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ARGUMENTS AND THEIR CLASSIFICATION

Abstract: The theory of argumentation has ever been the subject of interest of logicians. For some informal logicians the post-Fregean formal logic is not a proper tool of representing natural language and understanding of everyday argumentation. A new stimulus for the theory of argumentation is given by the development of Information and Communication Technologies and their employment in Artificial Intelligence. We will try to define the argument as generally as it is possible to encompass the intuitive notion of the argument involved in the natural language discourse. We develop the concept of the argument as the basis for developing a natural classification of arguments. The argument will be conceived as a pair of nonempty sets of propositions. Propositions will be characterized by their relation to a system of knowledge (a theory or a system of beliefs). The division of arguments conceived as a pair of sets of propositions will be based on the type of relation between the sets and the type of propositions being members of the sets. Finally, we try to clarify how the concept of the argument can assist in developing a classification of arguments.

Keywords: argumentation, structure of argumentation, assertion, rejection, suspension

1. Argument

In formal and mathematical logic the notion of argument is precisely defined and theoretically elaborated. But this notion does not comprise arguments as they are used in conversation, in metalanguage considerations and in social context as it is the case with juristic arguments. The general notion of the argument is far from clarity.

1.1. Propositions in argumentation

Propositions may have different status with respect to ($\mathfrak{B}$) – a particular system of knowledge, a theory or one's system of beliefs. Small Greek letters will be used to denote propositions (simple or compound). Large Greek letters will be used to denote sets of propositions.

Four types of relations between a proposition and $\mathfrak{B}$ can be distinguished. With respect to $\mathfrak{B}$ a proposition can be:

1. asserted: $\mathfrak{B} \vdash \phi$;[1]
2. rejected: $\mathfrak{B} \dashv \phi$;[2]
3. suspended: $\mathfrak{B} \mapsto \phi$,
4. neither asserted, nor rejected, nor suspended: ϕ.

In the case of assertion of a proposition the argumentation for the proposition fulfills the requirements imposed on $\vdash$. A proposition is rejected if there are some reasons that the requirements imposed on $\vdash$ would not be fulfilled. A proposition is suspended with respect to $\vdash$ if there are some arguments for or there are some arguments against the proposition and neither of the arguments is deciding; neither arguments for are satisfactory with respect to $\vdash$, nor arguments against are satisfactory with respect to $\vdash$. A proposition is neither asserted, nor rejected, nor suspended if there are no arguments for or against with respect to $\vdash$.

Any of the first three types of relations are graded. In the natural language discourse, the gradation is described qualitatively. For formal purposes quantitative description would be required. Let „*s-ϕ*" (**s**igned proposition) denote a proposition of any of the four types of propositions.

We may argue for[3]:
1. assertion,
2. rejection,
3. suspension

any of the *s*-propositions. An *s*-proposition with which the argumentation starts will be called *premiss* (of this argumentation). An *s*-proposition with which the argumentation ends will be called *conclusion* (of this argumentation). Both the notion of the premiss and the conclusion are relative: a proposition that is a premiss (conclusion) of an argumentation may be a conclusion (premiss) of another argumentation. Propositions of any type can be a premiss and can be a conclusion. We may argue, for example, to make higher the degree of assertion of a proposition, or we may argue for the rejection of an asserted proposition. As a premiss a rejected proposition as well as asserted one may be used.

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[1] The sign was introduced by G. Frege (1879). According to him, it serves to express a judgment.

[2] The notion of rejection has been introduced to formal logic by J. Łukasiewicz. Formal theory of rejection was developed, e.g. by J. Słupecki and his collaborators, see (1971, 1972).

[3] In the following, if it is clear from the context, $\mathfrak{B}$ will be assumed. Thus, eg. we will write: $\vdash \phi$ instead of $\mathfrak{B} \vdash \phi$.

Consider some examples of different types of *s*-propositions. Suppose that the expression „God exists" is a proposition, i.e. that the sentence has a meaning.

