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How Firms Can Hedge Against Market Risk

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Abstract. The article presents a problem of proper hedging strategy in expected utility model when forward contracts and options strategies are available. We consider a case of hedging when an investor formulates his own expectation on future price of underlying asset. In this paper we propose the way to measure effectiveness of hedging strategy, based on optimal forward hedge ratio. All results are derived assuming a constant absolute risk aversion utility function and a Black-Scholes framework.

Keywords: hedging, Black-Scholes model, derivatives, options, forward, utility function, effectiveness of hedging strategy.

1. Introduction

Firms take a financial risk in every economic activity. The majority of firm managers are aware that proper risk management is not only a way to increase their economic efficiency, but also ensures the existence on a market in general. The easiest method to limit or eliminate market risk is a proper use of derivatives. The simplest instruments are classified as so-called linear derivatives like forwards, futures and swaps. More complex derivatives such as options are conditional in the payment which is a non-linear function of underlying asset. They are very flexible and their combinations can generate desired by investor income profile.

The aim of this paper is to measure hedge effectiveness of popular hedging strategies. We use derivatives to hedge risk exposure against price risk. As a hedging strategy we understand position in derivatives which is matched in terms of the amount and maturity to investor risk exposure on the spot market. In this way the losses from spot market are offset by gains on the derivatives market and, therefore, the losses resulting from the market worst-case scenario are limited. The mismatch in quantity can generate an additional risk to investor. When investor underhedges or overhedges their exposure, then partially or entirely a speculative position is created.

The problem of wrongly constructed hedging strategies became widely publicised in Poland in 2008, when export firms used zero-cost strategies to hedge exchange rate. In fact the strategies had form of speculative positions and generated huge losses. In these transactions about 20 thousand firms i.e. 15% of companies (see Karkowski, 2009) were involved. We find the rationality conditions that must be fulfilled to an investor, who is guided by the principle of maximizing expected utility of wealth. We apply constant absolute risk aversion utility function and a Black-Scholes model to price options.

The problem of searching for optimal hedging strategy using derivatives in terms of maximizing expected utility was for the first time considered by Holthausen (1979). The author discusses the problem of the optimal level of hedge ratio using only forward contracts. For further discussion see (Battermann et al., 2000; Detemple and Adler, 1988; Moschini and Lapan, 1995). This paper shows that linear instruments, when the investor is exposed to market risk, play the dominant role in hedging strategy. Derivatives like forward contracts eliminate the risk associated with changes in prices, then hedging with forward (called perfect hedging) is an optimal strategy. Even the existence of basis risk case is not a sufficient condition for the insertion of non-linear instruments to the optimal hedging strategy (see Lapan, Moschini and Hanson, 1991). When investor expectations about future prices are taken into account, both linear and nonlinear instruments are necessary for the construction of effective protection against risk (Lapan, Moschini and Hanson, 1991). In the article (Echaust, 2007) it is shown how to substitute linear instruments by nonlinear in optimal hedging strategy when investor formulates his or her own expectation about future prices or volatility of underlying asset. Optimal strategies consisting of options and forward together, maximize expected utility of wealth, but create speculative position and only partially reduce risk of cash flow. In this paper we consider only these strategies which fully hedge investor risk exposure.

2. Specification of investment problem

Let's consider the case of an investor holding a long position in the underlying asset, which the price S (share price, exchange rate, interest rate, commodity price) at $t = 0$ is known, but it is not known in the future at $t = T$, when the asset will be sold. Lack of knowledge about the future price level causes that the investor is exposed to market risk, which can significantly affect the attractiveness or profitability of the investment. At a sale moment investor will realize an income equal to:

$$V_T = S_T \times Q \tag{1}$$

where:

Q – quantity (deterministic value),
S_T – asset price at maturity T (random variable).

In order not to be dependent on market movements investor hedges his exposure using derivatives. Income after hedging can be presented as follows:

$$V_T^H = S_T \times Q + H(S_T) \tag{2}$$

where:

$H(S_T)$ – profit or loss of hedge instrument at maturity.

