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A PEIRCIAN RECONCILIATION OF THE OLD AND THE NEW LOGIC*

Abstract. With the ascendancy of Peter Strawson's account of the categorical sentences of traditional logic, Sterling Lamprecht offered an alternative. The present paper attempts to bolster Lamprecht. It does this by distinguishing and offering different logical forms for each of the categorical sentences of traditional logic. One of the three does fit with the form now in almost all logic texts. A second is really a matter of plural quantification. However a third, suggested in Lamprecht and found in Peirce, should be dealt with in terms of restricted quantifiers when these are instantiated in a suitable way. These are instantiated in a suitable way. Quine cited Peirce to this effect. It is this restricted quantification scheme that does yield a full Square of Opposition and benefits beyond that as well. In this paper I offer a formal account thereby saving the traditional claims with special restricted quantifiers. These quantifiers have rules that parallel unrestricted ones.

Keywords: Expanding the Pierce-Quine, Account/Defense of Traditional Logic, Pierce's Defense of Traditional Logic

1. Distribution – Quine's Use of Peirce Against Geach

The doctrine of distribution is a central theme in the traditional logic of categorical syllogisms. One speaks of the subject and predicate terms as to whether they are distributed or not. Keynes says that "a term is said to be distributed when reference is made to all of the individuals denoted by

* I am particularly indebted to the Polish philosophical tradition in a personal as well as a professional way. My most important doctoral advisor was Henry Hiz. His friend George Krzywicki-Herbut was a C.U.N.Y. colleague as well as my sometime ski instructor. They served in the Polish armed forces in the early days of the second World War and met in a prisoner of war camp. Unfortunately I did not spend as much time with Professor Grzegorczyk. However, he was generous in meeting with me in Warsaw. I gained much from his conversation, and later from his correspondence and his publications. His History of Logic should be better known. It is a notable addition to the unique Polish tradition of writing on this history. Andrzej Grzegorczyk was unconventional. He maintained both the most rigorous approach in logic and an existentialist stance on broader philosophical questions.

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it ... undistributed when they are referred to only partially ..." [Geach, 1962, p. 28]. There are rules of quantity and of quality for evaluating the validity of such syllogisms. Those of quantity apply to the distribution of terms. "Illicit process" is the name for violating the rule which requires that a term distributed in the conclusion must be distributed in a premise. The fallacy of "Undistributed middle" violates the rule requiring that the middle term (the term occurring once in each premise) be distributed in one of these premises. I shall anachronistically argue that these rules constitute an algorithm, a decision procedure, for testing the validity of standard categorical syllogisms.

In a somewhat well known piece, Peter Geach claimed that the doctrine of distribution is seriously flawed. Then in a much less well known piece Quine defended Peirce's version of the doctrine. In his review of Geach's *Reference and Generality* [1962], Quine [1964] maintained that Geach had failed to show that the doctrine of distribution is defective. What a surprise it is to see Quine defending a doctrine of traditional Aristotelian logic. He comments that the purported flaws Geach cites are not in the doctrine itself, but in flawed accounts of it. Quine likens the situation to one in which Boolean Algebra would be condemned because of flaws in Boole's manner of explaining it. "One might as well denounce Boolean Algebra by fastening on Boole's mistakes and confusions" [Quine, 1964, pp. 100–1]. Geach followed a deplorable practice of reading authors, especially past ones, in a narrow unsympathetic, if not biased, spirit. The material on distribution appears in the first chapter of Reference and Generality. It had appeared as an article and was reprinted in at least one collection. Quine's review is short. He does not spend much time on the topic and his rejection of Geach's claim consists of citing some lines he says are derived from Peirce.[1]

[1] I could not find this material at the place in Peirce where Quine locates it.

It is somewhat ironic that Quine should cite Peirce. Peirce was aware of the history of logic in ways that Frege, Russell, Quine himself and other proponents of the "existence is what existential quantification expresses" approach were not. Unlike Quine and his precursors, Peirce connected quantification with products and sums. His notation for the universal and the "existential" quantifier were "Π" and "Σ". As far as I know he did not speak of the "existential quantifier" or link existence and quantification. The historian Bocheński comments on Peirce's view as a "rediscovery" of ideas found in the Terminist Albert of Saxony [Bochenski, 1961, p. 349]. Peirce wrote on Ockham and this Terminist tradition. They treated categoricals in terms of a descent to conjunctions and disjunctions of singular sentences. This aspect of supposition theory furnished part of the motivation for adopting an all/some – and/or material adequacy condition for the quantifiers. As Albert of Saxony stated