1. There are people who believe in God. Thus these people affirm the existence of God. They assert the proposition: *God exists.*
2. There are people who do not believe in God. Thus these people deny the existence of God. They reject the proposition: *God exits.*
3. There are people who are skeptical about God and there are people – called agnostics – who deny the possibility of finding an answer for the question of existence of God. As well skeptics as agnostics neither affirm nor deny the existence of God. They suspend judgment about whether or not God exists. They suspend the proposition: *God exists.*

Some propositions of the language of mathematics are:

1. asserted: $2 + 2 = 4$;
2. rejected: $2 + 2 = 5$
3. suspended: the Goldbach conjecture – *Every even integer greater than 2 can be expressed as the sum of two primes.*

In physics the proposition:

1. $E = mc^2$ – is asserted,
2. *heavy objects fall faster than lighter ones, in direct proportion to weight* – is rejected,
3. any proposition that with perfect accuracy states position and momentum of a particle is neither asserted nor rejected.[4]

1.2. Structure of argumentation

Some premisses as well conclusions are not be written (spoken) directly in a text. These are enthymeme's premisses and conclusions. The set of premisses as well the set of conclusions is conceived as formed by all the written (spoken) and enthymeme's premisses and conclusions. By a text we conceive the sequence of all the sentences that are written (spoken) and that are given implicite (enthymeme). It means a text is a set of sentences indexed by natural numbers.

Argumentation is built out of simple arguments. In text $\mathfrak{T}$ an argument $\langle \Sigma, \Gamma \rangle$, where Σ is the set of premisses and Γ is the set of conclusions, is a simple argument if and only if:

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[4] According to the *Heisenberg uncertainty principle* there is a limit on the accuracy with which certain pairs of physical properties of a particle cannot be simultaneously known.

1. in $\mathfrak{T}$ no element of Σ is taken as a conclusion or a premiss of other elements of Σ,
2. any propositions that in $\mathfrak{T}$ is used as a premiss for Γ is an element of Σ,
3. in $\mathfrak{T}$ no element of Γ is taken as a premiss or a conclusion of other elements of Γ,
4. any propositions that in $\mathfrak{T}$ is used as a conclusion of Σ is an element of Γ.

A simple argument $\langle \Sigma, \Gamma \rangle$ includes:

1. only all premisses of a given set of conclusions Γ,
2. only all conclusions of a given set of premisses Σ,
3. in $\mathfrak{T}$ no subset of Σ is divided into a set of premisses and a set of conclusions,
4. in $\mathfrak{T}$ no subset of Γ is divided into a set of premisses and a set of conclusions.

The simple argument may be described as the largest fragment of $\mathfrak{T}$ that can be divided into a set of premisses and the set of their conclusions and it is the only such division.

There are different relations between the set of premisses, i.e. the set of s-propositions with which the argumentation starts and the set of conclusions, i.e. the set of s-propositions with which the argumentation ends. We distinguish the following directions:

1. *direction of argumentation*: from the set of premisses to the set of conclusions;
2. *direction of entailment*: Σ entails Γ (Σ logically implies Γ, or Γ is the set of logical consequences of Σ);
3. *direction of justification*: Σ gives evidence that (supports, grounds) Γ.

The places of premisses and conclusions in the text can be different but the direction of argumentation is determined by the context and special words.

In logic the relation of entailment is defined for propositions (not for s-propositions). From Σ follows Γ if and only if the conjunction of propositions of Σ and the negation of disjunction of propositions of Γ are inconsistent, i.e. there is no possible situation in that both the propositions would be true. It could be that neither Γ follows from Σ nor Σ follows from Γ. Thus the relation of entailment is not total.

For Łukasiewicz the division of reasonings into deductive and reductive is more proper than the division into deductive and inductive reasonings.[5]

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[5] See (Bocheński 1980, p. 75) or (Bochenski 1992) – in Polish.

The relation of justification holds between sets of *s*-propositions Σ and Γ if any of *s*-propositions of Σ is used in $\mathfrak{T}$ to give evidence for (to support, to ground or is a reason for) assertion, rejection or suspension of at least one of the *s*-propositions of Γ.

In complex argumentation *s*-propositions are used to support other *s*-propositions. *s*-Propositions that are supported may also be used to support other *s*-propositions.

