the set ***S'*** of all its *well-formed expressions*, and its subset ***S*** *of meaningful expressions*, is determined, satisfying the requirements of *categorial syntax* and *categorial semantics.*

3.1. Categorial syntax – Theory *T*

3.1.1. General characteristics of the *categorial language*

The theory ***T*** of the syntax of the language *L* is built on the basis of Husserl's idea of pure grammar (1900–1901) and in accordance with the general assumptions of Leśniewski's (1929, 1930) and Ajdukiewicz's (1935, 1960) theories of syntactic (semantic) categories. The language *L*, syntactically characterized in it, can be precisely defined as a *categorial language*; that is, as a language all of whose *well-formed expressions* of the set ***S'*** (briefly *wfes* of ***S'***) are generated by a *categorial grammar*, the idea of which originated from Ajdukiewicz (1935, 1960) and which has already had a long history (see Bar-Hillel, 1950, 1953, 1964; Lambek, 1958, 1961; Hiż, 1960, 1961, 1967; Montague, 1970a, 1974; Geach, 1970; Cresswell, 1973, 1977; Gamut, 1991; Marciszewski, 1988; Buszkowski, 1988, 1989, 1994, 2003; van Benthem, 1986, 1988; Simons, 1989, 2006; Tałasiewicz, 2010; Duži, Jespersen & Materna, 2010; Wybraniec-Skardowska, 1985, 1991, 2006; Wybraniec-Skardowska & Rogalski, 1998). On the basis of the theory ***T***, it is possible to reconstruct the classical *categorial grammar*.

A characteristic feature of the categorial language *L*, generated by the classical categorial grammar, is that each *wfe* of the set ***S'*** has a *functor-argument* structure, that it is possible to distinguish in it the main part – the so-called *main functor*, and the other parts – called *arguments* of this functor, yet each constituent of a meaningful expression of ***S*** has a determined *syntactic category* and *semantic categories* (*extensional* and *intensional*), can have a *meaning* assigned to it, and thus also a *category of knowledge* (the *category of constituents of knowledge*), and also *denotation*, and thus – an *ontological category* (the *category of ontological objects*).

The syntactic categories of *wfes* of *L*, and also the indicated categories corresponding to them, are determined by attributing to them *categorial* indices (*types*) which were introduced by Ajdukiewicz (1935) into logical semiotics with the aim of determining the syntactic role of expressions and of examining their *syntactic* connection, in compliance with the *principle of syntactic connection* (*Sc*), which will be discussed below.

The categorial indices are, however, useful not only while establishing and examining *syntactic connection* of *wfes* of *L*. They appear simultaneously in the role of a tool coordinating meaningful expressions and metalanguage objects (see Suszko, 1958, 1960, 1964; Ajdukiewicz, 1960; Stanosz & Nowaczyk, 1976); they also serve to describe ***categorial adequacy*** – a main aspect of language adequacy.

The principle of syntactic connection (*Sc*), which makes reference to the principle applied by Ajdukiewicz, can be formulated freely in the following way:

(*Sc*) *If e is a functor-argument expression of the language L, f is the main functor of the expression e, and* $e_1, e_2, \ldots, e_n$ $(n \geq 1)$ *are subsequent arguments of the functor f, then if a is a categorial index of the expression e, while* $a_1, a_2, \ldots a_n$ *are categorial indices of subsequent arguments of the functor f, then the categorial index of the functor f is formed out of the index a of the expression e, which the functor forms, as well as out of the subsequent indexes* $a_1, a_2, \ldots, a_n$ *arguments of this functor.*

In the *quasi*-fractional notation applied by Ajdukiewicz, the index of the functor f is the following fraction:

$$a/a_1a_2\ldots a_n.$$

And thus, for example, the expression:

Warsaw is the capital of Poland,

in which '*is*' is distinguished as its main functor, with the categorial index s assigned to sentences, satisfies the principle (Sc), since the functor *'is'*, with the subsequent arguments which are the names *'Warsaw'* and *'the capital of Poland'*, and the categorial indices n and n, as a sentence-forming functor with arguments being names, has the categorial index s/nn, formed out of the index s and the indices of its subsequent arguments.

In the formal definition of a *wfe*, it is required that each complex functor-argument constituent of the given expression should satisfy the principle (Sc). As regards our instance of the sentence, this principle must also be satisfied by the expression *'the capital of Poland'*.

The set ***S'*** of all *wfes* of L is defined in the axiomatic theory ***T*** of categorial syntax, with the help of primitive notions of this theory.

3.1.2. Two levels of formalization of categorial syntax

Formalization of the theory ***T*** runs on two levels. In accordance with the distinction by Peirce (1931–1935): *token–type* of signs,[4] the double ontological nature of signs of the language L is taken into account in it.

On the ground of the theory ***T***, the language L is syntactically characterized as:

– a language of expression-*tokens* – on the first level, the *level of tokens* and

– a language of expression-*types* – on the other level, the *level of types*.[5]

Tokens of the signs of L are a starting point in formalization of the theory ***T***. They are intuitively understood as concrete, material, empirical, spanning over time and space, objects perceived through senses. Usually, though not necessarily, they are graphical signs. They can appear on paper, on a school blackboard, on computer screens. They can be illuminations of light on advertising billboards, smoke signals, arrangements of objects, e.g., configurations of stars, compositions of flowers, stones, and the like. The method of conceptualization, which leads to formalization of knowledge about language within an independently fixed temporal range of considerations and a freely-established area of language-based communication, allows isolating (extracting) a set-*universe* of sign-*tokens* which are used in this communication.

Types of the signs of the language L are its secondary objects. In the theory $\boldsymbol{T}$ they are defined by means of *tokens* of a determined *universe*. They are abstract objects, whose concrete realizations are *tokens*. The *types* are understood as set-theoretical sets, classes of *tokens* remaining in a broadly-understood *indentifiability relation* between one another (defined, obviously, on the given *universe*). The notion of *identifiability* is the result of the conceptualization process (*notioning*) of knowledge, with the same manner of use of sign-*tokens* (making use of these signs) in a selected fragment of the system of communication between human beings.

3.1.3. The foundations of the formal theory $\boldsymbol{T}$ – the *level of tokens*

The theory $\boldsymbol{T}$ built on the *level of tokens* is an axiomatic theory, including the ***concretistic*** categorial characteristics of the language L. Its primitive notions on the *level of tokens* are:

- the universe U of all sign-*tokens* of L,
- the binary relation $\sim$ of *identifiability of tokens* of the set U,
- the ternary relation c of concatenation, defined on *tokens* of the set U,
- the initial vocabulary V_0^1 of L,
- the auxiliary initial vocabulary V_0^2 for L, containing a set of categorial indices,
- the binary relation i of indicating indices to word-*tokens* of L,
- the binary relation r_1 of forming functor-argument expression-*tokens* of L,
- the binary relation r_2 of forming indices of functor-*tokens* of L.

The system of axioms which characterize the primitive notions of the theory $\boldsymbol{T}$ are given in the author's works (1985, 1989, 1991, 2006). It is postulated about the *universum* U of sign-*tokens* of L that it is a non-empty set, about the relation $\sim$ of *identifiability* – that it is an equivalence relation in the universe U. It is not assumed about the *concatenation* relation c that it is a function: a concatenation of two *tokens* is a complex *token*, formed out of two tokens *identifiable* with them, respectively, and also each token *identifiable* with it. For example, the *concatenation* of two word-*tokens*:

semiotics

l o g i c a l

the right and the left ones, of different fonts, thickness and size of type, is both:
the complex word-*token*:

Logical Semiotics

and the word-*token*:

LOGICAL SEMIOTICS

and also each word-*token identifiable* with the two complex words.

As regards the initial vocabularies V_0^1 and V_0^2 of L, it is postulated that they are non-empty subsets of the *universe* U, out of which the set W^1 of all word-*tokens* of L and the set W^2 of all auxiliary word-*tokens* for L are formed, respectively. The initial vocabularies may contain structural symbols, e.g., brackets or punctuation marks.

Sets of word-*tokens* W^1 and W^2 are defined as set-theoretical intersections of all sets including, respectively, the vocabulary V_0^1 and the auxiliary vocabulary V_0^2, which are closed with respect to the concatenation relation c.

The relation i of indicating the indices of word-*tokens* of L (in short: the *indexation* or *typification* relation) is defined on the subset of the Cartesian product $W^1 \times W^2$:

$$i \subseteq W^1 \times W^2.$$

Its left domain is a set of word-*tokens* possessing categorial indices (*types)*, the right one – the set I of indices of such words.[6] This relation is not a function – however, to a word-*token* there corresponds, with the accuracy to *identifiability*, one categorial index of the set I.

We read the expression $i(w, a)$: *a is a categorial index* (*type*) *of the word-token w*.