Let's suppose that following hedge strategies are available: only forward contract, only put option with various strike prices and the most reasonable and the most popular for hedging purpose, option combinations like Collar and Participator. Many different hedging strategies can be found in (Baird 1998, Hull 1997). We use ATM[1] option, 10% – OTM[2] and 10% – ITM[3] option. Collar in our experiment is a combination of one long 10% – OTM put and one short call with strike calculated in the way to obtain zero-cost strategy. Participator is a combination of one long put and half short call. Strike prices of both options are the same and strategy is also zero-cost. For each considered strategy profit – loss profile and plot of effective price after hedge are shown in the Figure 1. We assume current spot price equal to one, one year to maturity, volatility 20% and risk free interest rate equal to zero. The adaption of such an interest rate is a simplification of the reality, but there is no significant effect on presented results. Options are priced in Black-Scholes model (Black and Scholes, 1973). Let c and p denote the price of European call and put options respectively, the Black-Scholes formula for option pricing states that

$$c = S_0 N(d_1) - Xe^{-rT} N(d_2), \tag{3}$$

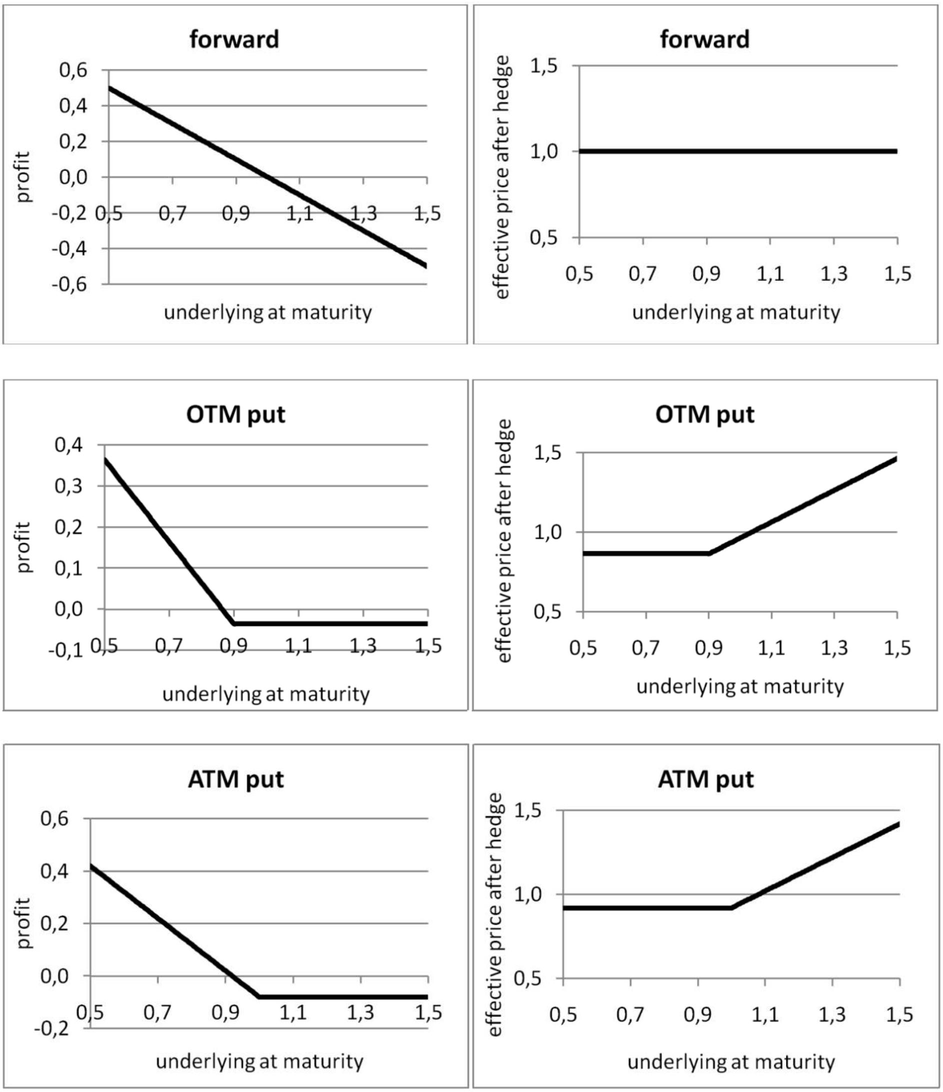
$$p = -S_0 N(-d_1) + Xe^{-rT} N(-d_2), \tag{4}$$

where:

$$d_1 = \frac{\ln\frac{S_0}{X} + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}, \tag{5}$$

$$d_2 = d_1 - \sigma\sqrt{T}, \tag{6}$$

S_0 – initial underlying asset price,
X – strike price,
r – risk-free interest rate,
T – time to expiration in years,
σ – volatility of underlying asset price,
$N(\cdot)$ – the cumulative standardized normal distribution function.