> A sign of universality is one through which a general term to which it is adjoined is denoted to stand, in a conjunctive manner, for every one of its values (supposita).
>
> A sign of particularity is one through which a general term is denoted to stand, in a disjunctive manner, for every one of its values [Moody, 1953, p. 45]

Distribution comes to be registered by the word "every" when we paraphrase the four forms into terms of identity and distinctness thus:[2]

> All S are P: Every S is identical with some P or other.
> Some S are P: Some S or other is identical with some P or other.
> No S are P: Every S is distinct from every P.
> Some S are not P: Some S or other is distinct from every P.
> [Quine, 1964, p. 100]

Quine does not say more, but his point should be clear. The presence or absence of something like universal quantification with respect to the traditional subject and predicate terms coincides with what the doctrine of distribution claims. In other words, the logical form of the four types of categorical sentences coincide with whether the quantifications (quantifier phrases) or their equivalents are universal or particular (so-called "existential" generalizations).

2. Canonical Notation, Paraphrase, and Regimentation

A few words are in order about views on canonic notation, paraphrase and regimentation. Quine 1960, pp. 157–61 offers a distinctive view of the artificial language of first order predicate logic supplemented by sentences of English (natural language) which serve as paraphrases of the artificial notation and at the same time are supposed to improve upon, i.e., regiment other English locutions. Quine's official canonical notation consists of certain predicate logic sentences and their English paraphrases. These sentences contain unrestricted universal and particular/"existential" quantifiers, truth functional sentence connectives, predicates, individual variables (no names-names are Quinized into definite descriptions, and then Russelled away), and an identity sign. No claims are made about synonymy being a requirement for paraphrasing into the artificial symbolic notation, or for paraphrasing from non-canonical English forms to canonical ones.

There are three goals served by Quine's conception of a canonical notation:

1. as an aid to communication,
2. for deduction, and

[2] The idea is from [Peirce, 1931–1935, 2.458].

3. for delineating a philosophical set of categories ("the inclusive conceptual structure of science – called philosophical, because of the breadth of the framework concerned" – The quest of a simplest clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality") [Quine, 1960, p. 161]

ad 1. As an aid to communication one function of the canonic notation is to resolve ambiguity. The ambiguity of the sentence "Dogs are friendly" can be resolved by supplying a universal or a particular quantifier.

ad 2. The notation of predicate logic and its canonic English paraphrase constitutes the content of logical theory, the science of deduction.

ad 3. A conspicuous example of the philosophical utility of this canonical notation for Quine is the use of the particular/existential quantifier to express existence claims and to help to determine ontological commitment. In one respect I would add an element to Quine's account of the aims of a canonic notation. For the purposes of communication and deduction it is necessary to include an appeal to linguistics. Linguistic differences are connected with logical differences. Moreover, this addition is in keeping with Quine's naturalistic methodology, ensuring among other things that we are conservative and do not mutilate linguistic data.

In his *Methods of Logic* [Quine, 1982, pp. 93, 95; p. 81] three types of each categorical sentence are taken as paraphrasable into predicate logic notation as unrestricted generalizations of conditionals and conjunctions. For simplicity's sake, I concentrate on the A form sentences. There is the plural form with a restricted quantifier, 'All A are B', the restricted quantifier singular copula form, 'Every A is a B', and the unrestricted quantifier conditional form, 'If anything is an A then it is a B'. To repeat, Quine does not claim that the relation between these three sentences is that of synonymy. Outside of the Geach review Quine seems to hold the view that there need not be any one right solution as to which of the three is to be preferred. Given a sentence and a context any one of these three forms may serve as canonical English versions of a predicate logic unrestricted universal generalization over a conditional. They are put on a par in *Methods of Logic*. But in the review, for the purpose of defending the doctrine of distribution, he seems to give special prominence to a slight variant, the singular copula form. This occasions a problem. Whereas on Quine's official view of canonical notation each of these English sentences is on a par, from the standpoint of the review (trying to capture the notion of distribution and its use in logic) we have to single out the singular copula restricted quantifier sentence for special consideration.