The proper vocabulary V^1 of L is defined as a set of word-*tokens* of the initial vocabulary V_0^1 possessing a categorial index (*type*), whereas the proper vocabulary V^2 auxiliary to L – as a set of auxiliary word-*tokens* of the vocabulary V_0^2, being indices of words of the vocabulary V^1.

The left domains of the relations r_1 and r_2 are, respectively, a set of finite tuples of word-*tokens* of the set W^1 possessing indices from the set I and a set of finite tuples of indices of such words. The relations r_1 and r_2 are not functions, but assign to any finite tuple of word-*tokens* possessing indexes, or, respectively, to any tuple of indices of word-*tokens*, with the accurate to *identifiability*, one complex word-*token* called *functor-argument expression-token*, or, respectively, one *index of the functor*.

We read the expression

$$(e) \qquad r_1(f, e_1, e_2, \ldots, e_n;\ e)$$

as follows: e is a functor-argument expression-*token* composed of the main functor f and its subsequent arguments $e_1, e_2, \ldots, e_n$.

The expression

$$(i) \qquad r_2(a, a_1, a_2, \ldots, a_n;\ a_f)$$

is read: a_f is an index of the functor f, formed out of the index a and subsequent indexes $a_1, a_2, \ldots, a_n$.

The expression e in (e) can be treated as a schema representing any expression-*tokens* of L, formed from the functor f and its subsequent arguments $e_1, e_2, \ldots, e_n$, irrespective of the concrete rules of the syntax of L, independent of the position which these constituents take in the expression e, and independent of the applied notation, type, etc.

Similarly, the expression a_f in (i) replaces any index of the functor formed from the index a and indices $a_1, a_2, \ldots, a_n$, irrespective of the applied notation of the functor indices, e.g., *quasi*-fractional, or with the use of brackets, or still any other, applied by researchers of categorial grammars.

The set E^1_{f-a} of all the functor-argument expression-*tokens* of the language L (complex expressions of L) is defined as the right domain of the relation r_1, and the set E^2_{f-a} of all the indices of functors (complex indices) – as the right domain of the relation r_2, contained in the set I of index-*tokens*.

The set E^1 of all the expression-*tokens* of L and the set E^2 of all their index-*tokens* are defined, for $k = 1, 2$ as the following sets:

$$E^k = V^k \cup E^k_{f-a}.$$

In the theory T, the principle (Sc) of syntactic connection for the functor-argument expression e, satisfying the formula (e), is formalized by means of the formula:

$$(Sc_e) \qquad \forall_{1 \le j \le n}(i(f, a_f) \wedge i(e_j, a_j) \wedge i(e, a)) \Rightarrow (i).$$

In accordance with axioms of the theory $\boldsymbol{T}$, for the expression e satisfying the formula (e) we obtain the following rule corresponding to that of cancelation of indices, applied by Ajdukiewicz (1935) to examine the syntactic connection of expressions:

$$\forall_{1 \le j \le n}((i)) \wedge i(f, a_f) \wedge i(e_j, a_j)) \Rightarrow i(e, a).$$

In the notation applied by Ajdukiewicz to this formal rule there corresponds the following rule of cancelation indices (*types*):

$$a/a_1 a_2 \ldots a_n\ (a_1, a_2, \ldots, a_n)) \rightarrow a.$$

In our given example of the expression:

Warsaw is the capital of Poland

and checking whether it is a sentence, the rule takes the form:

$$s/n\ n\ (n, n) \to s.$$

A reconstruction of the classical categorial grammar on the ground of the theory $\boldsymbol{T}$ is the system of notions:

$$\Gamma = \langle U, c, \sim, V^1, V^2, i, r_1, r_2, (Sc)\rangle,$$

generating the set S' of all *wfe-tokens* of L. The set S' is defined as follows:

DEFINITION 1 (*the set of all well-formed expression-tokens*)

$$S' = \bigcap\{X \subseteq E^1 : V^1 \subseteq X \wedge \forall e \forall f, e_1, e_2, \ldots, e_n \in X(e) \wedge (Sc_e) \Rightarrow e \in X\}.$$

The set S' is, thus, the smallest set of expression-*tokens* containing the vocabulary V^1 of the language L and each of its functor-argument expression e such that, providing the structure (e) is preserved, satisfies the principle of syntactic connection (Sc_e).

Each *wfe-token* of S' possesses a categorial index which determines its *syntactic category*. On the *level of tokens*, the syntactic categories of *wfe-tokens* are determined by categorial indices of the set I and are defined as sets of *wfes* possessing, with the exactitude to *identifiability*, the same categorial index.

DEFINITION 2 (*syntactic category with the index* ξ)

$$SC_\xi = \{e \in S' : i(e, a) \Rightarrow a \sim \xi\}.$$

It is assumed that the set S' is a sum of the set B of *basic expressions* of L (with simple indices (*types*) of the auxiliary vocabulary V^2) and the set of *functors* F (with complex indices of the set E^2_{f-a}.

The basic expressions of categorial languages are usually sentences and names. The category of sentences is typically indicated by means of the index s, and the category of names by means of the index n. Complex indices which are assigned to functors are formed from these indices. And so, for instance, the index s/nn is attributed to sentence-forming functors of two nominal arguments (thus, in particular, the functor '*is*' in the sentence:

Warsaw is the capital of Poland); on the other hand, the index n/n – to name-forming functors of one nominal argument (thus, in particular, the functor '*the capital of*' in the name '*the capital of Poland*').

The semiotic-logical characteristics of L on the *level of tokens* is insufficient. *Tokens* of expressions indeed appear in the practice of human communication, in acts of language-based communication; nevertheless, in order to explain the very notion of language communication itself in logical pragmatics, it is necessary to have expression-*types*, and in logical semantics expression-*types* serve to define the notions of *meaning* and *denotation* of language expressions, in logical syntax – to describe grammatical rules.

3.1.4. Foundations of the formal theory $\boldsymbol{T}$ – the *level of types*

Each set of *tokens Set*, introduced into formalization of the theory $\boldsymbol{T}$ on the *level of tokens*, has – in the theory $\boldsymbol{T}$ on the *level of types* – its dual counterpart ***Set***, being a quotient family of equivalent classes of the $\sim$ *identifiability* relation, with representatives from the set *Set*. Thus:

$$\boldsymbol{Set} = Set/{\sim} = \{C : \exists e \in Set(C = [e]_{\sim}\}.$$

Each relation r, introduced into the theory $\boldsymbol{T}$ on the *level of tokens* and defined on the *tokens*, has – in the theory $\boldsymbol{T}$ on the *level of types* – its dual counterpart $\boldsymbol{r}$, determined on *types* and defined in the following way:

$$\boldsymbol{r}(\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n) \Leftrightarrow \exists e_1, e_2, \ldots, e_n$$
$$(\boldsymbol{e}_1 = [e_1]_{\sim} \wedge \boldsymbol{e}_2 = [e_2]_{\sim} \wedge \ldots \wedge \boldsymbol{e}_n = [e_n]_{\sim} \wedge r(e_1, e_2, \ldots, e_n)), \quad n > 1.$$

We will give some characteristics of the theory $\boldsymbol{T}$ on the *level of types*. Let us note that on the *level of types*

– to the relation of *identifiability* $\sim$, determined on *tokens*, there corresponds the relation $=$ of equality of *types* represented by these *tokens*,[7]
– to all the other relations of the *level of tokens*, on the level of *types*, there correspond relevant relations on *types*, being set-theoretical functions;
– all the dual counterparts of axioms, definitions and theorems of the theory $\boldsymbol{T}$, binding on the *level of tokens*, are theorems of the theory $\boldsymbol{T}$ on the *level of types*;
– the categorial language L on the *level of types* is characterized by categorial grammar

$$\Gamma = \langle \boldsymbol{U}, \boldsymbol{c}, \boldsymbol{V}^1, \boldsymbol{V}^2, \boldsymbol{i}, \boldsymbol{r}_1, \boldsymbol{r}_2, (\boldsymbol{Sc})\rangle,$$

the notions of which are sets of the *types* $\boldsymbol{U}$, $\boldsymbol{V}^1$, $\boldsymbol{V}^2$ and relation-functions $\boldsymbol{i}$, $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ determined for *types*,

