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SCIENTIFIC ARGUMENTATION AND THE VALIDITY OF RESULTS

Abstract. It might seem that in exact sciences, in particular in mathematics, there is nothing interesting to say about Argumentation Theory. We have a well defined language of logic, several rules and we can expect the results following these rules. However, if the context of investigations is not well known, if the language we choose is completely new, even following well established rules, we gain some conclusions, which are not easily accepted. Hence, we can pose a question: which arguments are necessary to convince others of the obtained results. In this paper we would like to focus on this problem taking as an example natural numbers.

Keywords: scientific argumentation, truth, language, relations, natural numbers

1. Introduction

Argumentation theory involves different aspects and fields of reasoning. If we think of philosophical argumentations they are distinguished because of their premises and rules of inference. The other type of argumentation – negotiations – set a goal to achieve a financial success. Every day negotiations, instead, are often hasty and mistaken because of their generality; in oral conversations, humans often fail at simple logical tasks, committing mistakes in probabilistic reasoning [4], [20]. Perhaps, this is because, as some authors state, inferential processes carried out by mechanisms for reasoning, are unconscious [12]. We cannot talk about a logical relationship between premises and conclusions.

The field where logic is very important is science. We apply a kind of argumentation consisting of true expressions. We base our argumentation on logical derivation, on the well established inferential mechanism. However, as we will show, sometimes logic is not strong enough to convince others of the obtained results.

To exemplify what we have said above, we will limit ourselves to the field of mathematics and show that to prove any kind of mathematical truth, we can choose any system of axioms, under the condition that it is consistent with established mathematical truths, and following rules of inference, we obtain results that necessarily follow them. Surprisingly, it does not matter whether the system is abstract or not, whether there is or there is not any relationship to classical theory. The point is that if the system is not inconsistent, we cannot deny either the truth or the validity of the results. Hence, we have no arguments to abolish a theory and we are not convinced enough to approve it. In such a case, what can be done? It has happened many times in science that an application of a theory was found much later, as in the case of imaginary numbers. The situation is different in experimental sciences, like physics. The first argument to accept a theory is its application, we have to verify it by experiment, but in mathematics – in an abstract field of reasoning, we cannot use this argument.

Hence, in the following paper, we will apply the example of natural numbers to follow the investigated line of reasoning. We will explicate Frege's idea of what a natural number is. Next, we will present two classical approaches how natural numbers can be introduced. In Section 4 we will propose a totally new language based on a concept of relation and apply it in order to define natural numbers. Section 5 will contain the formal proof of classical truths in this new relational framework. We will conclude with a discussion of results.

2. Frege's Concept of Natural Numbers

Numbers, according to Frege [6], (§45) are not obtained from objects by abstraction; they have nothing physical, nothing subjective; they are not a representation. Numbers do not take origin from the union of one object to another. Numbers, according to Frege, refer to concepts. Frege defines them by the use of the statement "the concept F is equinumerous with the concept G".

The definition of the concept of "**equinumerous**" states that there is a one-to-one correspondence between the objects which fall under F and the objects which fall under G if and only if every object falling under F can be paired with a unique and distinct object falling under G and, under this pairing, every object falling under G gets paired with some unique and distinct object falling under F. A one-to-one correspondence (i.e. function) takes objects as arguments and maps these arguments to values. With a function Frege associates the concept of "course-of-values" which is a record of the value of the function for each argument, it is also called **the extension**

of a concept F (objects which fall under the concept F), known as *Frege's Basic Law V.*

Next, Frege describes the meaning of the statement: **the number of a concept F**', which is defined as the extension or set of all concepts that are equinumerous with F. In this way the number 0 is identified with the number of the concept of not being self-identical (§74), i.e. that nothing falls under this concept. The number 1 is identified with the class of all concepts for which exactly one thing falls, etc.

Continuing, to define natural numbers Frege introduces the relational concept **n follows immediately m** (§76) in the sequence of natural numbers if and only if:

1. there is a concept F and an object x such that: x falls under F,
2. n is the (cardinal) number which falls under the concept F,
3. m is the (cardinal) number which falls under the concept of the "object other than x falling under F".