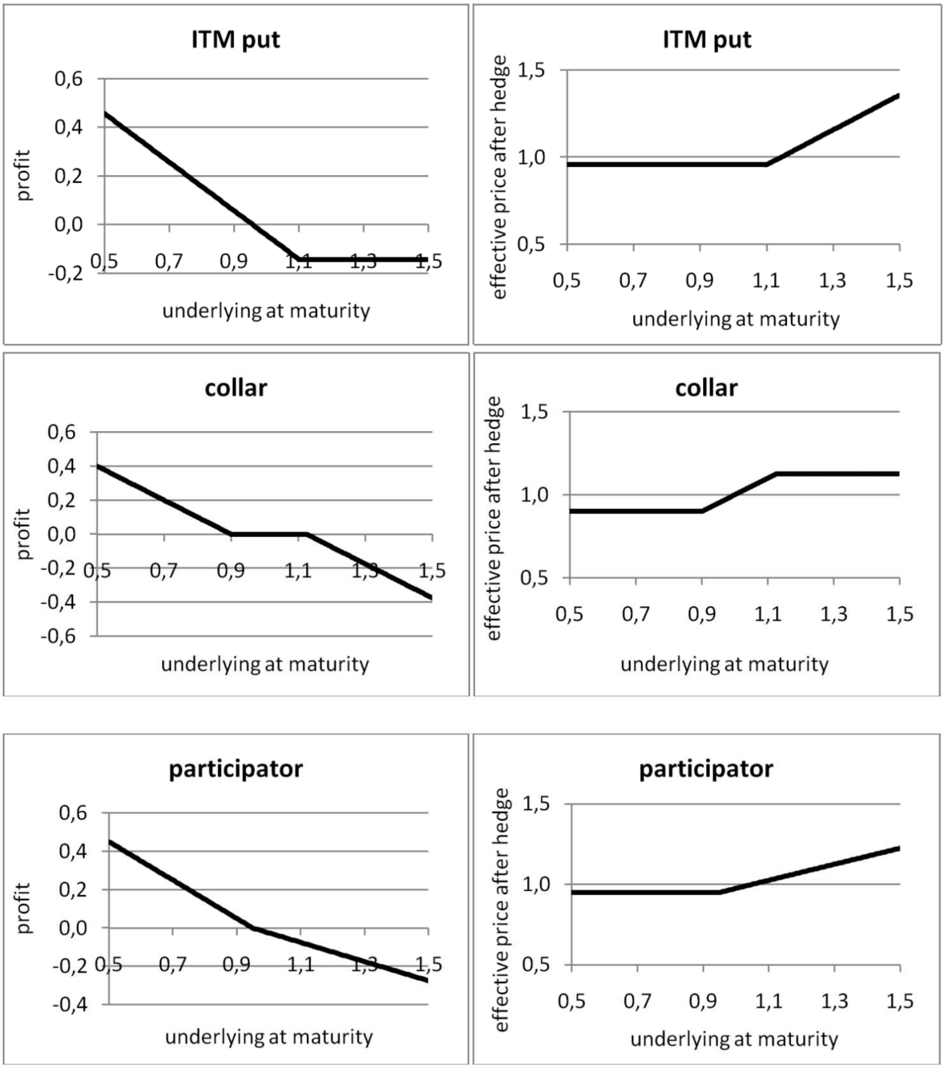


Figure 1. Profit-loss profile (left side) and effective price after hedge (right side) of considered hedging strategies

Source: own calculations.