- the principle ($\boldsymbol{Sc}$) of syntactic connection for functor-argument expression-*types* is defined in a way similar to that for principle (Sc) for expression-*tokens*;
- the set $\boldsymbol{S'}$ of all *wfe-types* (the set of equivalence classes, of *identifiable wfe–tokens* of the set S') is generated by grammar Γ.
- the functor-argument expression-*type* $\boldsymbol{e}$ satisfying the formula:

$$(\boldsymbol{e}) \qquad \boldsymbol{r}_1(\boldsymbol{f}, \boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n; \boldsymbol{e}),$$

  and thus built from *types*: the main functor $\boldsymbol{f}$ and its arguments $\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n$, can be written in the function-argument form:

$$(\boldsymbol{e_f}) \qquad \boldsymbol{e} = \boldsymbol{f}(\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n),$$

  because each functor $\boldsymbol{f}$ can be treated as a set-theoretical function determined on finite tuples of word-*types* of the set $\boldsymbol{W}^1$, possessing categorial index-*types*, and taking values in this set (precisely in its subset $\boldsymbol{E}^1_{f-a}$);
- If the expression-*type* $\boldsymbol{e}$, having the form ($\boldsymbol{e_f}$), is a *wfe-type* (belongs to the set $\boldsymbol{S'}$), then in compliance with the principle of syntactic connection ($\boldsymbol{Sc}_e$) the index of its main functor $\boldsymbol{f}$, formed out of the index $\boldsymbol{a}$ of the expression $\boldsymbol{e}$ and of the subsequent indices $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_n$ of the subsequent arguments $\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n$ of the functor $\boldsymbol{f}$, can be written in the quasi-fractional form:

$$(\boldsymbol{i_f}) \qquad \boldsymbol{i}(\boldsymbol{f}) = \boldsymbol{i}(\boldsymbol{e})/\boldsymbol{i}(\boldsymbol{e}_1)\boldsymbol{i}(\boldsymbol{e}_2)\ldots\boldsymbol{i}(\boldsymbol{e}_n) = \boldsymbol{a}/\boldsymbol{a}_1\boldsymbol{a}_2\ldots\boldsymbol{a}_n.$$

- Syntactic categories of expression-*types* of the set $\boldsymbol{S'}$ are determined by index-*types* and by the indexation function $\boldsymbol{i}$ restricted to the set $\boldsymbol{S'}$ – the function $\boldsymbol{i_S}$:

$$\boldsymbol{SC}_\xi = \{\boldsymbol{e} \in \boldsymbol{S'} : \boldsymbol{i_S}(\boldsymbol{e}) = \xi\}.$$

The ***syntactic category with the index*** ξ is a set of all *wfe-types* which have the categorial index ξ.

- If $\boldsymbol{e}$ is a complex *wfe-types* of the set $\boldsymbol{S'}$, formed from the main functor $\boldsymbol{f}$ and its arguments $\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n$, satisfying the formula ($\boldsymbol{i_f}$), then the functor $\boldsymbol{f}$ and its index $\boldsymbol{i_S}(\boldsymbol{f})$ can be treated as set-theoretical functions which satisfy the equivalence:

  ($R1$) $\quad \boldsymbol{f} \in \boldsymbol{SCa}/\boldsymbol{a}_1\boldsymbol{a}_2\ldots\boldsymbol{a}_n$ if and only if

$(\boldsymbol{f})$ $\quad \boldsymbol{f}\colon \boldsymbol{SCa}_1 \times \boldsymbol{SCa}_2 \times \ldots \times \boldsymbol{SCa}_n \to \boldsymbol{SCa} \wedge \boldsymbol{f}(\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n) = \boldsymbol{e} \;\wedge$

$(\boldsymbol{i})$ $\quad \boldsymbol{i_S}(\boldsymbol{f}) : \{\boldsymbol{i_S}(\boldsymbol{e}_1)\} \times \{\boldsymbol{i_S}(\boldsymbol{e}_2)\} \times \ldots \times \{\boldsymbol{i_S}(\boldsymbol{e}_n)\} \to \{\boldsymbol{i_S}(\boldsymbol{e})\} \;\wedge$

(PCS) $\quad \boldsymbol{i_S}(\boldsymbol{e}) = \boldsymbol{i_S}(\boldsymbol{f}(\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n)) = \boldsymbol{i_S}(\boldsymbol{f})(\boldsymbol{i_S}(\boldsymbol{e}_1), \boldsymbol{i_S}(\boldsymbol{e}_2), \ldots, \boldsymbol{i_S}(\boldsymbol{e}_n)).$

We call the condition (PCS) the ***principle of syntactic compositionality***. Loosely speaking, this principle says that:

The syntactic category (categorial index) of the well-formed functor-argument expression-types $\boldsymbol{e}$ *of L is a function of syntactic categories (categorial indices) of arguments of its main functor* $\boldsymbol{f}$; this function is $\boldsymbol{i_S}(\boldsymbol{f})$.

3.2. Categorial semantics – the theory *ST*

The theory ***ST*** is an axiomatic theory, built over the theory of syntax ***T***. It describes both the *intensional semantics* and the *extensional semantics* of the categorial language *L*.

3.2.1. Intensional semantics

The basic notions of the *intensional categorial semantics* of *L* are the following:

- the notion of *meaning* (*intension*) of a *wfe-type* of *L*,
- the notion of a *category of knowledge* (constituents of knowledge), determined by means of the notion of *meaning*, and
- the notion of an *intensional semantic category*, defined by means of the previous notion.

In the semantic, formal characteristics of *L*, these notions are defined on the *level of types*. However, introducing into the formal theory ***ST*** the notion of *meaning* of a meaningful *wfe-type* of the set ***S***, and also that of *interpretation* of such an expression, as well as derivative notions, requires making references to some notions of the theory ***ST*** which are introduced on the *level of tokens*.

There exist various philosophical concepts concerning the nature of the *meaning* of a language expression, and also various theories of this notion. In the theory ***ST***, the formal concept of *meaning* is based on the general theory ***TM&I*** of meaning and interpretation, which were presented in the author's works (2005a, b; 2007a, b). This concept is a logical pragmatic-semantic one and has certain connections with the understanding of *meaning* as a ***manner of using language expressions***. It takes into account the so-called ***functional approach to language analysis*** represented by Pelc (1971, 1979).

According to the approach proposed by Pelc, we can speak of a double ***manner of using language expressions***:

1) regarding the first of them, the ***manner of using*** (*use*) takes place only in given conditionings, in determined situational-language contexts and concerns solely expression-*tokens*,
2) regarding the other one, the ***manner of using*** (*usage*, *Use*) characterizes the *meaning* of an expression; this manner is built into the *meaning* of an expression, while the very expression itself can be treated as isolated, static, torn out of context, e.g., as a dictionary entry; then it is an expression-*type*, a class of its concrete occurrences, a class of expression-*tokens*, either applied to represent some object or used in acts of communication and in given situations, with reference to only one broadly-understood object, or with reference to more than one object, still one of the same kind.

The difference between these two *manners of using* of expressions manifests itself in that two persons can use – in the sense of *Use*– the same expression-*type* by means of its two different *tokens*, thus using its different *tokens* in the sense of *use*.

In the set-theoretical formalization of the theory ***ST*** it is accepted that *use* is a relation dealing with real or potential physical acts of object references of *wfe-tokens*, already performed, being performed, or ones that may be performed by users of L in a determined communication process by means of these expressions. The relation *use* is a primitive notion of the theory ***ST***, whereas the relation *Use*, concerning the usage of expression-*types* by users of L, is a secondary notion of this theory. It is defined by means of the relation *use* and appears useful in the proposed, formal concept of *meaning* and *interpretation*, which makes references to certain ideas of Wittgenstein (1954) and Ajdukiewicz (1931, 1934). This concept is connected with understanding the *meaning* of expression-*types* as the *Use* manner of using them.[8]

The primitive notions of the theory ***ST***, with which the theory of syntax ***T*** is enriched are the following:

– the set *User* of all users of L,
– the set *Ont* of all extra-language objects, described by L,
– the binary operation *use* of *wfe-tokens* of the set S'.

It is assumed only axiomatically about the sets *User* and *Ont* that they are non-empty. A user of L, belonging to the set *User*, can be not only a current, but also a past or future user of it. On the other hand, objects of the set *Ont* can be not only concrete, material objects, but also fictional or abstract creations described by L. We do not assume anything, either, about categorization of the set *Ont*. Ontological categories can, but do not have to, be: a category of individuals, categories of sets of individuals, various categories of set-theoretical relations and functions, a category of situations (states of things), etc.

The relation *use* is understood in a very broad way, as well. It can be an operation of human production (not necessarily external) of expression-*tokens*, exposing them, or also interpreting with the aim to refer to determined objects of the set *Ont*. Such an operation conceived broadly – within a liberally fixed temporal space and any fixed area of language-based communication between people – is treated as all such physical activities of users of L, which are taking place currently, occurred in the past and may – potentially – happen in the future, and which are subject to referring concrete expression-*tokens* to determined objects of the set *Ont* in relevant situations. The operation *use* can be called a ***function of object reference of wfe-tokens of the language L by its users***.

