Mary Dziśko
Belarusian State University of Informatics and Radioelectronics

Andrew Schumann
Belarusian State University

CYCLIC PROOFS IN ARGUMENTATION. THE CASE OF EXCLUDING BORIS PASTERNAK FROM THE ASSOCIATION OF WRITERS OF THE USSR

Abstract: In the paper we consider some principal notions of non-well-founded proof theory in argumentation. This theory is based on the assumption of Anti-Foundation Axiom that every graph tree has a unique decoration. A decoration of a graph is an assignment of a derivable formula to each node of the graph in such a way that the premisses of the root-derivable formula assigned to a node are the derivable formulas assigned to the children of that node. According to Anti-Foundation Axiom in proof theory, cyclic graph and infinite graph trees have a decoration too. This means that there are cyclic and infinite proof trees. The natural interpretation of cyclic proofs in argumentation is their consideration as confirmation procedure, where premisses are compatible with a derivable statement, but they do not prove this in the standard meaning. As model example we use the case of excluding Boris Pasternak from the Association of Writers of the USSR.

Keywords: Non-well-founded proofs, cyclic proofs, Anti-Foundation Axiom, proof argumentation, confirmation argumentation.

1. Introduction

Boris Pasternak is a prominent Russian poet who wrote the best known Soviet novel *Doctor Zhivago* (Pasternak 1958) that brought him the Nobel Prize for Literature in 1958. Pasternak had to decline the honour because the protests in his home country. *Doctor Zhivago* was published first in Russian and in Italian translation by the publisher Feltrinelli in Milan in 1957, after the Italian journalist Sergio D'Angelo had smuggled the manuscript out of Russia. The English translation appeared in 1958. So, *Doctor Zhivago* was soon translated into 18 languages.

Pasternak probably completed the work in 1954. It was started in 1945, after the death of his father. He tried to publish his novel, but this effort turned out to be unsuccessful. It was expected taking into account that in the 1930s and 1940s Pasternak's works were criticized by the Russian

ISBN 978–83–7431–206–6 ISSN 0860-150X

Association of Proletarian Writers as the older literary type and they were not printed. However, at Stalin's time he did not die in the Gulag Archipelago as others because Stalin's respect for him.

Recall, at the Soviet time censor had a duty to inspect material before publication to ensure that it contains nothing offensive to government. As a result, everybody who lived in the Soviet Union knew that all scientists, artists, and other intellectuals were the objects of censorship. Official Soviet censorship, i.e. censorship that emanates from governmental authority determined all the information to be inspected.

As an example, Soviet literature had to find the "positive" hero among the builders of communism. This main character of Soviet literature had to reflect "socialist reality" and the "true traits of the new man." Soviet artists, members of registered associations, were obliged, as stated in one of the statutes, to "present life in light of socialist ideals." This meant that emotions and experiences that were not related to socialist development, from the socialist viewpoint, were not interesting enough to become the subject of art. It was one of the basic claims of socialist realism ("soc-realism"), the dominate Soviet treatment of authentic literature and painting.

The author's treatment of the early years of the October revolution presented by Pasternak in *Doctor Zhivago* formed a considerable contrast with the official propaganda on the period, which suggests only heroism and self-sacrifice. This treatment went against socialist realism. He explained the key idea of novel as follows: "When I wrote *Doctor Zhivago* I had the feeling of an immense debt toward my contemporaries. It was an attempt to repay it. This feeling of debt was overpowering as I slowly progressed with the novel. After so many years of just writing lyric poetry or translating, it seemed to me that it was my duty to make a statement about our epoch – about those years, remote and yet looming so closely over us. Time was pressing. I wanted to record the past to honor in *Doctor Zhivago* the beautiful and sensitive aspects of the Russia of those years. There will be no return of those days, or of those of our fathers and forefathers, but in the great blossoming of the future I foresee their values will revive. I have tried to describe them. I don't know whether *Doctor Zhivago* is fully successful as a novel, but then with all its faults I feel it has more value than those early poems. It is richer, more humane than the works of my youth."[1] This novel was banned in the Soviet Union. Pasternak was rehabilitated posthumously in 1987, which made possible the publication of his major work in Russia.

[1] It is an interview given by Pasternak to Olga Carlisle in 1960.

