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UNCONVENTIONAL PROBABILITIES AND FUZZINESS IN CADIAG'S COMPUTER-ASSISTED MEDICAL EXPERT SYSTEMS

Abstract: In the paper I sketched some applications of non-Kolmogorovian probabilities (including complex- and non-Archimedean-valued) and non-Archimedean fuzziness in the CADIAG expert system. This work I started this year.

1. Introduction

Recently, a huge number of implementations and developments of knowledge-based systems in medicine aiming at providing support for physicians in the decision-making process has been proposing. One of them is CADIAG's computer-assisted medical expert systems. The latter are fuzzy consultation systems in that fuzzy methods for almost all knowledge processing tasks are used: in them fuzzy inputs, operates on fuzzy sets with fuzzy rules, and produces fuzzy sets as output are accepted. The aim of CADIAG is to help in medical diagnosis. Notice that "medical diagnosis is the art of determining a person's pathological status from an available set of findings. Why is it an art? Because it is a problem complicated by many and manifold factors, and its solution involves literally all of a human's abilities including intuition and the subconscious" [8]. CADIAG is aimed to provide diagnosis with automatic inferences, i.e. to transform the art of diagnosis into knowledge.

In CADIAG-1, there were 187 diagnoses and 1,213 symptoms, signs, and findings within the rheumatological differential diagnostic group CADIAG-1/RHEUMA [5]. On the other hand, CADIAG-2s knowledge base contained more than 20,000 rules expressing causal relationships between patient's symptoms, signs, laboratory test results, clinical findings and diagnoses on the basis of the patient data and the knowledge base [3]. These relationships are formulated as IF–THEN rules, between symptoms, signs, test results and clinical findings on the one hand and diagnoses on the other hand. Since

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CADIAG-2 fuzzy logic has been used for defining IF–THEN rules. In particular, fuzzy logic has been aiming to (i) perform a formal consistency check of the rules of CADIAG-2 and CADIAG-4, (ii) formally justify the choice of the operators (t-norms) and the way of combining the rules of the systems and (iii) allow for computing satisfactory results in the presence of incomplete information.

CADIAG-2 and CADIAG-4 systems employ fuzzy set theory by using T-norm-based fuzzy logics to capture: uncertainty as to whether a patient's symptoms (signs, laboratory test results) are pathological or normal; uncertainty as to whether symptoms necessarily have to occur with a disease; and uncertainty as to whether symptoms sufficiently confirm or exclude a diagnosis [9]. A consequence relation for graded inference of those systems may be regarded within the frame of infinite-valued Łukasiewicz semantics [6].

In next sections we are proposing to apply hybrid fuzzy logics and hybrid probability logics to describe the knowledge representation and the knowledge acquisition procedures that support medical experts to add, edit and update their knowledge.

2. Non-Kolmogorovian Probabilities

Kolmogorov's probability theory is defined on $\{e_1, e_2, \ldots\} = E$, a collection of elements called elementary events, and F, a set of subsets of E called random events. This theory is based on the following axioms:

- F is a field of sets.
- F contains the set E.
- A non-negative real number $\mathbf{P}(A)$, called the probability of A, is assigned to each set A in F.
- $\mathbf{P}(E)$ equal 1.
- If A and B have no elements in common, the number assigned to their union is $\mathbf{P}(A \cup B) = P(A) + P(B)$; hence, we say that A and B are disjoint; otherwise, we have $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B)$.

Kolmogorov's probabilities have a huge number of applications to information science and computer technologies. For instance, such probabilities were applied in conditional probability interpretation of CADIAG-2's inferences, namely in identifying degrees of confirmation with probabilities and the rules themselves with conditional probabilistic statements [7]. This interpretation was proposed in terms of degrees of belief of the doctor (or doctors) on the truth of the consequent given that the antecedent of the rule holds is also possible though. However, it is possible to set non-Kolmogorovian probabilities, which have much better opportunities to express real life probabilities and to simulate medical data.

