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Roman Murawski

PHILOSOPHY OF LOGIC AND MATHEMATICS IN THE WARSAW SCHOOL OF MATHEMATICAL LOGIC*

Abstract. In the paper philosophical ideas concerning logic and mathematics developed in the Warsaw School of Mathematical Logic are considered. The views of two important representatives of this school – Alfred Tarski and Andrzej Mostowski – are analyzed in detail.

The Warsaw School of Mathematical Logic was a part of the Lvov-Warsaw School of Philosophy. It belonged to the most important centers of mathematical logic between the wars. It is natural to ask what were the philosophical views and attitudes of logicians in Warsaw towards mathematics and logic itself. One can also ask whether and to what extent those views influenced formal and technical research, whether that research had its source in philosophical considerations or was it independent of any philosophical presuppositions. Did the philosophical views bind the technical investigations or were they without meaning for them?

The attitude of Polish logicians and mathematicians towards the philosophy of mathematics can be shortly characterized as follows: they saw the mathematical and philosophical foundations of mathematics as independent although connected in a way and indispensable for understanding logical and mathematical activity. With two exceptions (Chwistek and Leśniewski) they represented a view guided by the following two principles:

- all commonly accepted mathematical methods should be applied in metamathematical investigations,
- metamathematical research cannot be limited by any *a priori* accepted philosophical standpoint.

On the other hand, logic and mathematics have their own genuine philosophical problems which should not be neglected. In particular, although

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metamathematical results do not solve philosophical controversies about mathematics and logic, yet the former illuminate the latter.

What were the sources of such an attitude? One can indicate two of them. The first one can be exemplified by Sierpiński's work on the axiom of choice (AC) and its applications in mathematics. In his French paper [1918] on the role of AC, Sierpiński distinguished two independent questions:

- philosophical controversies around this axiom,
- its place in proving mathematical theorems.

According to Sierpiński the second issue should be investigated independently of the philosophical inclinations concerning the problem whether the axiom of choice is to be accepted or not. This opinion was included in all editions of Sierpiński's books on set theory from 1923 (*An Outline of Set Theory*, 1923) to 1965 (*Cardinal and Ordinal Numbers*, 1965). In [1965, p. 94] he wrote:

> Still, apart from our personal inclination to accept the axiom of choice, we must take into consideration, in any case, its role in set theory and in calculus. On the other hand, since the axiom of choice has been questioned by some mathematicians, it is important to know which theorems are proved with its aid and to realize the exact point at which the proof has been based on the axiom of choice; for it has frequently happened that various authors have made use of the axiom of choice in their proofs without being aware of it. And after all, even no one questioned the axiom of choice, it would not be without interest to investigate which proofs are based on it and which theorems are proved without its aid – this, as we know, is also done with regard to other axioms.

This means simply that one should disregard philosophical controversies (and treat them as a "private" matter) and investigate (controversial) axioms as purely mathematical constructions using any fruitful methods.

The second source of the discussed attitude of Polish mathematicians and logicians towards philosophy was the tradition of Polish analytic philosophy originated by Kazimierz Twardowski in Lvov. According to Twardowski and his students, we must clearly and sharply distinguish world-views and scientific philosophical work. This idea was particularly stressed by Łukasiewicz, the main architect of the Warsaw school of logic. He regarded various philosophical problems pertaining to the formal sciences as belonging to the world-views of mathematicians and logicians, but the work consisting in constructing logical and mathematical systems together with metalogical and metamathematical investigations constituted for him the subject of logic and mathematics as special sciences. Hence philosophical

views cannot be a stance for measuring the correctness of formal results. Yet philosophy may serve as a source of logical constructions.

One of the consequences of the described attitude of Polish logicians and mathematicians was the fact that they did not attempt to develop a comprehensive philosophy of mathematics and logic (Stanisław Leśniewski and Leon Chwistek were here the exceptions!). They formulated their philosophical opinions concerning mathematics or logic only occasionally and only on problems which just interested them or on which they actually worked. Consequently there were in Poland no genuine philosophers of mathematics. Philosophical remarks were formulated by logicians and mathematicians only on the margin of their proper mathematical or logical works (and had no meaning for the results themselves).