In the paper we consider an argumentation model used at the session of the Association of Moscow Writers (held on October 31, 1958), which made the decision to expel Boris Pasternak from the Association of Writers of the USSR, to condemn his literature, and to evict him from the USSR. According to the records of this session (*Literaturnaya Gazeta*, 1 November 1958), 13 members are reported to have spoken but no indication was given of how many members attended. From the records one can read different statements made by writers against their colleague. These statements contained **cyclic proofs** for increasing the acceptability of the standpoint that Pasternak should be expelled from the Association of Soviet Writers and from the USSR. The notion of cyclic proofs in argumentation will be formally explicated in the third section. Its informal meaning is as follows:

- *there are no opponents, each speaker is proponent*,
- *speakers use just confirmation arguments and do not use proof arguments*,
- *the standpoint itself can be viewed as a reasonable argument.*

This session was chaired by S. Smirnov, the editor of Literaturnaya Gazeta (the best known literature newspaper in the USSR). All speakers who were prominent Soviet writers such as L. Oshanin, I. Zelenin, V. Pertsov, A. Bezymensky, A. Sofronov, S. Antonov, B. Slutski, G. Nikolayev, V. Soloukhin, S. Baruzdin, B. Polevoy and many others encouraged very negative attitudes towards Pasternak. At the end of Session Smirnov said: "I fully agree that the novel *Dr. Zhivago* is trash and I think that this internal emigrant, B. Pasternak, should be expelled from the USSR." He proposed to submit the request to the Soviet government to evict Pasternak from the USSR. The resolution demanding what amounts to Pasternak's deportation from the USSR was passed unanimously.

2. Non-well-founded proofs

Now consider some basic notions of proof theory further to formalize cyclic proofs used at the session of the Association of Moscow Writers. First of all recall that a proof (or derivation) of a well-formed formula S from a set of premisses U is a finite tree such that:

- S is the root of the tree and is called the end-formula.
- The leaves of the tree are all axioms or members of U.
- Each child node of the tree is obtained from its parent nodes by an inference rule, i.e. if S is a child node of $S_1, \ldots, S_n$, then $\frac{S_1,\ldots,S_n}{S}$ is an instance of a rule.

If we have a proof tree with the root S and $U = \emptyset$, then S is called a provable (or root-provable) formula. If we have a proof tree with the root S and $U \neq \emptyset$, then S is called a derivable (or root-derivable) formula from premisses U.

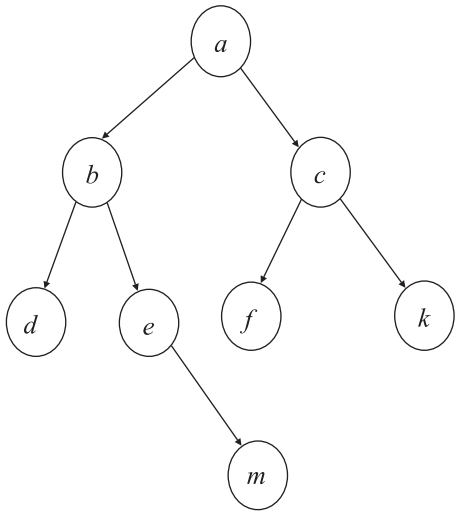

Figure 1. The example of well-founded graph tree decorated by the proof tree in Figure 2

$$1.\quad \frac{m}{e},$$

$$2.\quad \frac{d,\quad e}{b},\qquad \frac{f,\quad k}{c},$$

$$3.\quad \frac{b,\quad c}{a}.$$

Figure 2. The example of well-founded proof pictured by the graph in Figure 1. Each step is inferring

So, proofs may be pictured using downward growing trees of graphs; see Figure 1, where the root is a root-derivable formula. Each *graph* consists of a set of nodes and a set of edges. Every edge is an ordered pair of nodes (in Figure 1, we have a pair $\langle a, b\rangle$ as an example). If $\langle a, b\rangle$ is an edge then we will say that b is a child of a. A *path* is a finite or infinite sequence of nodes a, b, e, m linked by edges $\langle a, b\rangle$, $\langle b, e\rangle$, $\langle e, m\rangle$. A pointed graph is a graph together with a distinguished node called its point. This graph is

said to be accessible if for every node m there is a path from the point a to the node m. If this path is always unique then the pointed graph is a tree and the point is the root of the tree (the end-formula).

We will use accessible pointed graphs as our pictures of proofs. In the diagrams the point will always be located at the top. A *decoration* of a graph is an assignment of a derivable formula to each node of the graph in such a way that the premisses of the root-derivable formula assigned to a node are the derivable formulas assigned to the children of that node. A *picture* of a proof is an accessible pointed graph which has a decoration in which the root-derivable formula is assigned to the point.

Definition 1

A graph (tree) is called well-founded if it has no infinite path.

According to Mostowski's collapsing lemma (Aczel 1988), we can obtain the unique function d defined so that $dn = \{dn' : \langle n, n' \rangle \text{ is an edge}\}$ for each node n of the graph. The decoration d assigns the derivable formula dn to the node n. From this it follows that every well-founded graph has a unique decoration. As an example, a unique decoration of the graph in Figure 1 is obtained in Figure 2.