An elementary unit of argumentation is an argument, which is formed by premisses (Σ) and their conclusions (Γ). It means that:

- no *s*-proposition of the set of premisses Σ is (in considered argument) a premiss or conclusion of subset of Σ;
- no *s*-proposition of the set of conclusion Γ is a premiss or conclusion of the subset of Γ.

To describe the structure of argumentation diagrams can be applied.[6] The technique of argument diagramming is used to aid in the identification and analysis of argumentation as well in informal logic, as in legal logic and AI for the representation of knowledge and reasoning. Though the technique is well-established it is still not in an advanced state of development (Reed, Walton & Macagno 2007).

It should be decided which icons will be used to denote:

- direction: argumentation, entailment, giving evidence;
- goal of argumentation: to assert, to reject, or to support suspension;
- type of proposition: asserted, rejected, suspended or neither asserted, nor rejected or suspended.

2. Types of reasonings

The question of classifications of reasonings was discussed by Polish logicians, e.g. Łukasiewicz (1915) conceived reasoning as a mental proces of seeking of sentences which entail from a given sentences. In the case of deduction the direction of reasoning is the same as the direction of entailment. In the case of reduction the direction of reasoning is opposite to the direction of entailment. Czeżowski tried to improve Łukasiewicz's classification. Both classifications were criticized by Ajdukiewicz (1965).

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[6] Informal logic is mainly conceived and still is developed for educational goals. Thus it is natural to use some didactic improvements which could be helpful in teaching and mastering of reasoning and analyzing skills by students. The traditional square of opposition may be pointed as a device of this type. Frege employed diagrams as the formal language of his *Begriffsschrift*. The "language", due to its intricateness, has not been approved by logicians.

In the case of argumentation we want to show that the methodological requirements of a theory are fulfilled or that the argument is convincing. In the case of reasoning we want to show some reasons for truth-value. Reasoning can be described with the same diagrams that are used to describe argumentation.

In the case of reasoning, if the principle of bivalence is accepted, any argument for the rejection of proposition is equivalent to assertion of the negation of the proposition. From this assumption it follows that in a simple argument we may argue only for assertion of a proposition.[7]

Let Σ be a set of premises, Γ – set of conclusions. Let $\vdash \Delta$ be $\{\vdash \phi : \phi \in \Delta\}$. $\Delta \longrightarrow\!\mid \Lambda$ or $\Lambda \mid\!\longleftarrow \Delta$ means that the propositions of Δ are used to give evidence for (to support, to ground) the propositions of Λ.

There are four combinatorial possibilities:

1. $\vdash \Sigma \longrightarrow\!\mid \Gamma$,
2. $\vdash \Sigma \mid\!\longleftarrow \Gamma$,
3. $\Sigma \longrightarrow\!\mid \vdash \Gamma$,
4. $\Sigma \mid\!\longleftarrow \vdash \Gamma$.

The distinguished types of reasoning could be characterized dynamically.

1. (a) this reasoning starts with a set of asserted propositions which will give evidence;
   (b) a set of not asserted propositions is created, for the propositions will be given evidence;
2. (a) this reasoning starts with a set of asserted propositions for which will be given evidence;
   (b) a set of not asserted propositions is created, the propositions will give evidence;
3. (a) this reasoning starts with a set of propositions which are not asserted and will give evidence;
   (b) a set of asserted propositions is created, for the propositions will be given evidence;

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[7] In another simple argument we may argue for assertion of the negation of this proposition. Thus the complex argument may be conceived as an argument for the suspension of this proposition. E.g., there are some arguments for the presence of general Błasik in the cockpit and there are some arguments against his presence in the cockpit. The arguments for are not convincing for me and the arguments against are not convincing for me. For these reasons I suspend the proposition that general Błasik was present in the cockpit. Another example: there are arguments for and there are arguments against the existence of a civilization outside Earth. Neither of the arguments is deciding. For this reason the proposition that there is a civilization outside Earth may be suspended.

4. (a) this reasoning starts with a set of not asserted propositions for which will be given evidence;

   (b) a set of asserted propositions is created, these propositions will give evidence.

The reasoning 1 is named *inference*. In the case of deductive reasoning the truth of conclusion is guaranteed by the truth of premisses. In the case of inductive reasoning it is conversely, namely the truth of premisses is guaranteed by the truth of conclusions (and enthymeme's premisses). In the case of analogy neither the truth of conclusions is guaranteed by the truth of premisses nor the truth of premisses is guaranteed by the truth of conclusions.