3. Resolution

Let us follow the Peirce line taken in Quine's review. We will take the singular copula form for special consideration and try to follow Quine's practice in holding that logical forms are revealed in terms of a variant of first order predicate logic translations. To reveal the traditional patterns of distribution, i.e., universality and particularity (existential quantification), I suggest regimenting Quine's four Peircean categorical sentences, such as "Every A is some B or other", into restricted quantifier singular copula claims and then representing them in predicate logic as follows:

Every A is a B	as	$(x, Ax)[(\exists y, By)x = y]$
At least one A is a B	as	$(\exists x, Ax)[(\exists y, By)x = y]$
No A is a B	as	$(x, Ax)[\neg(\exists y, By)x = y]$
At least one A is not a B	as	$(\exists x, Ax)[\neg(\exists y, By)x = y]$

We now have a closer correlation between (a more perfect paraphrase of) these quantified singular copula English sentences and their predicate logic formulations. Both the predicate logic formulations and the English sentences consist of restricted quantifiers: 'Every A', 'At least one B'/'B, and a singular form of the copula dealt with in terms of identity. This regimentation and its predicate logic expression is the one that least mutilates the natural language singular copula generalizations. It might be that the identity involved is a version of the "dreaded" identity theory of the copula (or of predication). As such, it might provide a tool for re-examining the debates surrounding that notion.

Adhering to Quine's naturalistic methodology, and in particular, to the maxim of being conservative (his maxim of minimal mutilation) when choosing between different hypotheses, we note the following. The plural 'All A are B' involves restricted plural quantification. It differs from the standard unrestricted universal quantification over a conditional form. And along the same line of reasoning, this plural form should also be distinguished from the above singular copula form. So, instead of taking all three forms on a par as Quine [1982, p. 81] and many others do, we should distinguish them. Only the last, the conditional English sentence, is best taken as a canonical English paraphrase of the predicate logic unrestricted universal generalization of a conditional, '$(x)(Ax \to Bx)$'. As far as I know in the main body of his work, Quine did not follow this policy of distinguishing the three forms of categorical sentences. He equated all three forms and deals with them in terms of unrestricted quantification.

As an aside, here are some conjectures as to why one might be led with Geach to doubt the doctrine of distribution. If we take the plural 'All A are B' or the conditional 'If anything is an A, then it is a B', forms as basic, then there is no sign in English of a special quantifier phrase associated with the predicate. Doing this makes the doctrine of distribution for the predicate term highly questionable. When we concentrate on the plural and conditional forms, it seems intuitive that the subject term is distributed. But given scope considerations we might think that the predicate is distributed as well, since the English quantifiers in these cases can be taken as having scope over the predicate position. Matters are made even worse for seeing whether a traditional predicate involves a distribution pattern, when we follow the mutilating tradition of ignoring the logical contribution of the copula. This occurs when the predicate, e.g., 'is a human', is construed holophrastically with the copula and the indefinite 'a' parts of a fused predicate, e.g., 'is-a-human', and playing no distinct roles.

As noted above, Quine's favored canonical role for quantification allows only for unrestricted quantifiers. He holds the view that where there appears to be a need for restricted quantification, it can be restated as, i.e., reduced to, an unrestricted quantification by the expedient of treating restricted universal/existential quantifications as unrestricted universal/"existential" quantifications over conditionals/conjunctions. (Quine, Set Theory and its Logic p. 235) The desired restricted A form, $(x, Ax)(\exists y, By)[x = y]$, appears in canonic predicate logic notation in terms of unrestricted quantification as

$$(x)[Ax \rightarrow (\exists y)(By \& x = y)]$$

and the restricted I form, $(\exists x, Ax)[(\exists y, By)x = y]$, as

$$(\exists x)(Ax \& (\exists y)(By \& x = y)].$$

But doing so, creates problems. To begin with, the I form would not be a logical consequent of the A form as it is in traditional logic and its full square of opposition.[3] Another problem concerns distribution. The above treatment amounts to abandoning the quest to explicate the doctrine of

[3] Some (e.g., Dorothy Edgington, Gyula Klima) propose adding the conjunct $(\exists x)Ax$ to the unrestricted version of the A form, to yield

$$(x)[Ax \rightarrow (\exists y)(By \& x = y)] \& (\exists x)Ax.$$

The A form does imply the I form, but the term A is now both distributed and undistributed. The constraints for paraphrasing enlisted below in the paper mitigate against all such uses of unrestricted quantifications in connection with the singular 'Every A is a B' form. It violates the maxim of minimal mutilation to supply the English singular copula sentence with a predicate logic representation containing elements of conditionality and conjunction.

distribution. If we try to explain distribution in terms of the unrestricted quantifier and its scope, the predicate as well as the subject is included in that scope. This conflicts with the doctrine of distribution view that the predicate is not distributed. If we say that the predicate is both distributed and undistributed, we surrender the exclusive nature that the distinction is supposed to have.

