Coherence function in biomedical signal processing: a short review of applications in Neurology, Cardiology and Gynecology

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Abstract. The aim of this study is to present a coherence function, which can be used to find common frequencies of two signals and to evaluate the similarity of these signals. Another method is to use wavelet coherence function, which not only can find common frequencies of two signals, but also gives information when these frequencies appear. We would like to demonstrate the usefulness of coherence function in biomedical signal processing – in analysis of EEG, ECG, and uterine contraction activity signals. We have chosen four papers using coherence function in EEG analysis, four in ECG analysis and two in uterine contraction activity signals analysis (where we present some of our original work). Thus, these functions can be useful in analyzing two simultaneously recorded biomedical signals and they can provide some diagnostic value.

Introduction

The study of two signals, recorded simultaneously in one system, is a very interesting task. In case when the system under consideration is a part of human body and we record signals related to the activity of some organ, it can also provide a diagnostic value. For example – we can analyze EEG signals or signals representing uterine contractions and discover some abnormalities in the organ's functions.

To this end we can apply coherence function, which is based on Fourier transform. Word "coherence" is from the Latin word *cohaerentia* – it means natural or logical connection or consistency. The coherence function allows us to find common frequencies and to evaluate the similarity of signals. However, it does not give any information about time. There are two often used methods to calculate the coherence function: Welch method and MVDR (*Minimum Variance Distortionless Response*) method.

A transformation T is said to be linear if applied to linear combination of signals $ax + by$ gives linear combination of results $aT(x) + bT(y)$. Coherence function is based on Fourier transform, which is a linear transformation. However, the coherence function itself is not linear.

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Coherence function

Coherence function is defined as [4]

$$C_{xy}(\omega) := \frac{P_{xy}(\omega)}{\sqrt{P_{xx}(\omega)P_{yy}(\omega)}}, \tag{1}$$

where P_{xx} and P_{yy} are power spectra of signals x and y, P_{xy} is cross-power spectrum for these signals, ω is frequency. In case, when $P_{xx}(\omega) = 0$ or $P_{yy}(\omega) = 0$, then also $P_{xy}(\omega) = 0$ and we assume, that value $C_{xy}(\omega)$ is zero.

The power spectrum (also called periodogram) and cross-power spectrum are defined as [4, 20]

$$P_{xx}(\omega) := |\hat{x}(\omega)|^2 = \hat{x}(\omega)\overline{\hat{x}(\omega)}, \tag{2}$$

$$P_{xy}(\omega) := \hat{x}(\omega)\overline{\hat{y}(\omega)}, \tag{3}$$

where $\overline{x}$ is complex conjugate of x and

$$\hat{x}(\omega) := \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt \tag{4}$$

is the Fourier transform. It yields the information about frequencies occurring in signals and the dominant frequency for these signals.

Welch Method

A peak in the amplitude of cross-power spectrum means that there is a common frequency present in both signals. However, it is possible that components with that frequency appear in analyzed signals at different time. Thus cross-power spectrum in practical applications may not be subtle enough – we need a method to locate frequencies present in both signals at the same time. To this end we compute the coherence function (or magnitude squared coherence) on averaged estimates of power spectra and cross-power spectrum of segments of the initial signal [4]. It is known as the Welch method and it was proposed by Welch in 1967 [17].

For discrete signals (time-series) we calculate the power spectrum for coherence function in the following way. The signal is divided into K fragments of length M (possibly overlapping)

$$x^l(n) = x(n + lD); \quad 0 \le l \le K - 1, \quad 0 \le n \le M - 1, \tag{5}$$

where D is chosen lag [20]. Selected signal's fragments are multiplied by a window function $w(n)$ and for all of them we compute the power spectrum

$$P_{xx}^{i}(f) = \frac{1}{E_w}\left|\sum_{n=0}^{M-1} x^{i}(n)w(n)e^{-2i\pi(f/f_{pr})n}\right|^{2}, \tag{6}$$

where

$$E_w = \sum_{n=0}^{M-1} w^2(n). \tag{7}$$

Window most commonly used is the Hamming function defined in the following way

$$w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right). \tag{8}$$

Then these modified (obtained for signal multiplied by window) periodograms are averaged in time [7, 20]

$$P_{xx}^{w}(f) = \frac{1}{L}\sum_{i=0}^{L-1} P_{xx}^{i}(f). \tag{9}$$

In order to obtain estimation of cross-power spectrum, we need to average partial cross-power spectra

$$P_{xy}^{i}(f) = \frac{1}{E_w}\left(\sum_{n=0}^{M-1} x^{i}(n)w(n)e^{-2i\pi(f/f_{pr})n}\right)\cdot\left(\sum_{n=0}^{M-1} y^{i}(n)w(n)e^{2i\pi(f/f_{pr})n}\right). \tag{10}$$