At this point Frege does not write $n = m + 1$ because the equivalence relation will qualify $(m + 1)$ as an object, but according to him, a number is a concept and not an object.

Successively, he introduces the relation of successor in a sequence (§79) – "**y follows x** in φ-sequence" as follows: "for every concept F, y always falls under the concept F," if and only if:

1. every object for which x is in relation φ with it, falls under F,
2. for any object d, if an object d falls under F, every object for which d is in relation φ with it, also falls under F.

Formally, one can say that Frege introduces the relation $x < y$ in a sequence of natural numbers (in φ sequence). Precisely, the relation φ, for Frege, is a relation which is not necessarily thought of as a spatial or time order, even though this is not excluded.

Finally, Frege introduces a **sequence of natural numbers** and defines a **finite natural number**. We will stop here in Frege's investigations because only the main idea, what a natural number is, will be useful for our purpose in this paper.

3. Classical Constructions of Natural Numbers

The 20$^{\text{th}}$ century has witnessed several attempts to build mathematics on different grounds, not only those provided by classical logic. In non-classical logical frameworks, we have different systems describing the way of introducing natural numbers and as a consequence building arithmetic, such as the intuitionistic mathematics of Brouwer, Heyting's arithmetic, Church's arithmetic, etc. [1], [2], [18].

In this section I would like to present very briefly two classical approaches, in particular Peano's axiomatization and Von Neumann's construction of natural numbers.

3.1. Peano's Axiomatization

In the axiomatic system of natural numbers of Giuseppe Peano [15] we have three primitive notions: *a concept of natural number*, *a zero element* – 0, and a function called *a successor function which gives for n its successor* – n'. Hence, one can notice that Peano's idea is very close to Frege's intuition of natural numbers. Five axioms describe in a very simple and elegant way their properties.

Let us consider N as a set of natural numbers:

Axiom 1. $0 \in N$.

Axiom 2. $n \in N \longrightarrow n' \in N$.

Axiom 3. $n \in N \longrightarrow n' \neq 0$.

Axiom 4. $m, n \in N \wedge m' = n' \longrightarrow m = n$.

Axiom 5. $(Z \subseteq N \wedge 0 \in Z \wedge \forall k \in Z k' \in Z) \longrightarrow Z = N$.

Axiom 1 states that 0 is a natural number. Axiom 2 assures that every natural number has a successor and Axiom 3 that there is no model composed only of one natural number.

Similarly, Axiom 4 with Axiom 3 assure that we cannot create a model in which only a finite number of elements would be considered as natural numbers. The successor function maps in a bijective way (one-to-one) a set of natural numbers into its part. A one-to-one correspondence is assured by Axiom 4 and again we have a parallel to Frege's equinumerous concept. Due to Axiom 3 this one-to-one correspondence is into a proper part, because nothing has a successor equal to 0. Thus, the set of elements which fulfils Axioms 1–4 has to be equinumerous with its proper part, so it has to be infinite. If we threw out Axiom 4 we could construct the following model: 0 and 1 are natural numbers and 1 is a successor of 0 and 1 is a successor of 1, as well.

Finally, the *Induction Axiom* 5 assures that in the ordinal interpretation we cannot create a model including "infinite" natural numbers which is in agreement with Frege's idea of an infinite natural number.

There are various models of Peano's arithmetic [19], for example, let us call natural numbers all even numbers: $0, 2, 4, 6, 8, \ldots$. 0 will be considered as a *zero* element and $n+2$ – a successor of n. It is easy to verify that this model satisfies Peano's Axioms.

3.2. Von Neumann's Construction

Another construction of natural numbers was proposed by John von Neumann. Von Neumann [21] uses two primitive notions, exactly the same as those used in the axiomatization of the theory of sets of Zermelo-Fraenkel [9], [7], [17]: the *relation of being an element* – $\in$ and the concept of *set.*

The successor function of Von Neumann is defined in the following way:

Definition 1

For every set X: $\quad X' = X \cup \{X\}$.

which is a union of two sets: X and a singleton $\{X\}$. Such a set is called: *successor*. Additionally, Von Neuman states that there exists a null set – $\emptyset$ and for every set there exists its successor.