We postulate that the operation *use* is a two-argument partial function, whose first domain is the set *User* of users, the second – some proper subset of the set S' of all *wfe-tokens* of L, while the counter-domain – the subset of objects of the set *Ont*, to which these expressions are referred. And thus:

AXIOM 1 (sets: *User*, *Ont*)

$$\mathit{User} \neq \emptyset \text{ and } \mathit{Ont} \neq \emptyset.$$

AXIOM 2 (*use*)
use is a partial function:

$$\mathit{User} \times S' \to \mathit{Ont},$$
$$D_1(\mathit{use}) = \mathit{User} \text{ and } D_2(\mathit{use}) \subset S'.$$

We read the expression: $\mathit{use}(u, e) = o$, where $u \in \mathit{User}$, $e \in S'$, $o \in \mathit{Ont}$ as follows:[9] *uses(produces,exposes) the wfe-token e with reference to the object o*. The object o is called an ***object of reference*** or a ***referent*** or a ***correlate*** *of the expression e indicated by its user u.*

Thus, each user of L *uses* at least one *token* of an expression of this language with reference to some object, but not every language *token* must have some object reference (a referent, a correlate).

Let us note, formally, when an expression-*token* possesses an object reference:

DEFINITION 3 (*possessing a referent*)
e ***has an object reference*** iff $e \in S' \wedge \exists u \in \mathit{User}\, \exists o \in \mathit{Ont}(\mathit{use}(u, e) = o)$.

Thus: *Object reference is possessed only by such a wfe-token that is used by some user of L with reference to an extra-language object.*

DEFINITION 4 (*possessing the same manner of use of tokens*)

$e \approx e'$ iff $\exists o \in Ont[\exists u \in User(use(u,e) = o) \wedge \exists u \in User(use(u,e') = o))]$.

Thus: *Two wfe-tokens have the same manner of use* if and only if *they have the same object reference* (*they have the same referent*).

We introduce the ***relation of using expression-types*** in the sense *Use* in the following way:

AXIOM 3 (*Use*)

$$\emptyset \neq Use \subseteq User \times \boldsymbol{S}',$$

DEFINITION 5 (*Use*)

$$u\ Use\ \boldsymbol{e} \text{ iff } \exists e \in \boldsymbol{e} \exists o \in Ont(use(u,e) = o).$$

Therefore we postulate as follows: *There exists a user of L, who uses a wfe-type, and the user u uses the wfe-type* **e** *if and only if they use a token of the expression* **e** *with reference to a referent.*

The notion of *meaning* of an expression-*type* is determined by means of the ***relation*** $\cong$ ***of possessing the same manner of Use of expression-types***. The notion of meaning is thus defined only for expressions which belong to $D_2(Use) = \boldsymbol{S} \subseteq \boldsymbol{S}'$. It is only to such expressions that meaning is assigned.[10] We will call the set $\boldsymbol{S}$ the *set of meaningful expressions of L.*

DEFINITION 6 (*possessing the same manner of Use of types*)

$$\boldsymbol{e} \cong \boldsymbol{e}' \text{ iff } \forall u \in User[(u\ Use\ \boldsymbol{e} \Leftrightarrow u\ Use\ \boldsymbol{e}') \wedge$$
$$\wedge \forall o \in Ont(\exists e \in \boldsymbol{e}(use(u,e) = o) \Leftrightarrow \exists e' \in \boldsymbol{e}'(use(u,e') = o))].$$

The above-given definition states that: *Two meaningful expression types* **e** *and* **e'** *of* ***S*** *have the same manner of Use if and only if any user of L Uses one of them, when he/she Uses also the other of them and for each extra-language object it is a referent of some token of the wfe-type* **e** *if and only if this object is also a referent of some token of the other wfe-type* **e'***.*

The relation between the two different relations of possessing the same manner of using expressions of L is formulated by:

THEOREM 1.

$$\exists u \in User\ (u\ Use\ \boldsymbol{e}) \wedge \boldsymbol{e} \cong \boldsymbol{e}' \Rightarrow \exists e \in \boldsymbol{e}\ \exists e' \in \boldsymbol{e}'(e \cong e'),$$

in compliance with which: *If the two used expression-types* **e** and ***e'*** of ***S*** *have the same manner of using types* (in the other sense, the one of *Use*), *then there exist their relevant tokens e and e', which also have the same manner of using, but one that is proper to tokens* (the manner of using in the first sense, the one of *use*).

Let us note that in accordance with the introduced definition of the relation $\cong$ we can state that:

THEOREM 2.

Relation $\cong$ is an equivalence relation in the set $\boldsymbol{S}$.

We define the basic notion of intensional semantics for L, i.e., the notion of ***meaning*** (*intension*) ***of any meaningful wfe-type e*** of L as the equivalence class of relation $\cong$, determined by this expression:

DEFINITION 7 (*meaning of the expression-type* $\boldsymbol{e}$)

$$\mu(\boldsymbol{e}) = [\boldsymbol{e}]_{\cong} \text{ for every } e \in \boldsymbol{S}.$$

The *meaning* $\mu(\boldsymbol{e})$ of *wfe-type* $\boldsymbol{e} \in \boldsymbol{S}$ may be intuitively understood as a common property of all these *wfe-types* which possess the same manner of using (*Use*) as that of $\boldsymbol{e}$. This common property can be called the ***manner of using*** *Use* ***of expression-type e***.

The *meaning of the wfe* $\boldsymbol{e} \in \boldsymbol{S}$ can be determined also as an equivalence class of all expression-*types* being ***synonyms*** of the expression $\boldsymbol{e}$, and thus having the same meaning as that of $\boldsymbol{e}$, the same manner of using (*Use*) as that of $\boldsymbol{e}$.

It follows from the definition of *meaning* of a meaningful expression-*type* that there is exactly one meaning – the *global meaning* – that corresponds to such an expression. It needs, however, to be observed that since a *wfe-type* is a class of all *identifiable* expression-*tokens* (the fixed universe U), used in any time interval considerations and any established area of language communication, its global meaning can consist of several meanings determined by its *subtypes* – its subsets of *identifiable tokens*. For example, in the English language, the global meanings of the individual word-*types*: "logic", "key", "profession", or "leak" treated as classes of equiform, identifiable tokens, consist of, at least, two meanings ascribed to certain of their subtypes. These words are ambiguous and as such do not have one fixed meaning.

The notion of ambiguity is introduced into the theory $\boldsymbol{ST}$ by means of that of *denotation* – a notion of *extensional semantics*. The notion of *not possessing an established meaning*, on the other hand, is determined by the definition:

DEFINITION 8 (*not possessing an established meaning*)

***e* does not possess an established meaning** iff

$$\neg\forall \boldsymbol{e}' \subseteq \boldsymbol{e}\ (\mu(\boldsymbol{e}') = \mu(\boldsymbol{e})),$$

i.e.,

$$\exists \boldsymbol{e}' \subseteq \boldsymbol{e}\ (\boldsymbol{e}' \neq \boldsymbol{e}\ \wedge\ \mu(\boldsymbol{e}') \neq \mu(\boldsymbol{e})).$$

There follows from the definition, in particular:

THEOREM 3.

a. $\boldsymbol{e}$ *does not have an established meaning iff* $\exists \boldsymbol{e}_1, \boldsymbol{e}_2\ (\boldsymbol{e}_1 \subseteq \boldsymbol{e}\ \wedge \boldsymbol{e}_2 \subseteq \boldsymbol{e}\ \wedge \boldsymbol{e}_1 \neq \boldsymbol{e}_2\ \wedge \mu(\boldsymbol{e}_1) \neq \mu(\boldsymbol{e}_2))$, i.e., *there exist two subtypes of the wfe-type* $\boldsymbol{e}$ *with different meanings.*

b. *If* $\boldsymbol{e}$ *does not have an established meaning,* *then* $\exists u \in User\ \exists e_1, e_2 \in \boldsymbol{e}\ \forall o \in On\ \neg((use(u, e_1) = o = use(u, e_2)),$ i.e., *there exists a user of L, who does not use at least two tokens of the expression* $\boldsymbol{e}$ *with reference to the same extra-language object.*

c. *If* $\exists e_1, e_2 \in \boldsymbol{e}\ (\neg(e_1 \approx e_2))$, *then* $\boldsymbol{e}$ *does not have an established meaning.*

In compliance with condition c. of Theorem 3: *Expression-type does not have an established meaning when some two of its tokens are not used in the same manner.*

The given definition of *meaning* of an expression-*type* determines at the same time the ***operation of meaning*** μ as the following mapping:

$$\mu : \boldsymbol{S} \to 2^{\boldsymbol{S}}$$

of the set $\boldsymbol{S}$ of all meaningful expression-*types* of the language L into a family of all of its subsets. We call the image of the set $\boldsymbol{S}$ under the operation μ the ***set of constituents of knowledge*** and denote it by $\boldsymbol{K}$. Thus:

$$\boldsymbol{K} = \mu(\boldsymbol{S}).$$

The operation of meaning μ corresponds to the ***operation of interpretation*** ι defined as mapping:

$$\iota : \boldsymbol{S}^* \to 2^{\boldsymbol{S}}$$

defined by the formula:

$$\iota(\boldsymbol{e}) = [e]_{\cong \mathbf{i}}, \quad \text{for any } \boldsymbol{e} \in \boldsymbol{S}^* \subseteq \boldsymbol{S},$$

where $\cong_{\mathbf{i}}$ is a ***relation of possessing the same manner of interpreting meaningful expression-types*** and a sub-relation of the relation $\cong$ of possessing the same manner of using (*Use*) such expressions.[11]

Interpretation of a meaningful expression-type can be intuitively understood as a common property of all the meaningful expression-*types* which possess the same manner of interpreting.