The reasoning 2 is called *explanation*. Γ is the set of hypotheses. The hypothesis is used to explain facts stated by elements of Σ. This type of reasoning is an element of abduction. An abductive reasoning from Σ to Γ involves not simply a determination that, e.g., Γ gives evidence for Σ, but also that Γ is among the most economical explanations for Σ.

The reasoning 3 is named *verification*. The verification is used to confirm hypotheses. If the consequences of a hypothesis are confirmed, then the hypothesis is more probable.

The reasoning 4 is named *justification*. If from the set Γ logically follows Σ, i.e. if the truth of Γ guarantees the truth of Σ, then the justification is named *proof*. In mathematics a theorem ϕ is proved if and only if some already proved (asserted) theorems are found and it is shown that ϕ logically follows from these theorems.

Any premiss taken separately may give evidence (convergent argument) or to give evidence the premisses should be taken jointly (linked argument). The same is true about conclusions. The sign: ‿‿ or the sign: ⌢⌢ will be used to mark that propositions are taken jointly in the argument.

In diagrams to mark that Δ gives evidence for Λ we will write: $\Delta \longmapsto \Lambda$. To mark that disjunction of propositions of Λ logically follows from the conjunction of propositions of Δ we will write: $\Delta \longrightarrow \Lambda$. Instead of propositions in diagrams the numbers will be used.

There are three types of inference: deductive, inductive, analogical.

Deduction   Induction   Analogy
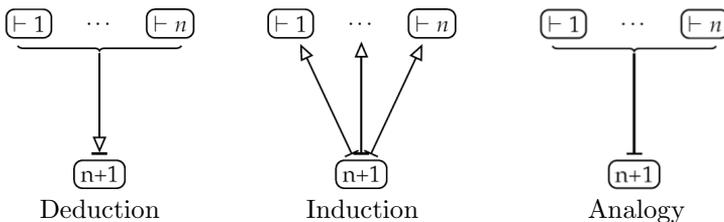
Using diagrams also other types of reasonings: explanation, verification and justification can be described.

The proposed description of arguments takes into account only pragmatic properties of propositions involved in argumentation and only logical relations between the set of premisses and the set of conclusions. It seems that the proposal is sufficiently rich to analyze a great variety of argumentations.

References

Ajdukiewicz, K. (1965), Klasyfikacja rozumowań, *in* 'Język i poznanie', Vol. 2, PWN.

Berka, K. & Kreiser, L., eds (1971), Akademie-Verlag, Berlin. Zweite durchgesehene Auflage, 1973.

Bocheński, I. M. (1980), *Die zeitgenössischen Denkmethoden*, 8 edn, München.

Bocheński, J. M. (1992), *Współczesne metody myślenia*, Wydawnictwo «W drodze», Poznań.

Frege, G. (1879), *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*, Halle. Introduction dated: 18 XII 1878. Reprinted in (Frege 1993). Shortened version in (Berka & Kreiser 1971). English translation in (Heijenoort 1967, pp. 1–82).

Frege, G. (1993), *Begriffsschrift und andere Aufsätze*, Hildesheim.

Heijenoort, J. v., ed. (1967), *From Frege to Gödel. A Source Book in Mathematical Logic 1879–1931*, 1 edn, Harvard University Press, Cambridge. 2nd ed., 1971.

Reed, C., Walton, D. & Macagno, F. (2007), 'Argument diagramming in logic, law and artifical intelligence', *The Knowledge Engineering Review* **22**, 87–109. doi:10.1017/S0269888907001051.

Słupecki, J., Bryll, G. & Wybraniec-Skardowska, U. (1971), 'Theory of rejected propositions, I', *Studia Logica* **29**, 75–123.

Słupecki, J., Bryll, G. & Wybraniec-Skardowska, U. (1972), 'Theory of rejected propositions, I', *Studia Logica* **30**, 97–145.

Łukasiewicz, J. (1915), O nauce, *in* 'Poradnik dla samouków', 2 edn, Warszawa.

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