Even assuming that the rules of quantity which pertain to the doctrine of distribution are accounted for, how would one begin to give a unified account of the rule of quality and a unified account of both the rules of quantity and of quality? The singular copula approach with its own rules of derivation (see the next section) will accomplish all of this. We assume the constraints on being a categorical syllogism, i.e., having three categorical sentences with exactly three terms arranged so that the two appearing in the conclusion, appear separately, one in each premise and the third term (the middle term) appears once in each premise. The rule for illicit process records the fact that one cannot derive a universal claim from an existential one. The remaining rule of quantity, i.e., undistributed middle, and the rules of quality pertain to the element of identity. Undistributed middle insures that the terms of two different identity claims provided by the premises allow for substitutivity which will yield the identity claimed in the conclusion. Considerations concerning the substitutivity of identity require that one of the premises be an identity claim when the other premise is an inidentity claim. We can't have two negative premises, since two inidentities in these contexts will not yield a conclusion. Lastly, if one premise involves an identity claim and the other doesn't, then a valid conclusion must be an inidentity claim which results from substitution in the inidentity premise in terms of the remaining identity premise.

4. Providing a Formal System

To some extent I am tempted to try to leave the matter of supplying a formal framework open, and allow that any number of different systems might be taken to fit in with the above approach. Restricted quantification is in keeping with work done on plural quantification along the lines either of Neale [1990] or Bach's [1989] restricted quantification treatments, or in some ways with the Oxford binary quantification accounts of Wiggins [1985] or Davies [1981]. Both these restricted quantifiers and the binary ones are grounded in a theory of generalized quantifiers derived from Mostowski's paper on generalized quantifiers [1967]. It has become better known in logic and linguistics through the work of Barwise and Cooper [1981]. However, I will put aside the above suggestions and take a different approach – one which will adapt Benson Mates' method of Beta-variants. This adaptation of Mates provides us with truth conditions for my special restricted quantifiers. Here is a sketch of a system of rules of inference along the lines of the tree method and some considerations motivating these rules.

The account I favor begins by utilizing the restricted quantification notation adopted for the categorical sentences given in the previous sections. We need an account of the instances appropriate to these generalizations. A key idea is that a restricted universal generalization can be instantiated with an appropriate sentence containing a demonstrative noun phrase. For example, 'Every human is a mammal', should imply by a universal instantiation rule 'This/that human is a mammal'. It should not imply 'That horse is a mammal'. The task then is to come up with a suitable predicate logic notation, rules of inference, and truth conditions.

I start with a notation and rules for trees which will fit in with the project so long as we confine ourselves to classic Aristotelian syllogisms and the square of opposition. These will be followed by a fuller system which takes into account more complex cases and relates restricted and unrestricted quantification.

Add to the language of predicate logic restricted quantifiers such as (x, Ax), $(\exists x, Bx)$. These quantifiers are the representations in predicate logic form of English quantified noun phrases: Every A, At least one B. Placing these in front of appropriate open sentences yields well formed formulas. We need symbolic counterparts of English demonstrative noun phrases, e.g., This A, That B, which will serve as the canonical substituends of restricted quantifiers and occur in the sentences serving as canonical instances of such generalizations. Since these canonical substituends are singular terms, use lower case letters with superscripts, e.g., a^1, b^2, etc., in a special way. Just as the English 'this A' somewhat formally indicates by the presence of the same noun that it is an appropriate substituend for the restricted quantifier 'Every A' use a lower case letter (with a superscript), e.g., a^1, a^2, that is the lower case version of the capital letter occurring in the quantifier phrase, e.g., (x, Ax). In this notation, Ba^1 would be a correct or canonical substituend for the formula $(x, Ax)Bx$, but Bb^1 would not. The former corresponds to the correct inference that (this apple) is red given that (Every apple) is red, while the latter would be like reasoning that (this bird) is red since (Every apple) is red. Using the same letter of the alphabet in a lower case as the restriction on the quantifier, mimics, in our notation, the relation, in English, of the restriction on the natural language quantifier to its canonical demonstrative noun phrase. The superscript on the singular term serves to distinguish the substituends, e.g., a^2, b^1, for restricted quantifiers, from the ordinary singular terms, such as, a, b, c, x, y, which serve as substituends for variables of unrestricted quantifiers.[4]

We adopt the method of trees/semantic tableaux for unrestricted quantifiers and also adapt it by formulating rules of universal and existential instantiation and quantifier exchange for restricted quantifiers as follows:

Tree Rules

The following rules of inference applying to restricted quantifiers are added to the standard tree rules for unrestricted quantifiers.