It is well-known that if an expression-*type* is intermediary in language communication, its *interpretation* can differ from its meaning. Let us note that formally we can merely state that for any meaningful expression-*type* $\boldsymbol{e}$

$$\iota(\boldsymbol{e}) \subseteq \mu(\boldsymbol{e}).$$

We can divide the set of constituents of knowledge $\boldsymbol{K}$ into *categories of knowledge*, like we have divided the set of *wfe-types* of L into syntactic categories. In order to do so we make use of categorial indices of the set $\boldsymbol{I}$ and introduce the ***function of indexation $\boldsymbol{i_K}$ of components of knowledge***:

$$\boldsymbol{i_K} : \boldsymbol{K} \to \boldsymbol{I}.$$

We define the ***category of knowledge with the index*** ξ in the following way:

$$\boldsymbol{K}_\xi = \{\boldsymbol{k} \in \boldsymbol{K} : \boldsymbol{i_K}(\boldsymbol{k}) = \xi\}.$$

If, in L, we have sentences, names, and functors-functions defined on them, then their meanings – as constituents of knowledge – determine, respectively, the category of logical judgments, the category of logical notions, and categories of operations on logical judgments and/or logical notions.

In the semantic, *intensional* description of L, we count *wfe-types* of L to suitable *intensional semantic categories* determined by categorial indices. And so, we are introducing the following definition:

$$\boldsymbol{Int}_\xi = \{\boldsymbol{e} \in \boldsymbol{S} : \boldsymbol{i_K}(\mu(\boldsymbol{e})) = \xi\} = \{\boldsymbol{e} \in \boldsymbol{S} : \mu(\boldsymbol{e}) \in \boldsymbol{K}_\xi\},$$

i.e., the ***intensional semantic category with the index*** ξ is a set of all these meaningful expression-*types* of L, whose *meanings* belong to the category of knowledge with the index ξ.

One of the conditions of *language adequacy* is an agreement of syntactic categories with *semantic categories*, and this of both *intensional* and *extensional* ones. We will introduce the latter formally on the second level of language semantics of the language L.

3.2.2. Extensional semantics

In compliance with Frege's distinction (1892): *Sinn–Bedeutung* and Carnap's distinction (1947): *intension–extension*, we distinguish the *meaning* of expression-*type* of L from a *denotation* of such an expression. We introduce the notions of *denotation* (*extension*) *of an expression-type* and that of *denotation of a constituent of knowledge*, corresponding to this expression, formally on the basis of the theory $\boldsymbol{ST}$, by means of respective notions of *denoting* (*reference*). All these notions belong to the semantics of the second level – the *extensional semantics* of L.

Denoting (*reference*) Ref_1 is a binary relation that holds between expression-*types* and extra-language objects of the set *Ont*. The notion of *denoting* can, however, also be introduced as the relation Ref_2 holding between constituents of knowledge and extra-language objects of the set *Ont*. Therefore, formally:

$$Ref_1 \subseteq \boldsymbol{S} \times Ont \text{ and } Ref_2 \subseteq \boldsymbol{K} \times Ont,$$

and the definitions of these relations are as follows:

DEFINITION 9 (*denoting*)

a. $\boldsymbol{e}\ Ref_1\ o$ iff $\exists u \in User\ \exists e \in \boldsymbol{e}\ (use(u, e) = o)$, where $\boldsymbol{e} \in \boldsymbol{S}$.

b. $\boldsymbol{k}\ Ref_2\ o$ iff $\exists \boldsymbol{e} \in \boldsymbol{S}\ (\boldsymbol{k} = \mu(\boldsymbol{e}) \wedge \boldsymbol{e}\ Ref_1\ o)$, where $\boldsymbol{k} \in \boldsymbol{K}$.

The ***expression-type e denotes the object*** o if and only if there exists a user of L, who *uses* any *token* of the expression $\boldsymbol{e}$ with reference to the object o, whereas the ***constituent of knowledge k denotes the object*** o, when there exists a meaningful expression of L determining $\boldsymbol{k}$ and denoting o. We will refer to the objects denoted by expression-*types* or constituents of knowledge as their ***denotates***.[12]

As an example, the *denotate* of the name "a computer" is each computer; any computer is also denoted by the notion 'a computer'; any computer is thus a *denotate* of this notion as well.

It is easy to notice that the *denotate* of an expression-*type* is, at the same time, an object reference of a *token*.

The set of all *denotates* of an expression-*type* or, respectively, a constituent of knowledge, is called its ***denotation*** or ***extension***. Thus:

DEFINITION 10 (*denotation*)

a. $\delta(\boldsymbol{e}) = \{o \in Ont : \boldsymbol{e}\ Ref_1\ o\}$, where $\boldsymbol{e} \in \boldsymbol{S}$.

b. $\delta_{\boldsymbol{K}}(\boldsymbol{k}) = \{o \in Ont : \boldsymbol{k}\ Ref_2\ o\}$, where $\boldsymbol{k} \in \boldsymbol{K}$.

The *denotation* of a meaningful expression-*type* or a constituent of knowledge corresponding to it does not have to be a non-empty set. It is such a set when a user of *L* *uses* the same expression-*type*; that is, he/she *uses* any of its *tokens* with reference to an extra-language object. Hence, we have:

THEOREM 4 (*the criterion of non-emptiness of denotation*)

$$\exists u \in User\ (u\ Use\ \boldsymbol{e}) \ \text{ iff } \ \delta(\boldsymbol{e}) \neq \emptyset \ \text{ iff } \ \delta_{\boldsymbol{K}}(\mu(\boldsymbol{e})) \neq \emptyset.$$

The definitions of *denotation* of a meaningful expression-*type* given below and the constituent of knowledge corresponding to it cover the so-called *global denotation*. Inasmuch as an expression-*type* is ambiguous,[13] its *global denotation* is composed of *denotations* determined by its unambiguous *sub-types*.[14]

When the *denotation* of a meaningful expression-*type* or a constituent of knowledge corresponding to it is a one-element set (a singleton), we identify it sometimes, in practice, with its sole *denotate*. This is so, for instance, when we come to deal with proper names. Let us note that in situational semantics, *denotates* of logical sentences are conceived as situations and frequently identified with *denotations* of such sentences. Also, in Frege's traditional semantics (1892), a *denotate* and – at the same time – *denotation* of a logical sentence is its logical value, i.e. truthfulness or falsity.

Let us note, too, that *denotations* of the so-called general names (predicative) and the logical notions corresponding to them, are called *scopes*, identifying the latter. For example, the scope (*denotation*, *extension*) of the name "a computer" – that is – the set of all computers, is identified with the scope (*denotation*) of the notion 'a computer'. This agreement of the denotations of names and notions corresponding to them is connected with *language adequacy*, and more precisely – with *semantic adequacy*, which is illustrated by Diagram 2.

In the theory $\boldsymbol{ST}$, there holds a theorem which frames this adequacy:

THEOREM 5 (*semantic adequacy*)

$$\delta(\boldsymbol{e}) = \delta_{\boldsymbol{K}}(\mu(\boldsymbol{e})), \qquad \text{for any } \boldsymbol{e} \in \boldsymbol{S}.$$

According to Theorem 5: *Denotations of any meaningful expression-type of L and the meaning (a constituent of knowledge) of this expression are in agreement.*

There follows immediately an important theorem from this theorem, pointing to the fact that the *meaning* of an *expression-type* determines its *denotation*:

THEOREM 6 (*dependence between the meaning and denotation*)

$$\mu(\boldsymbol{e}) = \mu(\boldsymbol{e}') \Rightarrow \delta(\boldsymbol{e}) = \delta(\boldsymbol{e}'), \quad \text{for any } \boldsymbol{e}, \boldsymbol{e}' \in \boldsymbol{S}.$$

According to this theorem: *If two expression-types have the same meaning* (*intension*), *then they also have the same denotation* (*extension*). In other words: *If two expressions are synonymous, then they are extensionally equivalent.*

The reverse theorem does not hold: two expressions can have the same *denotation* (be extensionally equivalent), but may not have the same *meaning* (may not be synonymous). Instances of such expressions are: the "Morning Star" and the "Evening Star" (see Frege, 1892).