The current trends and views in the philosophy of mathematics, i.e., logicism, intuitionism and formalism, were of course well known (and there appeared papers discussing those tendencies, their meaning and development). But none of them was represented in the Warsaw School. Moreover, it did not represent any other trend; it had no official philosophy of logic and mathematics. This followed from the belief that logic and mathematics are autonomous with respect to philosophy. Opinions in the field of the philosophy of logic and mathematics were treated as "private" problems and philosophical declarations were made reluctantly and seldom. If they were made then it was stressed, directly or indirectly, that these were personal opinions.

Though some of the logical investigations were motivated by philosophical problems – e.g. the many-valued logics by Łukasiewicz – the formal, logical constructions were always separated from their philosophical interpretations. Another example is the investigation of intuitionistic logic carried out among others by Tarski without accepting intuitionism as the philosophy of mathematics. The programme of Janiszewski [1917] and the Polish School of Mathematics created set-theoretical foundations of mathematics in a methodological and not philosophical sense.

What were the separate philosophical opinions formulated by Polish logicians, philosophers and mathematicians? We shall answer this question considering the philosophical views of two representatives of the Warsaw School of Mathematical Logic: Alfred Tarski (1901–1983) and Andrzej Mostowski (1913–1975). Tarski belonged to the first generation of the Warsaw School; Mostowski, to the second generation.[1]

[1] For logic and the philosophy of logic and mathematics in Poland between the wars see the basic monograph [Woleński, 1989] as well as [Woleński, 1992; 1993; 1995; Murawski and Woleński, 2008; Murawski, 2004; 2011].

Alfred Tarski was interested in philosophical problems and very actively participated in the philosophical life of his time. He was convinced of the philosophical significance of his works, in particular of his work on truth [Tarski, 1933]. He described himself as [Tarski, 1944]:

> Being a mathematician (as well as a logician, perhaps a philosopher of a sort) [...]

Tarski's philosophical attitude was anti-metaphysical; he supported the idea of scientific philosophy. He accepted a programme of "small philosophy" which aims at detailed and systematic analysis of the concepts used in philosophy. Such a philosophy is minimalistic, anti-speculative and sceptical towards many fundamental problems of traditional philosophy. This attitude was inherited by Tarski from the Lvov-Warsaw School and strengthened by contacts with the Vienna Circle. He maintained also empiricism and abandoned the analytic/synthetic distinction. He stressed that logical and empirical truths belong to the same generic category. Influenced by Leśniewski and Kotarbiński he was inclined to a rather strongly nominalistic understanding of expressions. One finds many places in which he confirmed this. E.g. during a symposium organized by the Association for Symbolic Logic and the American Philosophical Association held in Chicago on 29^{th}–30^{th} April 1965 and devoted to the philosophical implications of Gödel's incompleteness theorems, he said (cf. [Feferman and Feferman, 2004, p. 52]):

> I happen to be, you know, a much more extreme anti-Platonist. [...] However, I represent this very [c]rude, naïve kind of anti-Platonism, one thing which I would describe as materialism, or nominalism with some materialistic taint, and it is very difficult for a man to live his whole life with this philosophical attitude, especially if he is a mathematician, especially if for some reasons he has a hobby which is called set theory.

In the biography of Tarski written by Fefermans one finds more such quotations, for example (cf. [Feferman and Feferman, 2004, p. 52]):

> I am a nominalist. This is a very deep conviction of mine. It is so deep, indeed, that even after my third reincarnation, I will still be a nominalist. [...] People have asked me, 'How can you, a nominalist, do work in set theory and logic, which are theories about things you do not believe in?' ... I believe that there is value even in fairy tales.
> [I am] a tortured nominalist.

They write also: "Elsewhere Tarski has said more specifically that he subscribed to the reism or concretism (a kind of physicalistic nominalism) of his teacher Tadeusz Kotarbiński".

Mostowski wrote about Tarski so (cf. [1967, p. 81]):

> Tarski, in oral discussions, has often indicated his sympathies with nominalism. While he never accepted the 'reism' of Tadeusz Kotarbiński, he was certainly attracted to it in the early phase of his work. However, the set-theoretical methods that form the basis of his logical and mathematical studies compel him constantly to use the abstract and general notions that a nominalist seeks to avoid. In the absence of more extensive publications by Tarski on philosophical subjects, this conflict appears to have remained unresolved.