2.1. Probabilities in Complex Dimension

Now let us define probabilities on $\mathbf{C}$, the set of complex numbers. Each member of $\mathbf{C}$ is a number comprising a real and imaginary part. Let $\mathbf{R}_C$ be the set of real numbers playing role of real parts of complex numbers and $\mathbf{I}_C$ the set of imaginary numbers playing role of imaginary part. We add the following statements to Kolmogorov's list of axioms

- Let $\mathbf{P_I}(A) = i(1 - \mathbf{P}(A))$, where $i^2 = -1$, be the probability taking values in $\mathbf{I}_C$. It is an imaginary part associated to the probability $\mathbf{P}(A)$ of event A taking values in $\mathbf{R}_C$. The probability $\mathbf{P_I}(A)$ is called imaginary.
- The complex number $c = \mathbf{P_C}(A) = \mathbf{P}(A) + \mathbf{P_I}(A) = \mathbf{P}(A) + i(1 - \mathbf{P}(A))$ has an absolute value $|c|^2 = \mathbf{P}(A)^2 + (1 - \mathbf{P}(A))^2$. The probability $\mathbf{P_C}(A)$ is called complex.

Interpretation of complex probabilities:

Complex probabilities allow us to consider hidden variables, called imaginary probabilities, as unknown forces causing the output as well as known forces, called real probabilities. As a result, it is possible to examine the same phenomena not only within realized possibilities, but also taking into account unrealized possibilities.

Advantages of complex probabilities:

1. We could calculate the degree of knowledge by considering $|c|^2$ as an appropriate degree. Let us exemplify this opportunity by the game of coin tossing. Hence, $P(\text{'getting head'}) = 1/2$ and $\mathbf{P}(\text{'getting tail'}) = 1/2$. Then $|c|^2 = 1 - 2 \cdot P(\text{'getting head'}) \cdot (1 - P(\text{'getting head'})) = 1 - (2 \cdot 1/2) \cdot (1 - 1/2) = 1/2$.
2. $\mathbf{C}$ can be presented as a two-dimensional real vector space. Continuing in the same way, we could define complex probabilities as 2-dimensional quantities, too. In the vector representation, the rectangular coordinates are typically referred to simply as x and y and we may offer geometric images of probabilities.

In CADIAG's fuzzy medical consultation system, uncertainty and probability concerning the confirmation or exclusion of diagnoses D_j in a patient P is modelled by degrees of their compatibility. *Using complex pro-*

babilities and fuzziness, we may present the confirmation or exclusion of diagnoses as a real and imaginary parts of complex quantities. This way is much simpler.

2.2. Probabilities in non-Archimedean Dimension

Now consider non-Archimedean probabilities. The ultrapower $\Theta^I/\mathcal{U}$, where $\mathcal{U}$ is ultrafilter that contains all complements for finite subsets of I, is said to be a proper nonstandard extension of Θ and it is denoted by ${}^*\Theta$. There exist two groups of members of ${}^*\Theta$: (1) functions that are constant, e.g. $f(\alpha) = m \in \Theta$ for an infinite index subset $\{\alpha \in I\} \in \mathcal{U}$; a constant function $[f = m]$ is denoted by ${}^*m$; (2) functions that are not constant. The set of all constant functions of ${}^*\Theta$ is called standard set and it is denoted by ${}^\sigma\Theta$. The members of ${}^\sigma\Theta$ are called standard. It is readily seen that ${}^\sigma\Theta$ and Θ are isomorphic: ${}^\sigma\Theta \simeq \Theta$. If Θ was a number system, then members of ${}^*\Theta$ are called hypernumbers. If $\Theta = \mathbf{R}$, then the new numbers are called hyperreal numbers and if $\Theta = \mathbf{Q}$, then the new numbers are called hyperrational numbers.