The two conclusions below follow from the above theorem, in particular:

COROLLARY 1.

If $\exists o \in Ont\ (\boldsymbol{e}_1\ Ref\ o \wedge \neg \boldsymbol{e}_2\ Ref\ o \vee \boldsymbol{e}_2\ Ref\ o \wedge \neg \boldsymbol{e}_1\ Ref\ o)$,
then $\mu(\boldsymbol{e}_1) \neq \mu(\boldsymbol{e}_2)$.

COROLLARY 2.

Any expression-type does not possess an established meaning, when there exist two such subtypes of it that an object is the denotate of only one of them.

Following Corollary 1: *Two expression-types do not have the same meaning as long as an object is the denotate of only one of the expressions.*

In accordance with the other conclusion, for example, the ambiguous name "a key" does not possess an established meaning, since there exists "a key" which is the *denotate* of a certain subtype of this name, yet which is not the *denotate* of another subtype of this name.

The given definitions of the denotation of an expression-*type* and the denotation of a constituent of knowledge determine simultaneously two ***denotation operations***: δ and $\delta_{\boldsymbol{K}}$. They are the following mappings:

$$\delta : \boldsymbol{S} \rightarrow 2^{Ont} \ \text{ and } \ \delta_{\boldsymbol{K}} : \boldsymbol{K} \rightarrow 2^{Ont},$$

respectively: of the set $\boldsymbol{S}$ of all meaningful expression-*types* of L into the family of all subsets of the set of extra-language objects *Ont* and of the set $\boldsymbol{K}$ of all constituents of knowledge into this family.

Thus, it follows from Theorem 5 of semantic adequacy that the denotation operation δ is a composition of denotation operation $\delta_{\boldsymbol{K}}$ and the meaning operation μ that is (see Diagram 2): $\delta = \delta_{\boldsymbol{K}} \circ \mu$.

The image of the set $\boldsymbol{S}$ with respect to the operation δ and the set $\boldsymbol{K}$ with respect to the operation $\delta_{\boldsymbol{K}}$, are called *a set of ontological objects* and denoted by $\boldsymbol{O}$. Thus (see Diagram 2):

$$\boldsymbol{O} = \delta(\boldsymbol{S}) = \delta_{\boldsymbol{K}}(\boldsymbol{K}).$$

We can divide the set $\boldsymbol{O}$ of ontological objects into ontological categories, in a similar way as we divided the set $\boldsymbol{S}$ of meaningful expressions of L into syntactic categories, and the set of constituents of knowledge $\boldsymbol{K}$ into categories of knowledge. For this purpose we use the categorial indices of the set $\boldsymbol{I}$ and introduce the ***function of indexation*** $\boldsymbol{i_O}$ ***of ontological objects***:

$$\boldsymbol{i_O} : \boldsymbol{O} \to \boldsymbol{I}.$$

We define the ***ontological category with the index*** ξ in the following way:

$$\boldsymbol{O}_\xi = \{\boldsymbol{o} \in \boldsymbol{O} : \boldsymbol{i_O}(\boldsymbol{o}) = \xi\}.$$

If, in L, we have sentences, individual names, and functor-functions defined on them, then the ontological objects corresponding to them – as their denotations – determine, respectively, a category of states of things (in Frege's semantics – a category of logical values), a category of individuals, and a category of operations on states of things (resp., on logical values), on individuals, on the former and/or the latter, etc.

In the semantic, *extensional* description of L, the meaningful expression-*types* of this language count into respective *extensional semantic categories*, determined by categorial indices. And so:

$$\boldsymbol{Eks}_\xi = \{\boldsymbol{e} \in S : \boldsymbol{i_O}(\sigma(\boldsymbol{e})) = \xi\} = \{\boldsymbol{e} \in S : \sigma(\boldsymbol{e}) \in \boldsymbol{O}_\xi\},$$

i.e., the ***extensional semantic category with the index*** ξ is a set of all the expression-*types of* L, whose denotations (*extensions*) belong to the ontological category with the index ξ.

One of the conditions of *language adequacy* is an agreement of *syntactic categories* with applied *semantic categories*, and this both *intensional* as well as *extensional*. This agreement is not ensured by the agreement of both levels of the semantics of L: *intensional* and *extensional*.

In the next part of the paper, we will consider, with more precision, the problem area of *language adequacy*, discussing its various aspects.

4. Language adequacy and its aspects

In the *Introduction*, we defined the problem area of *language adequacy* in a most general manner, as a compatibility of language syntax and its bi-level semantics: *intensional semantics* and *extensional semantics*. Formal consideration of the problem of *language adequacy* can be conducted on the basis of the theory of syntax $\boldsymbol{T}$ and its extension to the semantic theory $\boldsymbol{ST}$ for the categorial language L. Taking into account the bi-level semantics of L, we have already established an important theorem which characterizes the ***semantic adequacy*** for this language and states that for any expression-*type* $\boldsymbol{e} \in \boldsymbol{S}$ of L:

$$\delta(\boldsymbol{e}) = \delta_{\boldsymbol{K}}(\mu(\boldsymbol{e})) \in \boldsymbol{O},$$

that is, the same object of the reality described by L corresponds to the denotation of any meaningful expression-*type* $\boldsymbol{e}$ of L and the denotation of its counterpart which is a constituent of knowledge (see Diagram 2). *Semantic language adequacy*, like certain *intensional* and *extensional* agreement with reality described by the language, is the starting point in the consideration of various aspects of *language adequacy*.

In compliance with the understanding of the adequacy of language syntax and semantics provided by Frege (1879, 1892), Husserl (1900–1901), Leśniewski (1929, 1930) and Suszko (1958, 1960, 1964, 1968),[15] ***language adequacy*** assumes, primarily, that the categories of language expressions – syntactic and semantic (*extensional*), with the same indices – should be the same. Extending this agreement onto the identity of all distinguished kinds of categories of meaningful expression-*types* of L: syntactic, semantic extensional, as well as semantic intensional, with the same categorial indices, we will use the term ***categorial adequacy***. In order to determine it, we postulate the following:

POSTULATE (*categorial adequacy*)

$$\boldsymbol{SC}^{*}_{\xi} = \boldsymbol{Int}_{\xi} = \boldsymbol{Eks}_{\xi}, \qquad \text{for any } \xi \in \boldsymbol{I},$$

where $\boldsymbol{SC}^{*}_{\xi} = \{\boldsymbol{e} \in \boldsymbol{S} : i_{\boldsymbol{S}}(\boldsymbol{e}) = \xi\}$.[16]

We can formulate the postulate of *categorial language adequacy* given above in two equivalent ways imposed by conditions a and b of the following theorem (see Diagram 3):

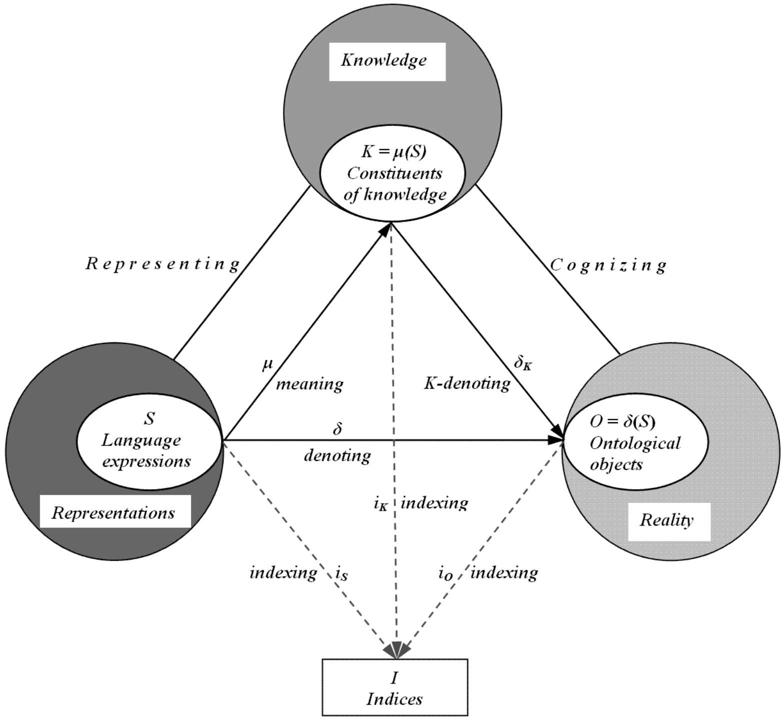

Diagram 3. Categorial adequacy

THEOREM 7 (*categorial adequacy*)

a. $\boldsymbol{e} \in \boldsymbol{SC}^{*}_{\xi}$ iff $\mu(\boldsymbol{e}) \in \boldsymbol{K}_{\xi}$ iff $\sigma(\boldsymbol{e}) \in \boldsymbol{O}_{\xi}$, for any $\boldsymbol{e} \in \boldsymbol{S}$.

b. $\boldsymbol{i_S}(\boldsymbol{e}) = \boldsymbol{i_K}(\mu(\boldsymbol{e})) = \boldsymbol{i_O}(\sigma(\boldsymbol{e}))$, for any $\boldsymbol{e} \in \boldsymbol{S}$.