In practice the overlap of signal fragments is often chosen as 50%, i.e. $D = M/2$ [20], while the number K of intervals is chosen as a compromise between spectral resolution and precision of localization.

MVDR Method

Another method – the MVDR (*Minimum Variance Distortionless Response*) – was proposed by Capon in 1969 [3]. This method in many cases gives more precise results than the Welch method [1–2, 19]. It is based on a specific filter designed to minimalize the power of the output signal [1–3, 18–19]. More information about the MVDR method can be found in the papers by Benesty [1–2] and Capon's paper [3].

Example of the coherence function

In [Fig. 1] we show an example of application of the coherence function. In parts A) and B) of [Fig. 1] we can see chosen signals:

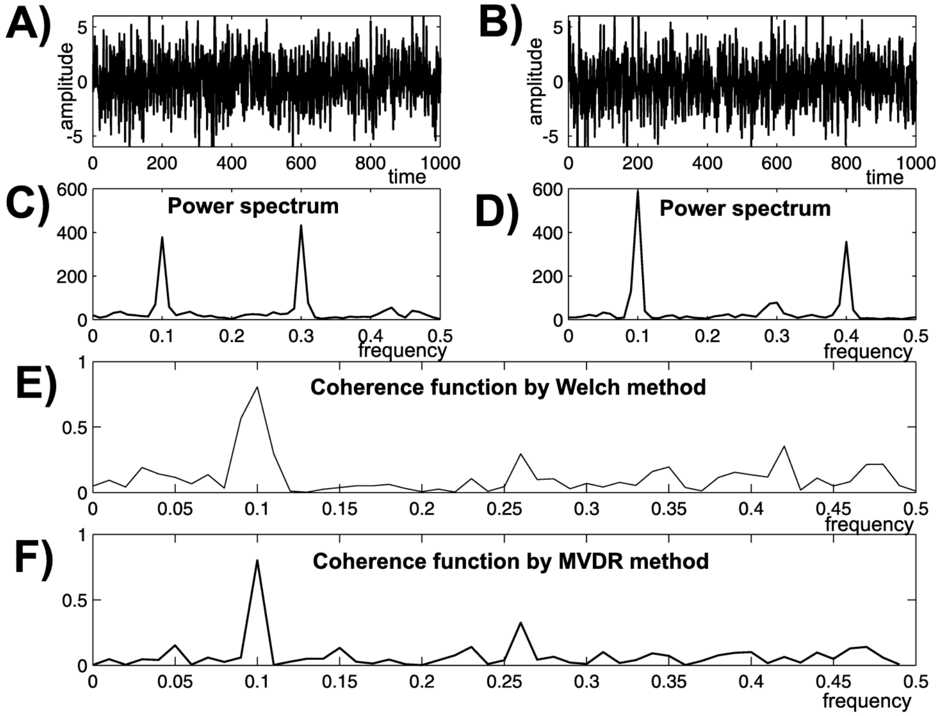

Fig. 1. Example of coherence function: A) signal $x(n)$, B) signal $y(n)$, C) power spectrum of signal $x(n)$, D) power spectrum of signal $y(n)$, E) coherence function by Welch method, F) coherence function by MVDR method (common frequency for two signals 0.1 Hz)

$$x(n) = k_1(n) + \cos(2\pi \cdot 0.1n) + \cos(2\pi \cdot 0.3n), \tag{11}$$

$$y(n) = k_2(n) + \cos(2\pi(0.1n + \psi)) + \cos(2\pi(0.4n + \varphi)), \tag{12}$$

where k_1 and k_2 are white noise, ψ and φ are phase shifts.