We assume further, that the investor is strictly risk averse, which is expressed in a utility function von Neumann-Morgenstern (Neumann von and Morgenstern, 1944). It is increasing and strictly concave because of his aversion to risk. The investor is interested in maximizing expected utility of cash-flow realized at maturity. Additionally he or she formulates his own expectation about future price movements which need not be coincident with market expectations. Stated problem cannot be solved in general, and it is necessary to adopt additional assumptions about the form of the utility function. We assume constant absolute risk aversion (CARA) utility function of the form:

$$U(V_T^H) = -\exp(-\lambda V_T^H), \tag{7}$$

where:

$\lambda > 0$ – investor's risk aversion coefficient (Arrow, 1965; Pratt, 1964).

Expected utility of hedged cash-flow has following representation:

$$E[U(V_T^H)] = -\int_0^\infty \exp(-\lambda V_T^H) f(S_T) dS_T, \tag{8}$$

where:

$f(S_T)$ – investor's subjective density of price.

When we use log-normal distribution, then it is of the form:

$$f(S_T) = \frac{1}{\sqrt{2\pi}\sigma S_T} \exp\left(-\frac{1}{2}\left(\frac{\ln\frac{S_T}{S_0} - \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma}\right)^2\right), \tag{9}$$

where:

μ – subjective expected growth rate of the price.

Exponent in the density (9) we can rewrite in the following way:

$$-\frac{1}{2}\left(\frac{\ln\frac{S_T}{S_0} - \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma}\right)^2 =$$

$$= -\frac{1}{2}\left(\frac{\ln\frac{S_T}{S_0} - \left(\ln\frac{E(S_T)}{S_0} - \frac{\sigma^2}{2}T\right)}{\sigma}\right)^2 = \tag{10}$$

$$-\frac{1}{2}\left(\frac{\ln\frac{S_T}{E(S_T)} + \frac{\sigma^2}{2}T}{\sigma}\right)^2,$$

where:

$E(S_T) = \mu S_0$ – subjective expected future price of underlying asset.

Finally expected utility of hedged cash-flow is as follows:

$$E[U(V_T^H)] =$$

$$= -\frac{1}{\sqrt{2\pi}\sigma S_T}\int_0^\infty \exp\left(-\lambda V_T^H - \frac{1}{2}\left(\frac{\log\frac{S_T}{E(S_T)} + \frac{\sigma^2}{2}T}{\sigma}\right)^2\right) dS_T, \tag{11}$$

where V_T^H is given in (2).

We know that when the investor does not formulate his own expectations, only forward contracts will be used in optimal hedging, and selling forward is an optimal strategy. When his expectations of future price differ from forward contract price then optimal strategy does not exist. In order to measure effectiveness of considered strategies we must adopt a strategy as a benchmark. For this purpose it is convenient to use an optimal strategy with forward contract. It is the strategy which maximizes the expected utility of income realized in the future fixed time T, by finding an optimal hedge ratio. Mathematically, the problem can be expressed as follows:

$$h^* = \arg\max_h E\left(U(V_T^h)\right), \tag{12}$$

for

$$V_T^h = S_T \times Q + h(F - S_T), \quad \text{for} \quad 0 \le h \le 1, \tag{13}$$

where:

F – forward price,

h – hedge ratio.

The restriction on hedge ratio comes from the fact, that we do not allow to create speculative position in derivatives, but we allow to create a partially hedging strategy only for the benchmark. We measure effectiveness of hedging strategy as a percentage deviation expected utility of hedged cash-flow (2) from expected utility of strategy (13) determined by h^*

$$\zeta = \frac{E\left(U(V_T^H)\right)}{E\left(U(V_T^{h^*})\right)} - 1. \tag{14}$$

The smaller the value ζ, the better hedge strategy is considered, thus the optimal strategies have ζ equal to zero. Note, that the optimal strategy described in (12)–(13) does not satisfy the assumption of downside risk elimination and cannot be interpreted as a hedging strategy in our sense.

3. Effect of expectations for effectiveness of hedging strategies – simulation results

In this section we present results of effectiveness of hedging strategies. In all simulations we assumed quantity $Q = 1$. The parameters for derivatives are the same as presented in section 2. The zero risk free interest rate means the forward price is equal to the spot price. Therefore, if the $E(S_T)$ differs from one, an investor formulates his own expectation and perfect hedging must not be the optimal strategy in this case. We carried out simulations for different investor risk aversion coefficients.