Tarski was inclined to identify mathematics with the deductive method. He maintained that there is no hard borderline between formal and empirical sciences. He admitted the rejection of logical and mathematical theories on empirical grounds. He claimed also that there is no sharp demarcation between logical and factual truth and that the concept of tautology is unclear.

One must stress that all those were his "private" philosophical views which did not influence his logical and mathematical research; in other words, his research was independent of any philosophical presuppositions. In the paper "Über einige fundamentale Begriffe der Methodologie der deduktiven Wissenschaften" [1930] he explicitly wrote:

> [...] it should be noted that no particular philosophical standpoint regarding the foundations of mathematics is presupposed in the present work.

This was typical for him and for the whole Warsaw School in logic. This independence of logical and mathematical studies and philosophical views explains the cognitive conflict and discrepancy between Tarski's nominalistic and empiricistic sympathies and his "platonic" mathematical and logical practice. Note that his attitude enabled him to contribute to various important foundational streams without the necessity of accepting their philosophical assumptions and attempting to reconcile the philosophy and the research practice. His programme of metamathematics can be summarized by his words from the paper [Tarski, 1954] where he wrote:

> As an essential contribution of the Polish school to the development of metamathematics one can regard the fact that from the very beginning it admitted into metamathematical research all fruitful methods, whether finitary or not.

Andrzej Mostowski inherited his general philosophical attitude from Tarski. He freely used infinitary methods and strongly insisted that no formal work should be limited by philosophical assumptions. However it seems that Mostowski felt himself obliged to a more extensive and systematic treatment of his views in the philosophy of mathematics. In a review of Mostowski's *Thirty Years of Foundational Studies* [1965] published in *Studia Logica* R. Suszko characterized him as a "mathematician-logician, to whom the philosophical aspect of logic and the theory of the foundations of mathematics is not alien" [Suszko, 1968, p. 169].

In many of his technical papers and works Mostowski stressed in the introductory sections or prefaces the importance and indispensability of certain philosophical presuppositions. He discussed also the possible philosophical consequences of technical mathematical results presented there. But such comments and remarks were always reduced to a minimum and had no influence on the technical considerations.

In the *Introduction* to the monograph *Teoria mnogości* (Set Theory) written together with K. Kuratowski.[2] they wrote [1952, p. vi]:

> There exists so far no comprehensive philosophical discussion of the basic assumptions of set theory. The problem whether and to what extent abstract concepts of set theory (and in particular of those parts of it in which sets of very high cardinality are considered) are connected with the basic notions of mathematics being directly connected with the practice has not been clarified so far. Such an analysis is needed because by Cantor, the inventor of set theory, basic notions of this theory were enwrapped with a certain mysticism.

On the other hand the authors are convinced that the meaning and importance of set theory for the foundations of mathematics were demonstrated also in connection with the philosophy of mathematics.

And they declare that the most important feature of set theory is the fact that it provides a *tool* for other parts of mathematics which are directly connected with applications.

The philosophical remarks were made only in the Introduction. One finds in the book no further philosophical declarations or statements. In the whole book the authors strictly distinguish (in the spirit of Sierpiński) the philosophy of the axiom of choice and its role in mathematics and set theory itself – all theorems in which AC is used are marked by a small circle.

[2] The discussed remarks were reprinted in the second and third Polish editions of the book.

Mostowski considered also philosophical problems in connection with Gödel's incompleteness theorems. As in the case of set theory he indicated only the philosophical problems connected with the discussed mathematical issues and showed possible solutions but avoided any fixed and definite philosophical declarations. Moreover the philosophical comments were reduced to a minimum.

He stressed that we do not have a precise notion of a correct mathematical proof. In the paper [1972, p. 83] Mostowski emphasizes that: "A mathematical proof is something much more complicated than a simple succession of elementary rules contained in the so called inference rules. [...] Therefore one must necessarily show moderation in stressing the role of logical rules in [mathematical] proofs". On the other hand the author is sure that despite the fact that the old program of formalization of mathematics has been practically waived "the collaboration of logic and mathematics was fruitful and probably will still bring important results" [p. 83].