The categorial adequacy is therefore ensured by the identity of categorial indices: of any meaningful expression-*type* of L, its *meaning* and its *denotation*.

There follow from Theorem 7 of categorial adequacy theorem-equivalents of the theorem (*R1*), permitting one to state that it is not only a functor and its index, but also the semantic equivalents of the functor – its *meaning* and its *denotation* – that can be treated as set-theoretical functions (see Wybraniec-Skardowska, 2009):

THEOREM 8 (*meaning and denotation of functor*)

If $\boldsymbol{e} = \boldsymbol{f}(\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n)$ *is a meaningful expression of the set* $\boldsymbol{S}$, *satisfying the formula* ($\boldsymbol{i_f}$), *then the following equivalences are satisfied*:

(*R2*) $\mu(\boldsymbol{f}) \in \boldsymbol{Ka}/\boldsymbol{a}_1\boldsymbol{a}_2\ldots\boldsymbol{a}_n$ iff
($\mu(\boldsymbol{f})$) $\mu(\boldsymbol{f}) : \boldsymbol{Ka}_1 \times \boldsymbol{Ka}_2 \times \ldots \boldsymbol{Ka}_n \to \boldsymbol{Ka} \wedge$
(*PCM*) $\mu(\boldsymbol{e}) = \mu((\boldsymbol{f}(\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n)) = \mu(\boldsymbol{f})(\mu(\boldsymbol{e}_1), \mu(\boldsymbol{e}_2), \ldots, \mu(\boldsymbol{e}_n))$
and
(*R3*) $\sigma(\boldsymbol{f}) \in \boldsymbol{Oa}/\boldsymbol{a}_1\boldsymbol{a}_2\ldots\boldsymbol{a}_n$ iff
($\sigma(\boldsymbol{f})$) $\sigma(\boldsymbol{f}) : \boldsymbol{Oa}_1 \times \boldsymbol{Oa}_2 \times \ldots \boldsymbol{Oa}_n \to \boldsymbol{Oa} \wedge$
(*PCD*) $\sigma(\boldsymbol{e}) = \sigma((\boldsymbol{f}(\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n)) = \sigma(\boldsymbol{f})(\sigma(\boldsymbol{e}_1), \sigma(\boldsymbol{e}_2), \ldots, \sigma(\boldsymbol{e}_n))$

We call the condition (*PCM*) the ***semantic principle of compositionality of meaning***, and the condition (*PCD*) the ***semantic principle of compositionality of denotation***. These principles were already known to Frege (1892).[17]

Loosely speaking, these principles state, respectively, that:

The meaning (resp. *denotation*) *of a well-formed functor-argument expression of L is the value of the function of meaning* (resp. *denotation function*) *of its main functor, defined by meanings* (resp. *by denotations*) *of arguments of this functor.*

The categorial character of the language *L* under consideration allows speaking also about ***structural adequacy*** as an agreement of the structure of any expression composed of a functor and its arguments, with the structure of the constituent of knowledge that corresponds to it and with the structure of the object of the cognized reality that corresponds to it. ***Structural adequacy*** is obtained through holding three *principles of compositionality*:[18] one syntactic – the principle (*PCS*) *of compositionality of syntactic forms* – and two semantic principles: (*PCM*) and (*PCD*), *of compositionality of meaning* and *compositionality of denotation*.[19]

The three principles of compositionality mentioned above, for $h = \boldsymbol{i_S}$, μ, σ and any meaningful expression $\boldsymbol{e} = \boldsymbol{f}(\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n)$, have the following common schema:

$$h(\boldsymbol{e}) = h(\boldsymbol{f}(\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n)) = h(\boldsymbol{f})(h(\boldsymbol{e}_1), h(\boldsymbol{e}_2), \ldots, h(\boldsymbol{e}_n)),$$

which can be treated as a schema of three conditions of the homomorphism of partial algebra $\boldsymbol{L}$ of *L* in the algebra of its images $h(\boldsymbol{L})$, i.e.,

$$\boldsymbol{L} = \langle \boldsymbol{S}, \boldsymbol{F} \rangle \xrightarrow[h]{} h(\boldsymbol{L}) = \langle h(\boldsymbol{S}), h(\boldsymbol{F}) \rangle,$$

where $\boldsymbol{F}$ is a set of partial functor-functions defined by subsets of the set $\boldsymbol{S}$ and with values in the set $\boldsymbol{S}$, and $h(\boldsymbol{F})$, for $h = \boldsymbol{i_S}$, μ, σ, is a set of operations corresponding to operations of the set $\boldsymbol{F}$.[20]

We call the algebra

$$\boldsymbol{i_S}(\boldsymbol{L}) = \langle \boldsymbol{i_S}(\boldsymbol{S}), \boldsymbol{i_S}(\boldsymbol{F})\rangle$$

a ***syntactic model*** of L, and the algebras:

$$\mu(\boldsymbol{L}) = \langle \mu(\boldsymbol{S}), \mu(\boldsymbol{F})\rangle = \langle \boldsymbol{K}, \mu(\boldsymbol{F})\rangle \text{ and } \sigma(\boldsymbol{L}) = \langle \sigma(\boldsymbol{S}), \sigma(\boldsymbol{F})\rangle = \langle \boldsymbol{O}, \sigma(\boldsymbol{F})\rangle$$

the ***semantic models*** of this language; the first is called the ***intensional model***; the other – the ***extensional model***.

In the process of cognition of reality, *language adequacy* also consists in that sentences of language L should be true in its above mentioned models.

If for $h = \boldsymbol{i_S}$, μ, σ it is so that the sentence $\boldsymbol{e}$ of L is *true in the models* $h(\boldsymbol{L})$, then we can say that our *cognition is true* or that there occurs language ***cognitive adequacy***.

The notions of *truthfulness* in respective models are introduced in the theories $\boldsymbol{T}$ and $\boldsymbol{ST}$ by means of three primitive notions *Th*, satisfying at $h = \boldsymbol{i_S}$, μ, σ the axioms:

$$\emptyset \neq Th \subseteq h(\boldsymbol{S})$$

and understood intuitively, respectively, as: a singleton composed of the index of true sentences, a set of true judgments, a set of situations that take place (in Frege's semantics – a singleton composed of the value of truth).

DEFINITION 11 (*truthfulness*)

For $h = \boldsymbol{i_S}$, μ, σ

The sentence $\boldsymbol{e}$ of L *is true in the model* $h(\boldsymbol{L})$ iff $h(\boldsymbol{e}) \in Th$.

Language-related knowledge is passed in the process of inter-human communication. The transmitting and proper reception of it are connected with the proper *interpretation* of language expressions and ***communication adequacy***, based on the agreement of *meaning* and *interpretation* of language expressions which mediate in the communication (see Diagram 4).

Thus, if the expression-*type* $\boldsymbol{e}$ mediates in the communication between its sender and its receiver, then ***communication adequacy*** is secured by the condition:

$$\mu(e) = \iota(e).$$

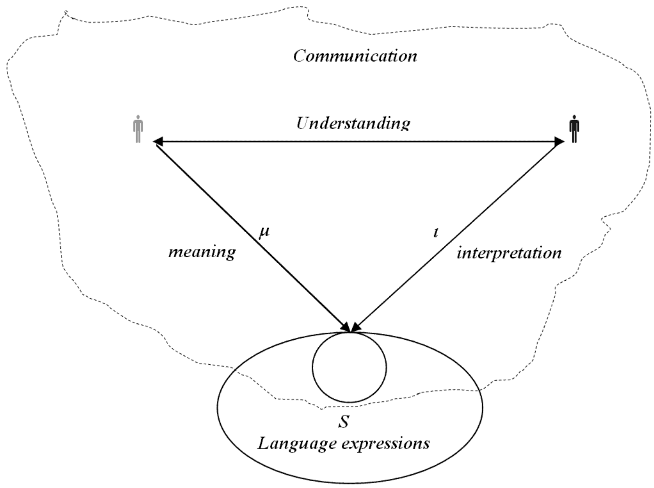

Diagram 4. Communication language adequacy

Let us pay attention to the fact that formal securing of the communication adequacy of L is based on such a formalization of it that uses the relations *Use* of using expression-*types* of this language, and therefore also the relation *use* of using its *tokens*, since in the presented theory $\boldsymbol{ST}$ the notions of *meaning* and *interpretation* of meaningful expression-*types* of L are defined by means of these relations. This fact implies the possibility of formalizing the notion of an inter-human *communication act* by means of *tokens* of language expressions and establishing formal conditions of its adequacy (see Wybraniec-Skardowska & Waldmajer, 2009).