Define a structure $\langle \mathcal{P}({}^*\Theta), \wedge, \vee, \neg, {}^*\Theta \rangle$ as follows (1) for any $A, B \in \mathcal{P}({}^*\Theta)$, $A \wedge B = \{f(\alpha)\colon f(\alpha) \in A \wedge f(\alpha) \in B\}$, (2) for any $A, B \in \mathcal{P}({}^*\Theta)$, $A \vee B = \{f(\alpha)\colon f(\alpha) \in A \vee f(\alpha) \in B\}$, (3) for any $A \in \mathcal{P}({}^*\Theta)$, $\neg A = \{f(\alpha)\colon f(\alpha) \in {}^*\Theta \backslash A\}$. Then a structure $\langle \mathcal{P}({}^*\Theta), \wedge, \vee, \neg, {}^*\Theta \rangle$ is not a Boolean algebra if $|\Theta| \geq 2$, because the set $\mathcal{P}({}^*\Theta)$ is not closed under intersection. Indeed, by definition of non-standard extension, some elements of ${}^*\Theta$ have a non-empty intersection that does not belong to $\mathcal{P}({}^*\Theta)$, i.e. they are not atoms of $\mathcal{P}({}^*\Theta)$ of the form $[f]$. The same situation is observed in the field $\mathbf{Q}_p$ of p-adic numbers.

Consequently, there is a problem how it is possible to define probabilities on non-Archimedean structures if we have no opportunity to put them on a field of subsets. A. Khrennikov who considered non-Archimedean probability theory on the set of p-adic numbers proposed a semi-algebra that is closed only with respect to a finite unions of sets, which have empty intersections. However, there is a better way if we get non-Archimedean probabilities on a class $\mathcal{F}(X)$ of fuzzy subsets A of the set X of non-Archimedean numbers (hypernumbers or p-adic numbers).

Interpretation of non-Archimedean probabilities:

Non-Archimedean probabilities are defined within non-Archimedean metric and taking into account that non-Archimedean structures are non-well-founded (the set-theoretic foundation axiom is violated there), in non-Archimedean probability theory we assume that reality is non-well-founded too and, as an example, we could accept phenomena with circular relations and calculate their probabilities.

Advantages of non-Archimedean probabilities:

1. we could set multi-hierarchical probabilities (higher-order probabilities), e.g. multi-hierarchical Bayesian networks,
2. we could analyze probabilities of overlapping phenomena (we are dealing with fuzzy probabilities there),
3. this theory allows us to regard probabilities of non-well-founded phenomena (phenomena with feedback and circular relations).

Medical cause-effect relationships (the relations between diagnoses and their symptoms) are hardly ever one-to-one, because diagnoses often share an overlapping range of symptoms[1] *and furthermore symptoms may assume feedback and circular relations among themselves*. As a result, fuzzy probabilities, on the one hand, and non-well-founded probabilities, on the other hand, are suitable tools for analyzing medical diagnosis. Notice that now there exist many researches showing advantages of non-Archimedean mathematics in mathematical modeling of natural (biological or medical) systems.

3. Non-Archimedean Fuzziness

Non-Archimedean valued fuzzy logics (including the p-adic case) may find many applications to rule-based systems and demonstrate their importance as a powerful design methodology. So, on the one hand, novel logics are complete and, on the other hand, they have been extended to *infinite hierarchy of fuzzy sets*, therefore they have more design degrees of freedom than do conventional fuzzy logics with values distributed in the standard unit interval $[0, 1]$.

Usually, the medical knowledge is presented as a flat relation between the set of symptoms and the set of diseases. Let S and D be the set of symptoms and diagnoses, respectively, and $\mathcal{F}(S)$ and $\mathcal{F}(D)$ be fuzzy power sets of S and D. Medical knowledge is then the relationship between symptoms and diagnoses expressed by a fuzzy relation $\mathcal{R} \subset \mathcal{F}(S) \times \mathcal{F}(D)$. However, there are tasks, when medical data need to be interpreted as a type-2 fuzzy set.

[1] For instance, we observe the overlapping syndromes in a patient in whom, on the one hand, "haemolytic anaemia, positive LE cells" and "proteinuria" and, on the other, "sclerodactyly, impaired oesophageal motion" and "lung fibrosis" are found, because there are symptoms present of both "systemic lupus erythematosus" and "progressive systemic sclerosis".

In this case we could use fragments of multi-hierarchies of non-Archimedean fuzzy logics.

Their multi-hierarchies are connected to properties of non-Archimedean numbers. Geometrically, we can imagine a system of these numbers in two ways:

- in case of hyperreal or hyperrational numbers they may be regarded as a homogeneous infinite tree with $[0,1]$-branches splitting at each vertex,
- in case of p-adic integers as a homogeneous infinite tree with p-branches splitting at each vertex.