Graphs C) and D) show power spectra for these signals. We can observe that in signal $x(n)$ there are two frequencies 0.1 and 0.3 present and in signal $y(n)$ – 0.1 and 0.4. On graphs E) and F) we have plotted the coherence function (using power spectrum obtained by Welch or MVDR method). Here we can see one peak – common frequency for signals $x(n)$ and $y(n)$ is 0.1Hz. All programs were written in MATLAB (ver. 5.2, MathWorks Inc., Natick, USA). We also used the *coherence_MVDR* function written by Benesty (http://www.mathworks.com/matlabcentral/fileexchange/9781-coherence-function/content/coherence_MVDR/coherence_MVDR.m).

Application in biomedical processing

Coherence function in neurology and cardiology studies

In this section we present some applications of the coherence function. We have chosen eight interesting papers on this matter.

Coherence function finds application mainly in neurology (mostly in EEG studies) [8, 10, 15–16] and in heart rate variability investigations [5, 11–12, 14].

Coherence function was applied in EEG signal processing as linear synchronization measure by Quiroga et al. [10]. They studied signals from left and right rat's hemisphere. Authors claim that coherence function can be useful in the investigation of common frequency of EEG signals. They discovered that common frequency for two selected EEG channels is in range 1Hz-10Hz.

Coherence function was analyzed in brain stimulation studies [8]. Miranda de Sá and Infantosi used coherence as a detection parameter of evoked responses to rhythmic stimulation.

Coherence function was also used in studies of synaptic activity using head models and simultaneous recordings of MEG (magnetoencephalography) and EEG [16]. Authors claim that coherence function is useful for finding information about functional interactions across brain regions and neocortical source activity.

In the paper by Sherman et al. [15] we can find that coherence can be a measure between field potentials and EEG channels. They studied a model of linear association between channels during epileptic seizures.

In papers by Ruchkin [13] and Pereda et al. [9] there are interesting reviews of applications of coherence function in neurophysiology and EEG signal studies.

In the paper by Ropella et al. [12] we can find an interesting application of the coherence function in cardiology. Namely coherence was used to quantify the relation between some spectral elements of electrogram, taken from two sites in either the atrium or ventricle during both fibrillatory and nonfibrillatory rhythms. Authors present that nonfibrillatory rhythms exhibited strong coherence throughout the 1–59Hz band and fibrillatory rhythms – weak coherence throughout this band, and moreover harmonics are not present. They claim, that comparison of two electrograms with coherence function can provide discriminate between fibrillatory and nonfibrillatory rhythms. They also investigated the coherence function in various ventricular tachyarrhythmias [11]. In that paper they show, that it is possible to distinguish between monomorphic ventricular tachycardia, polymorphic ventricular tachycardia, and ventricular fibrillation by means of coherence function. Ropella et al. [11] claim that coherence can be a measure of rhythm "organization".

Also Sarraf et al. [14] claim that coherence function can be used to quantify rhythms organization. They applied this function to the surface ECG studies and found out that it can discriminate between atrial fibrillation and nonfibrillatory atrial rhythms.

Coherence function can be also used in analyzing surface ECG as well as ECG taken directly from the heart [5]. Common frequency was found in 0Hz–60Hz band. They found that propagation of ventricular fibrillation became more organized during the first 10s. It can be an optimal window for defibrillation.

Coherence function in uterine contractility studies

In our studies coherence function was used to evaluate uterine contraction activity. In paper [7] we presented the coherence function obtained by Welch and MVDR methods for four selected pairs of signals from different patients (patient with primary dysmenorrhea, patient with fibromyomas, patient with endometriosis and healthy woman). Signals were obtained during examinations by means of intrauterine pressure method (during menstruation). We recorded two signals – one from fundus and one from cervix.

We found that coherence function can be useful in the analysis of the synchronization of the uterine contractility. The lowest values of coherence function were in case of patient with primary dysmenorrhea, the highest – healthy woman.

In our second paper [6] one can find analysis of the coherence function in groups of patients with primary dysmenorrhea, with fibromyomas, and with endometriosis. Values of common frequencies are in band 0.044Hz–0.056Hz. Values of the coherence function also point to the similarity between signals from fundus and cervix. We also shown that the lowest values are for patients with primary dysmenorrhea.