5. Summary

• This paper has offered a synthetic framework of the main ideas and theoretical considerations presented in earlier papers of the author, especially those dealing with the syntax and semantics of language characterized categorially:
– in the spirit of Husserl's idea of pure grammar (1900–1901),
– in compliance with the basic assumptions of the theory of syntactic categories of Leśniewski-Ajdukiewicz,
– according to Frege's (1892) ontological canons, and also
– to Bocheński's (1949) well-known statement: *syntax mirrors ontology.*

• The formal-logical framework of the theory of language syntax outlined in this work is based on Peirce's (1931–1935) distinction of sign-*tokens* from sign-*types*, on the assumption that expressions of language possess a double ontological nature: they can be physical concrete objects – *tokens* – or ideal abstract beings – *types*, classes of expression-*tokens*.

- Taking into account, in the theory of syntax, the double ontological character of language objects, as well as – following Pelc (1971, 1979) – the functional approach to the logical analysis of language, allows speaking about two manners of using language expressions. The first of them – applied in acts of inter-human language-based communication – concerns using expression-*tokens*; the other – using expression-*types* and determining on what, formally, correct *adequate* language communication depends. The other way allows also introducing, formally, basic semantic notions: the notion of *meaning* and that of *denotation* of expression-*type*, differentiating between them basically (similarly as was done by Frege, 1892, and Carnap, 1947), and also using means of logical pragmatics.
- A formal characteristic of semantic notions takes place in the theory of semantics of language, built over this theory of syntax. The formal-logical theory of language which is presented in this paper, is a result of conceptualizing inter-subjective knowledge about language communication in a liberally established time range, as well as a liberally determined area of such communication. The conceptualization includes the bi-level semantics of language: *intensional* and *extensional*. On the first of them, the *intensional* level of theoretical considerations, the notions of *meaning* and *interpretation* are introduced, making reference to the use (*Use*) of expression-*types* (through the use of expression-*tokens* by users of language) and preserving certain intuitions of Wittgenstein (1953) and Ajdukiewicz (1931, 1935), connected with the first of these notions. On the other, the *extensional* level of theoretical considerations, two notions of *denoting* and two notions of *denotation* are introduced. The notions of *denoting* and *denotation* (*extension*) of expression-*types* are differentiated from those of *denoting* and *denotation* of their *meanings* (*intensions*) treated as constituents of knowledge. All these notions are introduced as semantic-pragmatic ones, through referring to the two mentioned ways of using the expressions. This agreement of the two types of *denotation* is referred to as *semantic adequacy*, since it is connected with the bi-level semantics of language described by the theory under discussion in this paper.
- If – according to the ontological canons of Frege and Bocheński – language is to be a linguistic schema of ontological reality, and – at the same time – a tool of its cognition, its syntax should be in agreement with the bi-level semantics corresponding to it. This compliance has been called *language adequacy*, and its occurrence is guaranteed in the formal theory of lan-

guage by accepting the respective postulate (axiom) of *categorial adequacy*. There follows from it an important condition of *structural* (*compositional*) *adequacy*, connected with the *principles of compositionality of meaning* and *denotation*, which were known already to Frege, and also with their syntactic counterpart introduced in papers by the author.

• In the outlined theory of language there are also formally considered other aspects of *language adequacy*. They are connected with the effectiveness of human cognition and inter-human communication by means of language expressions.

NOTES

[1] The paper is an English translation (including slight transformations and complements) of the author's paper published in Polish in 2010, O adekwatności językowej. In J. Pelc (Ed.), *Deskrypcje i prawda*, BMS **51** (pp. 275–306). Warszawa: Polskie Towarzystwo Semiotyczne. The main assumptions of this work were presented at the VIIIth Philosophical Congress held in Warsaw on 16 September 2008, in the paper entitled *From syntax to bi-level semantics of language*. This paper also develops some ideas which were presented in the author's earlier papers (2007a, b, c) and (2009, 2010). I would like to kindly thank Professor Mieszko Tałasiewicz and Dr. Edward Bryniarski for their critical and valuable comments which contributed to introducing some complements to the first draft of this paper.

[2] Independently of Leśniewski, a theory of syntactic category was presented and developed for the needs of the so-called combinatory logics by Curry (1961, 1963). A somewhat complementary theory to ***ST*** is the so-called *Transparent Intensional Logic* presented by Duzi, Jaspersen and Materna (2010).

[3] Let us pay attention in this place to the fact that the notions of *intension* and *extension* introduced in the theory ***ST*** differ considerably from those introduced in Montague's pragmatics (1970b).

[4] In order to distinguish signs in such a way, Carnap (1942) applies the terms "sign-event" and "sign-disign".

[5] The theory ***T*** can be, in an equivalent way, formalized – first – on the *level of types*, and then – on the *level of tokens* (see Wybraniec-Skardowska, 1989, 1991, *Final Remarks*), representing the *Platonizing* approach to the description of language syntax.

[6] In the literature dealing with categorial grammars, it is accepted to refer to the categorial indices introduced by Ajdukiewicz as *types*. The categorial indices should not, obviously, be mistaken for the indices introduced by Montague (1970b) and applied as the ordered tuples of agent's factors which constitute the context of usage of expressions.

[7] Since, from the pragmatic point of view, equiform *tokens* may not be identifiable: equiform expression-*tokens* can have different functor-argument structures, then are treated as different language expressions of language. Thus, *types* of equiform expression-*tokens* do not have to be equal.

[8] The convergence between Ajdukiewicz's ideas and those of Husserl regarding the question of meaning of expressions as a manner of their usage is drawn attention to by Olech (2001). The very concept of meaning, deriving from Ajdukiewicz, is discussed in the book by Wójcicki (1999). A review of different concepts of meaning and a discussion on Ajdukiewicz's concept can be found, among others, in Maciaszek's copious monograph (2008).

[9] The way in which the expression: *use(u,e) = o* is read cannot be mistaken with the manner of interpreting this expression, in agreement with an intuitive, broad understanding of the operation *use*.

[10] In English, some not meaningful expressions are, for instance, sentences (well-formed expressions) like the following: *The computer gives the celling* or *The flowers are cooking dinner*.

[11] Relation $\simeq_{\mathbf{i}}$ is defined by means of the binary relation *Int* of *interpreting* expression-*types* (corresponding to the relation *Use*) and the binary operation *int* of *interpreting wfe-tokens*, about which it is assumed axiomatically that it is a non-empty reduction of the operation *use* of using expression-*tokens* (see Wybraniec-Skardowska, 2007a, 2007b). The set $\boldsymbol{S}^*\subseteq\boldsymbol{S}$ is the set of all meaningful expression-*types* that can be *Interpreted* by *Users* of *L*.

[12] Let us pay attention to the fact that – according to the assumptions of the theory ***ST*** – the notion of a *denotate*, as an object of the set *Ont* denoted by an expression-*type*, is broader than the notion of a *designate of such an* expression, usually accepted in the logic of language. In particular, it is accepted in logical semiotics that *designates* of the so-called concrete names can be material objects only. Such objects can be *denotates* of such names then, but they do not have to be; they can also be intentional, fictional objects. This explains, in particular, certain misunderstandings connected with so-called empty names. Such names as for instance, "Zeus", "Sphinx" or "Smurf" are acknowledged – on the one hand – to be *empty names* (as ones which do not denote any *designate*), on the other one – as *non-empty names* (as ones denoting their *denotates*).

[13] The formal definition of an ambiguous expression-*type* was given in the author's earlier paper (2007a).

[14] The *global denotation* can also be seen as the upper approximation of denotation of a vague expression, yet in this paper the problem of vagueness of language expressions will not be dealt with.

[15] See also Stanosz and Nowaczyk (1976).

[16] Let us notice that a formalization of the notion of categorial adequacy does not require assuming that language expressions have to have a functor-argument structure. So if language is generated by another type of grammar than a categorial grammar, e.g. a phrase structure grammar or a dependency grammar (see Tensiére, 1959), then the postulate could be adapted.

[17] See also Gamut (1991).

[18] See Wybraniec-Skardowska (2001b, 2010).

[19] The problem of *semantic compositionality* is the subject of a heated discussion (see Montague, 1970; Partee, 1984; Janssen, 1996, 2001; Hodges, 1996, 1998, 2001; Pelletier, 2001; Kracht, 2011).

[20] Ideas connected with the algebraization of language can be found already in works by Leibniz. The algebraic approach to the syntax and the semantics of language can also be found in the works of Dutch logicians of language, especially in those by van Benthem (1980, 1981, 1984, 1986). However, the algebraic approach presented here differs significantly from that given by van Benthem.

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