Theorem 1

There exists exactly one family of sets N with the following properties [9]:

(1) $\emptyset \in N$,

(2) $X \in N \longrightarrow X' \in N$,

(3) if K satisfies (1) and (2), then $N \subset K$.

Hence, Von Neumann's set of natural numbers is composed of the following elements: $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}, \}, ...\}$. Every element of this set defines a natural number: $\emptyset :=_{df} 0, \{\emptyset\} :=_{df} 1$, $\{\emptyset, \{\emptyset\}\} :=_{df} 2$, etc. The operation "′" corresponds to the operation of addition "+1" [9]. Every natural number n contains all numbers preceding it, $1 = \{0\}$, $2 = \{0, 1\}$, etc. At that point Frege considers numbers as classes.

4. Relational Language

Until now, natural numbers have always been introduced in terms of concepts and relations. Perhaps we can introduce them using only relations. Let us construct now a new language totally based on an abstract concept of relation. We will begin with two primitive notions: quality and binary relation as conceived by De Giorgi et al. [3], [5].

1. We will say that if q is a quality, then $q\,x$ means that the object x has the quality q. If nn is a quality to be a natural number then by writing $nn1$ we state that 1 is a natural number.
2. Given two objects x, y of any nature and a binary relation r, we will write $r\ x, y$ to say that "x and y are in the relation r". Sometimes instead of saying x and y are in the relation r we will say that x is in the relation r with y, for ex. $A \subseteq B$ states that $\subseteq A, B$, i.e. A and B are in a binary relation of inclusion – $\subseteq$.

Following [3], [5] we can introduce fundamental relations: $Rqual, Rrelb$, Rid which describe the behavior of qualities and relations.

Axiom 6

$Rqual$ is a binary relation.

1. if $Rqual\ x, y$ then $Qqual\ x$,
2. if $Qqual\ q$ then $Rqual\ q, x \equiv q\,x$.

Hence, q is a quality and x is a variable and they stand in a binary relation $Rqual$.

Axiom 7

$Rrelb$ is a ternary relation.

1. if $Rrelb\ x, y, z$ then $Qrelb\ x$,
2. if $Qrelb\ r$ then $Rrelb\ r, x, y \equiv r\ x, y$.

Axiom 8

Rid is a binary relation such that: $Rid\ x, y$ holds if and only if x and y are the same object.

$Rrelb$ is defined as a <u>ternary</u> relation which describes the behavior of a binary relation (cf. Axiom 7). On the other hand both $Rqual$ and Rid are originally defined as <u>binary relations</u> (Axioms 6, 8). We can note that the introduction of fundamental relations changes the perspective of considering the entities. In a certain sense it is an abstraction from the entities related in which both entities and the relation between them are considered on the same level. Hence, it would seem natural to consider *qual* and *id* as objects of a type we can call *unary relation* (for more details see [13], [14]) which can be defined as follows:

Definition 2

A unary relation is any "relation" $*$ such that $R*$ is a binary relation.

In the expression $Rrelp\ p, z$ $Rrelp$ indicates the fundamental relation (binary) connecting the unary relation p and its argument z (In [14] we called such types of relations *primary relations* to underlie their role in a system). One can notice, that unary relations underlie those binary relations in which at least one of the objects is defined in terms of the other. In literature [17] one can also find the term *unary relation* applied for subsets of a given set. In this perspective, the identity relation (cf. Axiom 8) will be defined as follows:

Axiom 9

id is a unary relation. Rid is a one-one binary relation for which $Rid\ x, y$ **iff** x and y are the same object.

Defining a concept of unary relation permits us to introduce another specific form of this relation, called tr relation and used for a definition of a kind of dynamic identity [14].

Axiom 10

tr is a unary relation. Rtr is a *one-one* binary relation such that: $Rtr\ x, y$ **implies** that x is not y (x and y are NOT the same object).