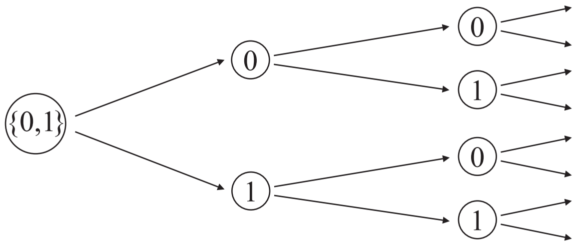

Figure 1. The 2-adic tree

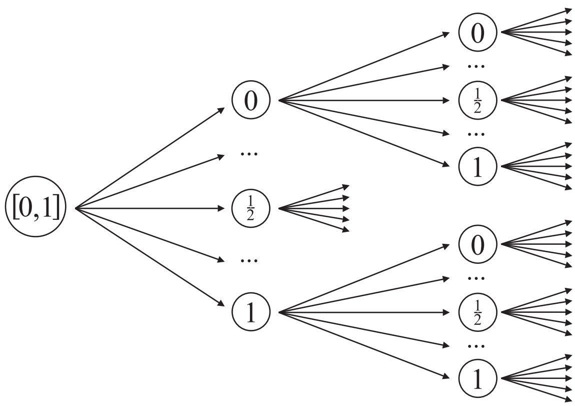

Figure 2. The non-Archimedean tree

The first one is said to be *non-Archimedean tree*, the second one *p-adic tree*. For instance, a 2-adic tree is pictured in Fig. 1 and a non-Archimedean tree with branches whose number runs over the set $[0,1]$ in Fig. 2. This geometrical presentation allows us to consider infinite hierarchies of conditional fuzzy (resp. finitely many-valued) data described in the standard form as that distributed in the set $[0,1]$ (resp. $\{0,\ldots,p-1\}$). In order to exemplify this feature, let us take some fuzzy (resp. finitely many-valued) data and build up using them *vectors of fuzzy information* generating a hierarchical structure between digits of these vectors. If $x = \langle x_1, x_2, \ldots, x_n, \ldots\rangle$, where $x_j \in [0,1]$ in the hypervalued case (resp. $x_j \in \{0,1,\ldots,p-1\}$ in the p-adic case), is a fuzzy information vector, then digits x_j have different weights. The digit x_0 is the most important, x_1 dominates over x_2, x_2 over x_3 in turn and so on. The distance between uncertain states $x = \langle x_1, x_2, \ldots, x_n, \ldots\rangle$ and $y = \langle y_1, y_2, \ldots, y_n, \ldots\rangle$ is determined just by the length of their common root: close uncertain states have a long common root (i.e. then there exists a large integer j such that $x_i = y_i$ for any $i = 0, \ldots, j$).

This distance between uncertain states could be regarded within ultrametric space $\langle X, \rho\rangle$, where the distance ρ satisfies the strong triangle inequality: $\rho(x,y) \leq \max(\rho(x,z), \rho(z,y))$. For $r \in R_+, a \in X$, we set

$$U_r(a) := \{x \in X\colon \rho(x,a) \leq r\}, \;\; U_r^-(a) := \{x \in X\colon \rho(x,a) < r\}.$$

Both sets are called *balls* of radius r with center a. It can be readily shown that balls in ultrametric space have the following properties:

- For any balls U and V in X, either they are ordered by inclusion (i.e. $U \subset V$ or $V \subset U$) or they are disjoint.
- Each point of a ball is a center.

Let us try to present results of a symptom recognition in the form of 2-adic valued logic. Suppose that our system fixes the observations of people to define diagnosis "asthma" and the first task is to detect "cough". Then $x_0 = 1$ if a person who is observed has cough and $x_0 = 0$ if does not. Notice that coughing may be caused by a respiratory tract infection, choking, smoking, air pollution, asthma, gastroesophageal reflux disease, post-nasal drip, chronic bronchitis, heart failure and medications such as ACE inhibitors, etc. Further, we have $x_1 = 1$ if a person has respiratory failure and $x = 0$ otherwise. Then $x_2 = 1$ if a person has chest tightness and $x_2 = 0$ otherwise. After that we set $x_3 = 1$ if (s)he has breathing difficulty and $x_3 = 0$ otherwise, etc. As a result, we will obtain a 2-adic tree if we continue. Results of such symptom recognition can be regarded as the following balls in 2-adic ultrametric space:

- the symptom of *cough* (tussis) $U_{1/2} = \{x = \langle x_0, x_1, \ldots, x_n, \ldots\rangle\colon x_0 = 1\}$,
- the symptom of *respiratory failure* $U_{1/4} = \{x = \langle x_0, x_1, \ldots, x_n, \ldots\rangle\colon x_0 = 1, x_1 = 1\}$,
- the symptom of *chest tightness* $U_{1/8} = \{x = \langle x_0, x_1, \ldots, x_n, \ldots\rangle\colon x_0 = 1, x_1 = 1, x_2 = 1\}$,

- the symptom of *breathing difficulty* $U_{1/16} = \{x = \langle x_0, x_1, \ldots, x_n, \ldots\rangle\colon x_0 = 1, x_1 = 1, x_2 = 1, x_3 = 1\}$,
- ...

For processing these results we can use operations of 2-adic valued logic $BL\forall_\infty$.

Thus, while the key notion of conventional fuzzy logics is that truth values are indicated by a value on the range $[0,1]$, with 0.0 representing absolute falseness and 1.0 representing absolute truth, in non-Archimedean and p-adic fuzzy logics we come cross non-Archimedean or p-adic trees (Fig. 1 and 2) and truth values are indicated by a value on the range ${}^*[0,1]$ of hypernumbers or on the range $\mathbf{Z}_p$ of p-adic integers. We build up infinite tuples of the form $\langle \mu_1, \mu_2, \ldots, \mu_n, \ldots\rangle$, where μ_j is a fuzzy measure such that μ_j depends on μ_i for any natural number $i < j$. For instance, we can interpret this dependence as types of higher-order logic. Let us take a fuzzy set $\mathcal{P}_i \in \mathcal{F}_i(D)$, where $\mathcal{F}_i(D) := \underbrace{[0,1]^{\cdots^{[0,1]^{[0,1]^D}}}}_{i+1}$, then a membership function $\mu_{\mathcal{P}_i}(x)$ is defined as the degree of membership of x in $\mathcal{P}_i$.

The further advantages of non-Archimedean and p-adic valued fuzzy logics consist in a possibility to be considered as behavior fuzzy logics. Let us recall that behaviors can be viewed as a labelled transition system. The set of finite sequences over a set A will be denoted by A^*, and the empty sequence by ϵ. The transition system is understood as a tuple $\Upsilon = \langle S, S_0, L, \longrightarrow\rangle$, where S is a potentially infinite collection of states, S_0 is the set of initial states, L is a potentially infinite collection of labels, $\longrightarrow \subseteq S \times L \times S$ is a transition relation that models how a state $s \in S$ can evolve into another state $s' \in S$ due to an interaction $\alpha \in L$. Usually, $\langle s, \alpha, s'\rangle \in \longrightarrow$ is denoted by $s \xrightarrow{\alpha} s'$. A state s' is reachable from a state s if $s \xrightarrow{\alpha} s'$.

The *finite trace of transition system* is a pair $\langle s_0, \sigma\rangle$, where $s_0 \in S_0$ and $\sigma = \alpha_1\alpha_2 \ldots \alpha_n$ is a finite sequence of labels such that there are states $s_0, s_1, \ldots, s_n$ satisfying $s_i \xrightarrow{\alpha_{i+1}} s_{i+1}$ for all i such that $0 \leq i < n$. The *infinite trace of transition system* is a pair $\langle s_0, \sigma\rangle$, where $s_0 \in S_0$ and $\sigma = \alpha_1\alpha_2 \ldots$ is an infinite sequence of labels such that there are states $s_0, s_1, \ldots$ satisfying $s_i \xrightarrow{\alpha_{i+1}} s_{i+1}$ for all $i \geq 0$. The set of all finite (resp. infinite) traces is denoted by $trace^*(\Upsilon)$ (resp. by $trace^\omega(\Upsilon)$). Notice that non-Archimedean and p-adic trees may be naturally interpreted as sets of traces of transition system, where the set of states runs either over $[0,1]$ or $\{0, \ldots, p-1\}$. In other words, non-Archimedean and p-adic trees may be regarded as behavior trees.

