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ON KINDS OF TEMPORAL LOGIC AND SOME OF ITS APPLICATIONS

Temporal Logic belongs to non-classical logics. Its systems started to be constructed at the end of the first half of the previous century. Up to now, a great variety of temporal systems have been constructed. The following systems are referred to as temporal logic systems: tense logic systems (whose precursor is A. N. Prior); logic of empirical time (containing a time variable) initiated by J. Łoś, and developed (as chronological logic) by N. Rescher and A. Urquhart, in particular; G. H. von Wright's temporal logic systems; systems of interval time logic (for instance, J. van Benthem's systems, Y. Venema's systems); temporal logic constructed in various versions of hybrid languages (for example, C. Areces's systems); and, at present, especially temporal systems using the notion of time in computer programmes (from standard systems of Linear Temporal Logic to various systems of Branching Temporal Logic). The enumerated types of temporal logic are independent of each other (what makes them different from each other are specific temporal functors appearing in them); however, they have a mutual property: they formalize (in various ways) some of time expressions.[1]

The diversity of temporal logic systems can lead to certain terminological confusion. More so because, despite the significant formal development of temporal logic as well a number of philosophical analyses devoted to it, an attempt at classifying temporal systems has not been made. Therefore, such classification will be the purpose of this paper. Since the issue of usefulness of such systems is often raised, this paper also aims to show (without analyzing them) their prospective applications.

The paper is divided into four parts. In each of them, there will be a brief outline of one of the four different formalisms referred to as temporal

[1] Therefore, it is not the question of arithmetic independence because there are several structural analogies between the systems discussed below.

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logic. The first three ones could be applied in natural sciences (physics, in particular), the last one was constructed to meet the needs of information science. The paper does not discuss all the kinds of temporal logic, although it depicts the most important ones (non-standard approaches are omitted).

I

One of the kinds of temporal logic are systems combining research on logical structure of grammatical tenses (tense logic) with logical analysis of temporal relationships undertaken by the philosophy of science (logic of time).

This integration took place because the analysis of grammatical tense entails applying a certain model of physical time.[2] The theses of this kind of temporal logic determine the meaning of various temporal functors appearing in them and these theses describe the properties of time in different ways[3] (in physics time can be treated as set of moments arranged linearly by the relation earlier/later $<$; the theses of tense logic describe a number of properties of this relation).

In the formal sense, tense logic is derived from modal logic, which is why it is often referred to as modal temporal logic. Its originator was A. N. Prior. He interpreted possibility functor and necessity functor temporally, introducing four sentence-forming functors, derived from a sentence argument, to the systems constructed by himself. These functors correspond to different grammatical tenses in natural language:

Hp – It has always been the case that p,

Pp – It has at some time been the case that p,

Gp – It will always be the case that p,

Fp – It will at some time be the case that p.[4]

The basic, minimal tense logic system is denoted by the symbol K_t. It was created by E. J. Lemmon. This system describes the basic properties of

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[2] Cf. J. Wajszczyk. *Logika a czas i zmiana.* Olsztyn WSP: 1995. pp. 7–8.

[3] Cf. R. P. McArthur. *Tense Logic.* Synthese Library. Vol. 111. Dordrecht/Boston 1976. pp. 1–51.

[4] Cf. A. N. Prior. *Time and Modality.* Oxford 1957. pp. 9–54. These functors can be interpreter in the following way: F (replaced the modal possibility functor) – possible in the future, G (replaced the modal necessity functor) – necessary in the future and P – possible in the past, H – necessary in the past (past equivalents of modal functors). Cf. E. Hajnicz. *Reprezentacja logiczna wiedzy zmieniającej się w czasie.* Warszawa: Akademicka Oficyna Wydawnicza PLJ 1996. p. 152.

the functors introduced by Prior and is independent from any assumptions concerning the properties of time. Every K_t thesis is basically the shortened notation of the corresponding thesis of classical sentential calculus interpreted temporally. Tense logic systems for the time which exhibits the appropriate properties are extensions of K_t systems. They include: linear tense logics, circular tense logic and branching tense logics.

The simplest extension of K_t system is CR system, also referred to as K_t4 system. It was constructed by N. B. Cocchiarella. The theses of this system signify only transitivity of earlier/later $<$. Because CR system describes only this property of this relation, it is the base system both for linear tense logic and for circular tense logic and branching tense logic.