Now, we will give very briefly an outline of a definition of dynamic identity which will be used to construct natural numbers.

Definition 3

The dynamic identity triple DIT is composed of three distinct tr relations: tr_1, tr_2, tr_3.

Axiom 11

tr_1 is described by the binary relation Rtr_1 or, alternatively, by the unary operation[1] $Optr_1$ which acts in the following way: If $Rtr_1\ x, y$ then $Optr_1(x) = y_x$ where y_x means "y with x in y", which is to be interpreted in the (*mereological*) sense [10] of x being a part of y.

Axiom 12

tr_2 is described by the binary relation Rtr_2 or, alternatively, by the unary operation $Optr_2$ which acts on the result of $Optr_1$ in the following way: If $Rtr_2\ y, x$ then $Optr_2(y_x) = x_{y_x}; Optr_2$ transforms the result of

[1] We apply the concept of one-to-one correspondence or unary operation as it is conceived in a classical approach (see [17]) and describe in an axiomatic way the behavior of these three types of tr relations. We have adopted the original way of describing it.

$Optr_1(x)$, i.e. y_x, into a singular (mereological) class, (see [14], [10]) and gives as a result x_{y_x}.

Finally,

Axiom 13

Given tr_1 and tr_2, there is another tr relation tr_3 such that $Rtr_3\ z, tr_1$ iff $Rtr_3\ z, tr_2$.

Summarizing, $Optr_1$ acts as a function mapping y into x. $Optr_2$ transforms the result of $Optr_1$ into a class composed of only one element – itself. The image of $Optr_2$ is the generator with what has been generated by it in it. tr_3 is a pure relationship between tr_1 and tr_2, it is not a map. The main idea how these three relations behave is that any change of one's features causes changes in two other entities because of their mutual dynamic relationship. Such a model was thought for physical phenomena, in particular for elementary particles.

5. Natural Numbers in Relational Language

Now we will try to adopt this relational framework to construct natural numbers. We will show that the statements of Peano's system become theories in this new language. This part will be slightly formal.

Let us abbreviate some expressions. Let us put at the place of $Optr_1$ – f, $Optr_2$ – φ and ($Rtr_3\ z, x$ iff $Rtr_3\ z, y$) – $x \Diamond y$

If $tr_1 = x$ then by Axioms 11, 12, 13

1. $f(x) = y_x$ – which means x is a proper part of y.
2. $\varphi(f(x)) = \varphi(y_x) = x_{y_x} = x_{f(x)}$ – which represents a singular (mereological) class in which $f(x)$ is a proper part of x and is "identical" to x in a sense that there is no difference between them [14] in this universe.
3. $y_x \Diamond x_{y_x} \Leftrightarrow f(x) \Diamond x_{f(x)}$. Hence, we can substitute $x_{f(x)}$ by $f(x)$ because of the equivalence relation.

We can construct natural numbers in the following way:

$x :=_{df} 0$
$f(x) = f(0) = y_x :=_{df} 1$
$\varphi(f(x)) = \varphi(f(0)) = \varphi(1) = x_{y_x} = x_{f(x)} = x_1 = 0_1$
$1 \Diamond x_1$

$f(x_1) = f(1) :=_{df} 2$
$\varphi(f(x_1)) = \varphi(f(1)) = \varphi(2) = x_{f(x_1)} = x_2 = 0_2$
$2\Diamond x_2$

$f(x_2) = f(2) :=_{df} 3$
$\varphi(f(x_2)) = \varphi(f(2)) = \varphi(3) = x_{f(x_2)} = x_3 = 0_3$
$3\Diamond x_3$, etc. ...

We obtain the following sequence, beginning with 0, composed of natural numbers (tr_1 relations) and singular classes (tr_2 relations).

$x = 0 \rightarrow_f 1 \rightarrow_\varphi x_1 \rightarrow 2 \rightarrow x_2 \rightarrow 3 \rightarrow x_3...$

We can notice that the natural number n and its corresponding singular class x_n form an equivalence class[2].

We shall introduce now some simple definitions and assumptions in order to define natural numbers.

Axiom 14

0 is a natural number.