Now let us construct a *fuzzification of transition system* Υ by defining the following functions

- in case S is infinite, $fuz^*\colon trace^*(\Upsilon) \mapsto [0,1]^*$, where $[0,1]^*$ denotes the set of finite tuples of members of $[0,1]$.
- in case S is infinite, $fuz^\omega\colon trace^\omega(\Upsilon) \mapsto {}^*[0,1]$.
- in case S is finite and $|S| = |\{0,\dots,p-1\}|$, i.e. they are of the same cardinality, then

$$fuz^*\colon trace^*(\Upsilon) \mapsto \{0,\dots,p-1\}^*,$$

  where $\{0,\dots,p-1\}^*$ denotes the set of finite tuples of members of $\{0,\dots,p-1\}$.
- in case S is finite and $|S| = |\{0,\dots,p-1\}|$, i.e. they are of the same cardinality, then

$$fuz^\omega\colon trace^\omega(\Upsilon) \mapsto \mathbf{Z}_p.$$

Suppose that all these functions are injections such that

- $fuz^*(\epsilon) = \langle 0,\dots,0\rangle$, where ϵ is the empty sequence of $trace^*(\Upsilon)$,
- $fuz^\omega(\epsilon) = {}^*0$, where ϵ is the empty sequence of $trace^\omega(\Upsilon)$,
- an i-th projection of $fuz^*(\langle s_0,\sigma\rangle)$ (resp. $fuz^\omega(\langle s_0,\sigma\rangle)$) is a fuzzy measure of s_i for any $i = 0,1,2,\dots$

Thus, we can analyze an evolution of transition system Υ by setting its fuzzification and further by using the non-Archimedean valued logic $Ł\Pi\forall_\infty$ and the p-adic valued logic $BL\forall_\infty$ for processing results.

Let us assume that the following n ($n > 6$) attributes should be identified for a medical diagnosis: palpation of an appropriate part of body P, result of X-ray X, body temperature T, blood pressure P, rate of heartbeats H, rate of breathing B, etc. Each experiment consists of obtaining data for each attribute in the fuzzy diapason: [norm, pathology]. Let us denote by S^n the set of all possible data (signs) that could be obtained in those experiments. By S^n_0 we will denote results of the first experiment. Our diagnosis model could be presented as a transition system $\Upsilon = \langle S^n, S^n_0, L^n, \longrightarrow\rangle$, where L^n is a set of labels for each attribute. Suppose that L^n consists of the following items [5]:

- OC: a relation between two signs if the first sign shows obligatory occurrence with a second one, and if it occurs it is confirming for that second sign. *Example*: The X-ray finding "endoprothesis of the knee" is obligatory occurring and confirming for the diagnoses "arthroplasty of the knee".

- FC: a relation between two signs if the first sign shows facultative occurrence with a second one, but if it occurs it is confirming for that second. *Example*: The lab result "intracellular uric acid crystals in joint effusion" is facultative occurring yet confirming for the diagnosis "gout".
- ON: a relation between two signs if the first sign has obligatory occurrence with a second one, but does not confirm it. *Example*: The clinical finding "HEBERDEN's nodes" is obligatory occurring and not confirming for the diagnosis "HEBERDEN's arthrosis".
- FN: a relation between two signs if the first sign is both facultative occurring with a second one and not confirming for that sign. *Example*: The lab finding "elevated ESR" isfacultative occurring and not confirming for the diagnosis "rheumatoid arthritis".
- E: a relation between two signs if the first sign excludes a second sign. *Example*: The lab finding "WAALER ROSE titre 1:128" excludes the diagnosis "seronegative rheumatoid arthritis".

Thus, in a diagnosis model Υ we could deal with labels defined as fuzzy relations between states s_i and s_{i+1} of an appropriate attribute, where s_i and s_{i+1} are states obtained in the i-th experiment and $i+1$-st experiment respectively. The sequences of such fuzzy labels could be regarded as traces of Υ within the non-Archimedean valued logic $Ł\Pi\forall_\infty$.

Thus, unconventional probabilities and fuzziness may find a lot of applications in medical diagnosis within fuzzy logical approach.

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