Restricted Universal Instantiation

$$\frac{(x, Ax)\phi x}{\phi a^1}$$

(as individual constants use the lower case letter of the restriction on the quantifier with superscripts to distinguish these canonical instances for restricted quantification from instances associated with unrestricted quantification)

Particular "Existential" Instantiation

$$\frac{(\exists x, Ax)\phi x}{\phi a^i}$$

where a^i is new to the tree

Quantifier Interchange (Duality)

$$\frac{\neg(x, Ax)\phi x}{(\exists x, Ax)\neg(\phi x)}$$

$$\frac{\neg(\exists x, Ax)\phi x}{(x, Ax)\neg(\phi x)}$$

This system will do for classic Aristotelian logic. Here is an example of how it works for **Darii**: Every C is an A, Some B is a C, so Some A is a B. The tree closes:

[4] More sophisticated non-syllogistic reasoning requires a more sophisticated type of mimicking. See below **The Full System**.

	1. $(x, Cx)[(\exists y, Ay)x = y]$	
$\surd$	2. $(\exists x, Bx)[(\exists y, Cy)x = y]$	
$\surd$	3. $\neg(\exists, Ax)[(\exists y, By)x = y]$	
	4. $(x, Ax)\neg[(\exists y, By)x = y]$	3 Quantifier Interchange
$\surd$	5. $[(\exists y, Cy)b^1 = y]$	2 E.I., i.e. Existential Instantation
	6. $b^1 = c^1$	5 E.I.
$\surd$	7. $[(\exists y, Ay)c_1 = y]$	1 U.I.
	8. $c^1 = a^1$	7 E.I.
$\surd$	9. $\neg[(\exists y, By)a^1 = y]$	4 U.I.
	10. $(y, By)\neg a_1 = y$	9 Quantifier Interchange
	11. $\neg a^1 = b^1$	10 U.I.
	12. $\neg c^1 = b^1$	8, 11 Identity
	13. $c^1 = b^1$	6 Identity
	$\times$	

However, a full system also has to

a. connect restricted and unrestricted generalizations and

b. take cognizance of complex restrictions on quantifiers and reasoning involving them.

I offer the following notation and rules.

Let '$(tx^n)\Psi x^n)$' be a singular term. In quasi English it says 'this/that Ψ'.

III. The Full system

The **full** rule of **Restricted Universal Instantiation**

$$\frac{(x, \Psi x)\Phi x}{\Phi x^n(tx^n)(\Psi x^n)}$$

We also have **full Restricted Particular "Existential" Instantiation**

$$\frac{(\exists x, \Psi x)\Phi x}{\Phi x^n(tx^n)(\Psi x^n)}$$

where n is new to the tree

and **full Quantifier Interchange (Duality)**

$$\frac{\neg(x, \Psi x)\Phi x}{(\exists x, \Psi)\neg\Phi x}$$

$$\frac{\neg(\exists x, \Psi x)\Phi x}{(x, \Psi x)\neg\Phi x}$$

This will still only do the job that the notation and rules for "simple/fused" predicates did. As it stands, it does not cover reasoning with complex predicates. We arrive at a quite natural solution by focusing on the demonstrative noun claims serving as instances. To begin, note that the English expressions 'this' and 'that' serve two roles. They serve as determiners in noun phrases: '(That oak desk) is heavy', and they can stand alone as demonstratives: '(That) is heavy'. In the right circumstances, both '(That oak desk)' and '(That)' demonstrate the same item. So when standing alone, t^3, can be thought of quite naturally as being like an individual constant, a name. The following is a familiar equivalence pertaining to names.

$Fa \leftrightarrow (\exists x)(x = a \& Fx)$

So for $Hx^1(tx^1)(Dx^1)$, i.e., That desk is heavy, we put

$(\exists x)(x = t^1 \& Dx^1 \& Hx^1)$, i.e. That is a desk and it is heavy.

And in general we take as a contextual definition:

$$\Psi x^n(tx^n)(\Phi x^n) =_{def.} (\exists x)(x = t^n \& \Phi x) \& \Psi x)$$

It is quite simple showing that the reasoning from 'That brown dog is friendly' to 'That dog is friendly' is valid.

The truth conditions for this system can be a variant of Mates' method of beta variants [Orenstein, 1999; 2000]. One might also be able to provide some other method, a substitutional quantifier account or perhaps following the lines of generalized quantifiers.

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