Definition 4

A natural number (beginning from 1) is a tr_2 relation.

[2] (1) $\Diamond$ is reflexive: $x\Diamond x$. By Axiom 13: Rtr$_3$ z,x $\Leftrightarrow$ Rtr$_3$ z,x, which is a tautology.

(2) $\Diamond$ is symmetric: $x\Diamond y \Rightarrow y\Diamond x$. Let us put a: Rtr$_3$ z,x $\Leftrightarrow$ Rtr$_3$ z,y, b: Rtr$_3$ z,y $\Leftrightarrow$ Rtr$_3$ z,x. Hence, we have to prove that $a \Rightarrow b$. By Axiom 13 we have even more than a simple implication: $a \Leftrightarrow b$.

(3) $\Diamond$ is transitive: $x\Diamond y$ and $y\Diamond r \Rightarrow x\Diamond r$. Let us put a: Rtr$_3$ z,x $\Leftrightarrow$ Rtr$_3$ z,y, b: Rtr$_3$ z,y $\Leftrightarrow$ Rtr$_3$ z,r, c: Rtr$_3$ z,x $\Leftrightarrow$ Rtr$_3$ z,r. We have to prove that $a \wedge b \Rightarrow c$. We can make the following substitution applying some substitution rules for equivalence, implication and De Morgan Laws [17] (for a moment, let us substitute $Rtr_3 z, x$ by zx, $Rtr_3 z, y$ by zy and $Rtr_3 z, r$ by zr):

$(zx \Leftrightarrow zy) \equiv (zx \Rightarrow zy) \wedge (zy \Rightarrow zx) \equiv \sim (zx \wedge \sim zy) \wedge \sim (zy \wedge \sim zx) \equiv (\sim zx \vee zy) \wedge (\sim zy \vee zx)$

Thus: $(zx \Leftrightarrow zy) \wedge (zy \Leftrightarrow zr) \equiv (\sim zx \vee zy) \wedge (\sim zy \vee zx) \wedge (\sim zy \vee zr) \wedge (\sim zr \vee zy) \equiv (\sim zx \vee zy) \wedge (\sim zy \vee zr) \wedge (\sim zr \vee zy) \wedge (\sim zy \vee zx) \equiv \sim (zx \wedge \sim zy) \wedge \sim (zy \wedge \sim zr) \wedge \sim (zr \wedge \sim zy) \wedge \sim (zy \wedge \sim zx) \equiv (zx \Rightarrow zy) \wedge (zy \Rightarrow zr) \wedge (zr \Rightarrow zy) \wedge (zy \Rightarrow zx) \equiv (zx \Rightarrow zr) \wedge (zr \Rightarrow zx) \equiv (zx \Leftrightarrow zr)$

Definition 5

Let $i \Diamond x_i$, and g be a successor function defined as follows:
$g(0) = f(0) = 1$
$g(1) = f(x_1) = 2$
$g(2) = f(x_2) = 3$
etc. ...
$g(n) = f(x_n) = n + 1$

If we consider P to be a set of natural numbers then we state that:

Corollary 1

$\langle P, 0, g \rangle$ is a model of Peano's axioms.

Proof.

1. The first Peano axiom (Axiom 1) is assured by Axiom 14.
2. For every natural number n, such that $n \geq 1$ if n is a tr_2 relation then by Definition 5: $g(n) = f(x_n)$, which by Axiom 11 is a tr_2 relation. Thus $g(n)$ is also a tr_2 relation.
3. 0, by construction, is a successor of no natural number.
4. Let k, l be natural numbers such that $g(k) = g(l)$.
   Let us notice that $g(k) = f(x_k)$ and $g(l) = f(x_l)$, $g(k) = g(l)$ implies that $f(x_k) = f(x_l)$. Because $x_k \Diamond k$ (x_k can be substituted by k [7]) then $f(x_k)$ becomes $f(k)$. This implies that $f(k) = f(l)$. By Axioms 10, 11, f is '1-1' which implies that $k = l$.
5. Let A be such that: 0 is an element of A and if n is an element of A then $g(n)$ is an element of A. In other words if n is a natural number then $g(n)$ is also a natural number. From general rules for quantifiers [17] we will have: $\forall n$ such that n is a natural number, $g(n)$ is a natural number.
   Summarizing: 0 is an element of A and for every n if n is a natural number then $g(n)$ is also a natural number (cf. (2)), so every natural number is an element of A. □