Linear tense logic system, in which the relation $<$ is transitive and bilaterally linear was constructed by Cocchiarella and is denoted by the abbreviation CL. Linear tense logic system, in which the relation $<$ is transitive, bilaterally linear and lacking the initial moment of time (unfinished in the past) and the final moment of time (unfinished in the future) was constructed by D. Scott. This calculus is denoted by the symbol SL. Prior is the originator of the linear tense logic calculus denoted by the symbol PL. It is a system whose theses describe the transitivity of the relation $<$, its bilateral linearity, lack of the initial and final moment of time as well as density. In the linear tense logic, a circular tense logic system has also been constructed, namely, the one in which the relation $<$ is transitive, reflexive and symmetric. The originator of this calculus denoted by the symbol PCr, is also Prior.[5]

Apart from linear tense logics, branching tense logics systems have also been constructed, whose base system, as already mentioned, is CR system. The originators of the best-known system of these were N. Rescher and A. Urquhart.[6] They constructed the calculus denoted by the symbol K_b, describing transitivity and backward linearity of the relation $<$. Therefore, it is possible for the temporal chain to branch into the future.

In order to develop the formalization of time expressions and to expand the application of functors P, F, G, H, several possibilities have been used. One of these involves connecting temporal functors with metric indexes, that is, the metric logic of time clauses.[7] Metric indexes appearing as superscripts at functors represent specific time intervals which, in turn, indicate

[5] Cf. A. N. Prior. *Past, Present and Future.* Oxford 1967. pp. 32–76, 176–179.

[6] Cf. N. Rescher, A. Urquhart. *Temporal Logic.* New York 1971. Chapter 4.

[7] Cf. A. N. Prior. *Past, Present and Future.* pp. 95–112.

the time of the utterance (whether the described event took place before uttering the sentence, or whether it will take place after it).[8]

Another possible extension of tense logic is to introduce modal functors into its systems. One of these is, for example, OT system, within which the modal functors: M (it is possible that ...) and L (it is necessary that ...) coexist with temporal functors.[9]

The above mentioned temporal systems were built on top of classical sentence calculus. There exist tense logic systems constructed as the extension of quantifiers' calculus. The minimal time logic of quantifiers is the system denoted by the symbol QK_t (its variants are $QK_t{}^*$, $QK_t{}^{**}$).[10] Other systems are its extension, including QCR, QCL etc.[11]

As already mentioned, tense logic systems are, in a specific way, connected with the physical time model. Therefore, some researchers (e.g. S. Kiczuk) maintain that this kind of temporal systems could find their application in natural sciences (mainly in physics and cosmology). Tense logic systems could be applied in natural sciences (in the philosophy of natural sciences as well) on condition that the properties of physical time are appropriately presented by the theses of these systems. Specific axioms and theorems of tense logic systems would have to be true sentences in the natural model of time. The systems that fulfils the condition is CL system (its theses describe properties of time, currently accepted in physics, namely, transitivity and linearity). Appropriate tense logic systems could provide these sciences (and philosophy of science as well) with the necessary tools, that is, the appropriate language and inference equipment. The elements of scientific language of the well-formed and appropriately used temporal systems could be used for specifying as well as accurately and precisely communicating some of the results of scientific cognition, connected with time. By applying appropriate logic, it could be possible to decide which reasonings expressed in a timed language are correct, depending on appropriate cosmological assumptions.[12]

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8 In order to, for example, formalize the sentence "It will rain in an hour's time from this moment" (more precisely than Fp), we use the following symbols to describe it: F^1p, assuming that in this example one hour is the basic time interval. Cf. R. P. McArthur. *Tense Logic*. pp. 5–6.

9 Cf. A. N. Prior. *Past, Present and Future*. pp. 113–136.

10 Ibid. pp. 137–174.

11 Cf. R. P. McArthur, H. Leblanc. *A Completeness Result for Quantificational Tense Logic*. "Zeitschrift für Mathematische Logik und Grundlagen der Mathematik" 1975. pp. 89–96.

12 This kind of logic would be connected not with the mathematical language of physics, but with its notional language (similar to the colloquial one, but enriched with the specialist terminology).

II

With the use of P, F, G, H functors the main sentences containing verbs of various grammatical tenses are formalized. Whereas, in order to formalize natural language time expressions as well as time expressions derived from numerous sciences, temporal logic systems, containing functors different from those used in tense logic, were constructed. To such systems belong, inter alia, systems with a time variable, containing R functor, interpreted in the following way:

R t p – p is realized at the time t (it is at time t that p).