Note also that the three trends in the philosophy of mathematics which dominated in the 20s and 30s of the 20$^{\text{th}}$ century (logicism, intuitionism and formalism) were the starting point of Mostowski's series of lectures *Thirty Years of Foundational Studies* [1965]. He stressed there that they gave rise to the development of three directions in logico-mathematical investigations: constructivism, metamathematical and set-theoretical ones. But in the main text one finds no further philosophical remarks.

So far we have shown that Mostowski was aware of philosophical problems connected with mathematics but avoided making any explicit philosophical declarations. There is however one paper by him in which he makes explicit declarations, namely the paper *The present state of investigations of the foundations of mathematics* (see [Mostowski, 1955a] and [Mostowski, 1955b]). Unfortunately there is a problem of interpretation: the paper was written in the first half of the fifties and the ideological atmosphere of that time could have had an influence on it. It is not possible now to decide to what extent outside factors influenced the paper. On the other hand the author could have restricted himself to purely mathematical issues and avoided entirely any philosophical remarks and declaration. If he did not do so we can treat his remarks as genuine.

He states there (cf. [Mostowski, 1955a, p. 42]):

> [...] An explanation of the nature of mathematics does not belong to mathematics, but to philosophy, and it is possible only within the limits of a broadly conceived philosophical view treating mathematics not as detached from other sciences but taking into account its being rooted in natural sciences, its applications, its associations with other sciences and, finally, its history.

Investigations on the foundations of mathematics by mathematical methods affect the formation of a broader philosophical view. The results obtained there confirm – according to Mostowski (cf. 1955a, p. 42]):

> the assertion of materialistic philosophy that mathematics is in the last resort a natural science, that its notions and methods are rooted in experience and that attempts at establishing the foundations of mathematics without taking into account its originating in the natural sciences are bound to fail.

Hence, Mostowski represents here an empirical point of view in the philosophy of mathematics. As mentioned above it is not quite clear what was the influence of outside factors (in particular of the then dominant ideology) on those views. Specific expressions used by him may suggest such an influence. Such statements could be, at least partially, the price that had to be paid to the official philosophy. On the other hand, note that empirical (or quasi-empirical) trends have been since the sixties of the last century still more and more vivid in the philosophy of mathematics.

Mostowski admitted in various places that constructivism (especially its aims, not necessarily its solutions) was always very attractive to him (cf. [Mostowski, 1959, p. 192]). The reason for that was the fact that (cf. [Mostowski, 1959, p. 192]):

> it wants to inquire into the nature of mathematical entities and to find a justification for the general laws which govern them, whereas platonism takes these laws as granted without any further discussion.

He stressed that constructivistic trends in the foundations of mathematics are nearer to nominalistic philosophy than to the idealistic (in the Platonic sense) one. This nominalistic character implies that constructivism does not accept the general notions of mathematics as given but tries to construct them. "This leads to the result that one can identify mathematical concepts with their definitions" [Mostowski, 1959, p. 178]. The advantage of nominalism is the fact that several important mathematical theories have been reconstructed in a satisfactory way on a nominalistic basis and those reconstructions have turned out to be equivalent to the classical theories.

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Polish logicians and mathematicians, being convinced of the importance of philosophical problems and knowing quite well the current philosophical trends, treated logic and mathematics as autonomous disciplines independent of philosophical reflection on them, independent of any philosophical presuppositions. Therefore they sharply separated mathematical and

logical research practice and philosophical discussion concerning logic and mathematics. Philosophical views and opinions were treated as a "private" matter that should not influence mathematical and metamathematical investigations. On the contrary, in the latter all correct methods could and should be used. This "methodological Platonism" enabled Polish logicians and mathematicians to work in various areas without being preoccupied by philosophical dogmas. In controversial cases, as for example in the case of the axiom of choice in set theory, their attitude can be characterized as neutral – without making any philosophical declarations they simply considered and studied the various mathematical consequences of both accepting and rejecting controversial principles, and investigated their role in mathematics.

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Roman Murawski
Adam Mickiewicz University in Poznań
Faculty of Mathematics and Comp. Sci.
ul. Umultowska 87, 61–614 Poznań, Poland
rmur@amu.edu.pl