6. Discussion of Results

Coming back to Frege, **the number** of the concept F is defined as the extension or **set of all concepts that are equinumerous with F**. This number **is also identified with the class** of all concepts under which **n** objects fall. In this way the number 0 is identified with the number of the concept of not being self-identical, the number 1 is identified with the class of all concepts for which exactly one thing falls under F, etc.

We have defined tr_1 as x: $tr_1 = x :=_{df} 0$, $tr_2 = f(x) = y_x :=_{df} 1$. In this sense 1 could be interpreted as a one-to-one correspondence between x and y. We can do that because of mapping between x and y. Moreover, we have showed that $\varphi(y_x) = x_{y_x} = x_1$, is a singular (mereological) class, a class composed only of its unique element – itself. Finally, $y_x \Diamond x_{y_x} \Leftrightarrow 1 \Diamond x_1$ – the number 1 is identified with this singular class x_1, etc. In this perspective natural numbers reflect entities describing a relationship between more basic concepts: *tr* relations. In this way the number 1 is not interpreted as a number of elements, but as a set of all concepts for which we have one-to-one correspondences. This interpretation is nothing else than Frege's concept of number. Hence, we have constructed natural numbers as conceived by Frege, but in a totally different way. How do we approach these results?

A great number of authors would base their evaluation on the correctness of the way of reasoning. Sometimes reasoning enables to produce arguments to convince others and to accept valuable information, but sometimes we point out a problem not to convince others of the truth of our opinion, but to meet the challenges of others [12]; the opinion of a group is always better than even that of its best member.

Hence, we can pose a problem in another way. In the example described above we have used arguments that are true, we followed the rules of classical logic. On the other hand the results might seem not to be strong enough to convince about the validity of the presented theory although our reasoning was based on general knowledge and we made decisions based on them. We have defined a language and rules, and we followed them. At this point we can behave like reviewers of scientific manuscripts, we can look for flaws in different papers either to justify or to reject results. The problem is that perhaps there is little empirical research on this topic or there is poor performance – the lack of elegance and beauty required in mathematical papers. By the way, the last one plays a great role in decision making [8].

The discussed example, hence, is an exemplification of a case where there is little empirical research on the topic. The relational language is new, very abstract, not well known enough, and this might also explain some lack of conviction in accepting results. There are so many different theories. How to gain the certaintity that we are not wrong? When the same problems are placed in a well known argumentative setting, a theory turns out to be appropriate and innovative which is not the case in a not well known context.

In the latter it seems that even the truth cannot support a theory. Even exact and formal argumentation cannot defend the opinions. Perhaps this explains some resistance of the scientific society in the face of new inventions, even if the motivation is pure curiosity about what is true. Nowadays, more often than in the past, authors are asked to give applications of their theories and this takes something away from the beauty of doing research and discovering the world. Like Johnson, I have to admit that scientists, which means also me, are interested in truths and when they discover something they would like to convince others of that truth. In general, "pure" mathematicians are not interested in applications of their theories but are attracted by the logic of the world and the beauty of discovery.

It happens that the only argument we can use to convince others of our results is time, which is not logical. Time will show whether results are worth to accept or not, as has happened many times in our history. However, argumentations motivated in this way can distort evaluations and attitudes and allow erroneous beliefs to persist. They are not in favour of approving of some decisions.

It might not be expected that even in science, argumentations consisting of truths sometimes are not strong enough to favour the conclusions for which they were found. Perhaps everything was calculated in our brain [16]. Perhaps the united forces of logicians, scientists, physicists and philosophers, one day, will transgress this limitation [11]. Anyway, it is comfortable to know that at the end truth should win, and this gives great satisfaction and courage to scientists to continue doing research.

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