First, from (12)–(13) we calculated numerically the optimal hedge ratios h^* for different expected prices. The results are shown in Table 1. As we can see, when the investor does not formulate his expectations or expects adverse lower price the optimal strategy is always perfect hedging. However, with the increase in the level of expectations of future spot prices, fewer and fewer forward contracts are sold in the optimal strategy. Such partial hedging does not hedge the entire risk exposed, and speculative position is created. The higher prices the investor expects, the smaller is the utility of hedging strategy. Moreover, the lower aversion to take a risk, the greater willingness to take advantage of favorable price changes and leave cash flow unhedged. We checked also the cases, when risk aversion coefficient is much bigger than 10. Then the perfect hedging becomes the optimal strategy at each level of expectations.

Table 1

Optimal hedge ratio h^* for different expected prices and risk aversion coefficients

	$E(S_T)$						
	0,5	0,8	0,9	1	1,1	1,2	1,5
$\lambda = 1$	1	1	1	1	0	0	0
$\lambda = 5$	1	1	1	1	0,532	0,104	0
$\lambda = 10$	1	1	1	1	0,766	0,552	0

Source: own calculations.

Having calculated benchmark strategy we can calculate expected utility of hedged cash-flow (11) and compare effectiveness (14) of considered hedging strategies. Results obtained numerically for different expected prices and different risk aversion coefficients are presented in Table 2. The bold type indicates the best of all analyzed strategies. Neither strategy is domi-

Table 2

The effectiveness of hedging ζ for different expected prices and risk aversion coefficients

(a) $\lambda = 1$	$E(S_T)$ 0,5	0,8	0,9	1	1,1	1,2	1,5
Forward	**0,00%**	**0,00%**	**0,00%**	**0,00%**	7,94%	18,77%	57,89%
OTM put	14,55%	11,55%	7,32%	1,23%	**1,47%**	**2,64%**	**3,58%**
ATM put	8,29%	7,12%	4,83%	0,79%	2,77%	5,28%	7,94%
ITM put	4,39%	3,95%	2,83%	0,45%	4,20%	8,44%	14,17%
Collar	10,51%	7,89%	4,58%	0,45%	4,10%	10,75%	40,36%
Participator	5,51%	4,53%	2,90%	0,26%	4,52%	10,40%	27,65%

(b) $\lambda = 5$	$E(S_T)$ 0,5	0,8	0,9	1	1,1	1,2	1,5
Forward	**0,00%**	**0,00%**	**0,00%**	**0,00%**	11,80%	50,40%	399,11%
OTM put	97,22%	78,76%	55,64%	26,17%	7,52%	**5,29%**	**17,04%**
ATM put	48,92%	43,05%	32,49%	15,60%	5,97%	10,69%	39,19%
ITM put	23,94%	21,96%	17,36%	8,15%	5,94%	18,06%	72,22%
Collar	64,83%	50,00%	32,32%	11,09%	**2,12%**	14,46%	181,48%
Participator	30,72%	25,69%	17,76%	5,80%	2,31%	15,58%	108,37%

(c) $\lambda = 10$	$E(S_T)$ 0,5	0,8	0,9	1	1,1	1,2	1,5
Forward	**0,00%**	**0,00%**	**0,00%**	**0,00%**	11,80%	50,38%	649,17%
OTM put	289,03%	235,62%	174,23%	102,31%	52,38%	28,35%	**20,30%**
ATM put	121,76%	108,74%	86,81%	53,83%	29,99%	22,23%	47,85%
ITM put	53,59%	49,85%	41,34%	25,64%	16,23%	20,30%	89,93%
Collar	171,72%	134,89%	93,22%	45,42%	15,09%	**7,72%**	153,63%
Participator	70,93%	60,22%	44,12%	21,19%	**6,92%**	7,75%	89,11%

Source: own calculations.

nated by the other one. If any of them gives higher effectiveness for certain expectation level, then its quality is weaker for the other expected price. For example, the perfect hedging guarantees optimal strategy when the investor does not expect price increases. However simultaneously it is the worst strategy for the investor who expects large or medium price growth.