Temporal logic systems of this kind were initiated by the Polish logician J. Łoś, who made the first attempt (which means earlier than Prior) to construct a formal system of temporal logic, by constructing, in 1947, the first modern (the logic theory of time variable dates back to Aristotle and Megarian-Stoic school, and was also developed in the Middle Ages) logic of empirical time.[13] The next systems containing time variable were constructed and developed, especially by Rescher, and also, to a some extent, by Urquhart and Garson.[14] Temporal systems in which R t p formula appears were denoted as temporal logic (or chronological logic) by them in order to distinguish of these temporal systems from tense logic systems.

It is worth mentioning that recently a minimal logic of empirical time (ET) was constructed, studied and presented by M. Tkaczyk in his work *Logika czasu empirycznego*. This logic includes the laws of usage of the expression "at the time" within the area of physical discourse. The functor of temporal realization denoted by the symbol R[15] is introduced for it.

Systems of empirical time are at present formally constructed and philosophically significant; they could also find their application in physics. However, logic of empirical time can be used not only in physical theories, but also everywhere where temporal relationships stated empirically are in question.

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[13] Cf. J. Łoś. *Podstawy analizy metodologicznej kanonów Milla*. "Annales Universitatis Mariae Curie-Skłodowska" 2 (1947) z. 5. pp. 269-301.

[14] Cf. N. Rescher, A. Urquhart. *Temporal Logic*. pp. 31–54 and N. Rescher. *Topics in Philosophical Logics*. Dordrecht, Holland: 1968. pp. 196–228.

[15] Cf. M. Tkaczyk. *Logika czasu empirycznego*. Lublin: Wydawnictwo KUL 2009. p. 5.

III

Apart from the systems given above, there also exist other ones, which are distinct from these systems, also referred to as temporal logic. These are systems which contain temporal functors whose equivalents in colloquial language are the following expressions: *and next*, *and then.* Temporal logic systems, within which such functors appear, were constructed by a Finnish logician G. H. von Wright. These systems are And Next and And Then.[16] The first one was constructed in 1965, and the other one in 1966. In these systems there appear temporal functors (sentence-forming from two sentence arguments) of the so-called timed conjunction (denoted by the symbol T).

Based on the system And Next, T functor is interpreted *and next.* Therefore:

pTq – p *and next* (in the moment directly following) q.

Based on the And Then, T functor is interpreted *and then.* Therefore:

pTq – p *and then* (sometime then) q.

The use of *and next* functor in the And Next system presupposes discreetness of the time structure. Whereas, the And Then system presupposes transitivity and linearity of time. Therefore, as can be seen, just as tense logic systems, von Wright's systems in a certain way are connected with the model of physical time. Hence, they also could find their application in natural sciences (mainly, in physics and cosmology). It particularly concerns the And Then system, which appropriately describes the properties of physical time. Its specific theses are sentences which are true in the natural model of time.[17] Therefore, this system could provide natural sciences (and philosophy of science as well) with the necessary tools, that is, the appropriate language (for specifying as well as accurately and precisely communicating some of the results of scientific cognition, connected with time) and inference equipment (for determining which reasonings expressed in a timed language are correct, depending on appropriate cosmological assumptions).

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[16] Cf. G. H. von Wright. *And Next.* "Acta Philosophica Fennica" 18 (1965). pp. 293–304 and Ibid. *And Then.* "Commentationes Physico-Mathematicae" 7: 32 (1966). pp. 1–11.

[17] Cf. S. Kiczuk. *Problematyka wartości poznawczej systemów logiki zmiany.* Lublin: Redakcja Wydawnictw KUL 1984. p. 226.

IV

With the increase in the number of uses of computers in many areas of life, the notion of time started to be used in computer programmes. In the 1970s completely different systems from the ones enumerated above appeared, which were also referred to as temporal logic system and which were used in computing sciences, in particular.

R. M. Burstall and A. Pnueli are considered to be the precursors of this kind of temporal logic, derived from modal logic. In 1974 R. M. Burstall for the first time suggested applying modal logic in computer science, whereas A. Pnueli systematized the logic presented by Burstall as TL (Temporal Logic) and it is the Pnueli's paper titled *The Temporal Logic of Programs* is regarded as a turning point in the use of modal logic in broadly understood reasoning about computer programmes (this kind of temporal logic was also constructed by, inter alia, Z. Manna, E. A. Emerson, A. Galton). In his paper, Pnueli introduced functors referring to the future tense:

□ p – p is satisfied in all the states (*always*),

◊ p – p is satisfied in at least one state (*sometime*).