Wavelet coherence function

In papers [6–7] we also present the wavelet coherence function. In [Fig. 2] we show an example of this function. First we obtain discrete wavelet decomposition (multi-resolution analysis) and find frequency band containing the dominant frequency ($D4$ band and $D7$ band in our example). To this end we reconstructed the approximation signal $A8$ and details D_j $(1 \leq j \leq 8)$. We obtain eight wavelet decompositions representing frequency bands (from $D1$

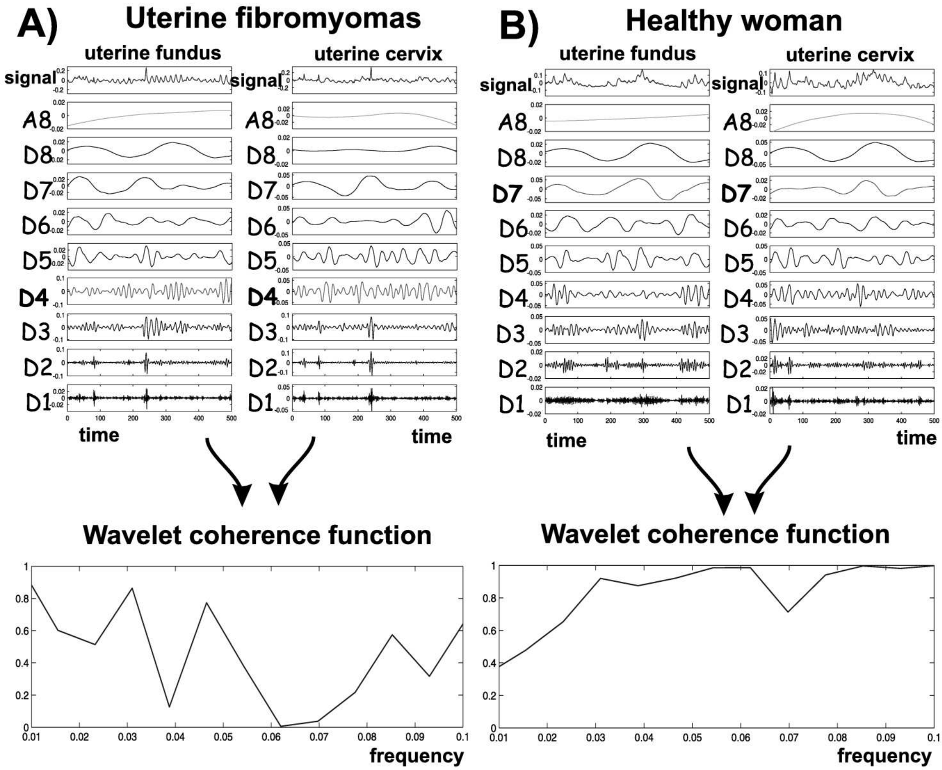

Fig. 2. Example of wavelet coherence function: A) discrete wavelet decomposition ($D4$ is selected frequency band) and wavelet coherence function for signals from patient with uterine fibromyomas, B) discrete wavelet decomposition ($D7$ is selected frequency band) and wavelet coherence function for signals from healthy woman

to $D8$), which are equivalent to period bands: 2–4s, 4–8s, 8–16s, 16–32s, 32–64s, 64–128s, 128–256s, 256–512s. We chose the frequency band which contained dominant frequency calculated by Fourier transform. Then we compute coherence function (in this chosen band, by means of the Welch method) and we receive the wavelet coherence function. By computation of this function we have obtained the information what are the common frequencies and when they appear. We were also able to estimate the similarity of two signals.

Examinations used in this example were conducted in the Department of Perinatology, The State Teaching Hospital of the Medical University of Bialystok.

Conclusions

In this paper we have presented a short review of applications of the coherence function in neurology, cardiology and uterine contractions activity studies. Coherence function is not a new concept, but in biomedical signal processing is rarely used, mainly in EEG signal studies. Since coherence is a frequency domain measure, it allows to find common frequencies in two signals and to evaluate the similarity of signals. Thus it can be useful in analyzing two simultaneously recorded biomedical signals and it can provide some diagnostic value.

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