The best strategy for weakly risk averse investor seems to be buying OTM put options. Such an option entirely hedges downside risk, it is relatively cheap and does not limit the profits from price increase. It is clear, that given the expected fall in prices, OTM put options have worse hedge properties than ITM or even ATM options. The superiority of this strategy over the other also disappears with the increase of investor risk aversion.

For the hedge purpose option strategies seem to be very reasonable. Collar is quite a flexible strategy because of the possibility to use different strikes ranges. In our case, however, when the strikes range is (0,9; 1,1268), this strategy is almost dominated by Participator strategy.

Participator is the best option strategy when investor does not specify any of his or her own expectation on the price movements in future. When he anticipates the fall in prices then hedging with ITM puts is slightly better. Only in the case, when investor expects high price increase, this strategy seems to be inadequate. Relatively to other strategies, hedging effectiveness of Participator rises when the investor becomes more averse to take a risk. It is because of similarity of its payoff profile to short forward payoff, although its break even point has to be worse (0,9464 in our experiment). Analyzing the whole range of expectations variability for different risk averse coefficients it seems to be reasonable to emphasize the Participator strategy. It protects the hedger against the risk of the underlying asset price falls below the fixed limit. At the same time hedger retains the possibility to benefit from the increase in the market price of the underlying asset. Besides, there are no costs associated with the acquisition strategy, which additionally supports the attractiveness of this strategy.

4. Summary

Hedging strategy is well designed when it eliminates entirely downside risk and does not create another risk to the investor by over-hedging. The best strategy does not exist in all circumstances and each of considered strategies can guarantee high effectiveness of hedging. The strategy choice depends on investor's subjective perception of future prices. If he or she is not interested in market state, the best strategy is always perfect hedging which entails the sale of all underlying assets in a forward contract. When the investor expects price increase and wants to take advantage of favorable price changes then more reasonable is to reach for options or more sophisticated hedging strategies. Such strategies allow the elimination of cost related to concluded options, better match of parameters to underlying position and taking advantage of the asset price growth.

NOTES

[1] An option is at the money (ATM) if the strike price is the same as current spot price of underlying asset.

[2] An option is in the money (ITM) if it has a positive intrinsic value. A call option is in the money when the strike price is below the spot price. A put option is in the money when the strike price is above the spot price.

[3] An option is out of the money (OTM) if it has no intrinsic value. A call option is out of the money when the strike price is above the spot price. A put option is out of the money when the strike price is below the spot price.

REFERENCES

Arrow, K. J. (1965). The theory of risk aversion. *Aspects of the Theory of Risk Bearing*, Helsinki: Academic Bookstores.

Baird, A. J. (1998). *Rynek opcji. Strategie inwestycyjne i analiza ryzyka.* Warszawa: Dom Wydawniczy ABC.

Battermann, H.L., Braulke, M., Broll, U., Schimmelpfennig, J. (2000). The preferred hedge instrument. *Economics Letters*, 66, 85–91.

Black, F., Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81 (3), 637–654.

Detemple, J.B., Adler, M. (1988). *Hedging with futures and options*, Studies in Banking and Finance, 5, 181–197.

Echaust, K. (2007). Hedging i spekulacja na rynku instrumentów pochodnych. *Rynek kapitałowy: skuteczne inwestowanie*, Wydawnictwo Uniwersytetu Szczecińskiego, Szczecin, 457–466.

Holthausen, D.M. (1979). *Hedging and the competitive firm under price uncertainty.* American Economic Review, 69, 989–995.

Hull, J.C. (1997). *Option, futures and Other Derivatives*, London: Prentice Hall International.

Karkowski, P. (2009). *Toksyczne opcje. Od zaufania do bankructwa.* Warsaw: Green-Capital.pl.

Lapan, C. H., Moschini, G., Hanson, S. (1991). Production, hedging and speculative decisions with options and futures markets. *American Journal of Agricultural Economics*, 73, 66–74.

Moschini, G., Lapan, H. (1995). The hedging role of option and futures under joint price, basic and production risk. *International Economic Review*, 36, 1025–1049.

Neumann, J. von, Morgenstern, O. (1944). *Theory of Games and Economic Behavior*, Princeton, NJ, Princeton University Press.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica* 32, January-April, 122–136.