Classic temporal logic in this version also contains the following functors:

○ p – p is satisfied in the next state after the reference state (*next*),

pUq – p is satisfied until q is not satisfied (*until*).[18]

In the first temporal logic systems of this kind, only the future states were analyzed. However, in the systems that were constructed later the past states started to be taken into consideration, and past functors were introduced to temporal logic (they are a mirror reflection of the future functors):

■ p – p has been satisfied in all the states (*has always been*),

♦ p – p was satisfied in at least one state (*once*),

● p – p was satisfied in the preceding state relative to the reference state (*previous*),

pSq – p has been satisfied since (recently) q was satisfied (*since*).[19]

[18] Cf. R. Klimek. *Wprowadzenie do logiki temporalnej.* Kraków: Wydawnictwa AGH 1999. pp. 16, 25.

[19] Ibid. pp. 47.

In this kind of temporal logic systems there can exist different modifications of the above-mentioned functors, and they can also differ in the system of written symbols used (notation).

Now the different systems of temporal logic presented in this section will be given; they are divided into different systems depending on the concept of time they assume.

Classic temporal logic was constructed on the basis of linear concept of time, which is why it is called Linear Temporal Logic – LTL. Depending on the assumed concept of time, past tense or future tense functors are introduced (or not) to temporal logic. Linear temporal logic which includes both types of functors is denoted by the symbol LTL(B). The logic whose formulae do not include past functors is denoted by the symbol LTL(F), and the logic whose formulae do not include future functors is denoted by the symbol – LTL(P).

On the basis of the concept of branching time, Branching Temporal Logic – BTL was constructed. There exist many different branching temporal logic systems, e.g. Unified System of Branching Time – UB and its versions: UB^+ and UB^-. Another branching logic (with a wider range of capabilities than UB) is Computation Tree Logic – CTL and its versions: CTL^+ and CTL^*. The extension of CTL is Fair Computation Tree Logic – FCTL, and Extended CTL – ECTL as well as Extended CTL^+ – $ECTL^+$.

Apart from the above-mentioned systems, there exist many other modifications and extensions of the above discussed temporal logic, including ITL – Interval Temporal Logic, ETL – Extended Temporal Logic, TLA – Temporal Logic of Action as well as different RTTL – Real-Time Temporal Logic systems. First-order temporal logic has also been constructed: First-Order Linear Temporal Logic – FOLTL as well as First-Order Branching Temporal Logic – FOBTL.[20] New temporal logic systems used in computing science are continuously being constructed.

The primary use of temporal logic in computer systems is especially specification and verification of programmes (the language of temporal logic can be applied to specification of the wide range of computer systems, the methods of this logic can be applied for verification), and synthesis of programmes and logic programming. Temporal logic is one of the most important formalisms used for the broadly understood reasoning on concurrent systems, and it also ranks high in all the development forecasts for formal tools used in the analysis of such systems.

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[20] Ibid. pp. 24–99.

It is worth adding that temporal logic systems can also find their application in the so-called artificial intelligence (e.g. formalization of such categories as events, actions, plans in the context of time). Such systems have been constructed by, inter alia, A. L. Lansky, E. Lafon and C. B. Schwind, R. S. Crouch and S. G. Pulman.[21] Among logicians in Poland involved in research into temporal logic and its applications, especially in computing science, are K. Trzęsicki[22] and R. Klimek.

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Rounding up the discussion included in this paper, it is worth noting that apart from the above-mentioned kinds of temporal logic, there exist other formalisms aspiring to be called temporal logic ones, e.g. systems of interval time logic (inter alia systems constructed by J. van Benthem, Y. Venema), or temporal logic constructed in different versions of hybrid languages (e.g. systems constructed by C. Areces); they are not discussed in this paper. Non-standard approaches have been omitted.

Summing up, it is worth noting that in our times temporal logic particularly refers to: tense logic, logic of empirical time (containing a time variable), von Wright's systems of temporal logic, and temporal logic systems that make use of the notion of time in computer programmes. All these systems are independent of each other and can find their application in various sciences. The issue of mutual relationships among the above-mentioned kinds of temporal logic and the analysis of their uses constitute an area of further research.

SUMMARY

The paper contains short characterization of various kinds of temporal logic. Possible fields of applications of them are also given. Parts I, II and III of the article present systems of temporal logic that may be used in natural sciences (mainly physics and cosmology): tense logic, logic of empirical time (containing a time variable) and von Wright's systems of temporal logic. In Part IV, the last part of the paper, systems of temporal logic that make use of the notion of time in computer programs are presented. Non-standard approaches have been omitted.

[21] These systems were thoroughly discussed in, e.g., E. Hajnicz. *Reprezentacja logiczna wiedzy zmieniającej się w czasie.*

[22] Cf. K. Trzęsicki. Logika temporalna. *Wybrane zagadnienia.* Białystok: Wydawnictwo Uniwersytetu w Białymstoku 2008.