Notice that in the standard proof theory we assume the well-foundedness of trees. This means that we accept just derivable formulas with finite paths. By this assumption, the notion of proof is understood as well-founded and proof theory considers only such well-founded proofs. However, we can suppose the existence of derivable formulas with infinite (e.g., cyclic) paths. For accepting these formulas we need to postulate the following axiom formulated first by Aczel[2]:

[2] Using the decoration of a graph for describing non-well-founded notions was proposed first by Aczel, see Aczel (1988), for more details see Barwise (1992). He defined a non-well-founded set by the decoration construction. More precisely, a *decoration of a graph* is regarded by him as an assignment of a set to each node of the graph in such a way that the elements of the set assigned to a node are the sets assigned to the children of that node. A picture of a set is an accessible pointed graph which has a decoration in which the set is assigned to the point. For example, the well-founded set $a = \{\{d, \{m\}\}, \{f, k\}\}$, where $b = \{d, e\}$, $e = \{m\}$, $c = \{f, k\}$, is pictured by the graph of Figure 1. The non-well-founded set $a = \{\{d, \{e\}\}, \{f, a\}\}$, where $a = \{\{d, e\}\} = \{d, e\}$, $e = \{e\}$, $c = \{f, k\}$, is pictured by the graph of Figure 3. This set has an infinite notation: $a = \{\ldots\{\{d, \{\{\{\ldots\}\}\}\}\}\ldots\} = \{\{d, \{\{\{\ldots\}\}\}\}, \{f, \{\{d, \{\{\{\ldots\}\}\}\}, \{f, \{\{d, \{\{\{\ldots\}\}\}\}, \{f, \ldots\}\}\}\}\}\}$.

Anti-Foundation Axiom

Every graph has a unique decoration.

According to this axiom, the graph in Figure 3 is decorated by the proof tree in Figure 4. However, this graph is not well-founded because of containing infinite (cyclic) paths, for example the cyclic path $\langle a, c\rangle$, $\langle c, k\rangle$, $\langle k, a\rangle$, $\langle a, c\rangle$, $\langle c, k\rangle$, $\langle k, a\rangle$, ... So, Anti-Foundation Axiom allows us to use non-well-founded trees of proofs in decorating graphs. According to **foundation axiom**, *every graph has at most one decoration*. As a result, the graph of Figure 1 has one decoration and the graph of Figure 3 have no decoration in the case foundation axiom holds true.

Definition 2

A proof (tree) is called non-well-founded if it is pictured by the graph containing at least one infinite path.

For instance, the non-well-founded proof of Figure 4 is pictured by the graph of Figure 3. It is an example of the proof tree with three cyclic paths.

The notion of cyclic proof tree first was introduced in Brotherston (2005), Brotherston (2006), Brotherston, Simpson (2007), and Sprenger (2003). The notion of non-Archimedean proof was introduced in Schumann (2007), Schumann (2008). The proof tree of latter kind is non-well-founded, too.

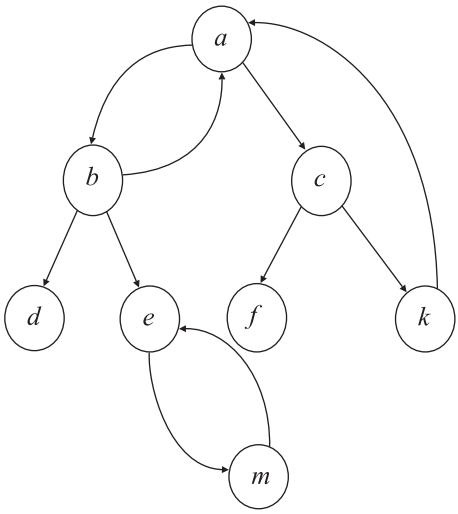

Figure 3. The example of non-well-founded graph tree decorated by the proof tree in Figure 4

$$1.\quad \frac{m}{e},$$

$$2.\quad \frac{e}{m},$$

$$3.\quad \frac{d,\quad e}{b},\qquad \frac{f,\quad k}{c},$$

$$4.\quad \frac{b,\quad c}{a},$$

$$5.\quad \frac{a}{b},\qquad \frac{a}{k}.$$

Figure 4. The example of non-well-founded proof pictured by the graph in Figure 3

3. Cyclic proofs

Argumentation is a kind of inferring from premisses. We will differ two argumentation ways:

- *proof argumentation* ("the standpoint A is provable by arguments $B_1, \ldots, B_n$"), it is a derivation in which the conjunction of premisses $B_1, \ldots, B_n$ implies a derivable statement A,
- *confirmation argumentation* ("the standpoint $\neg A$ is not provable by arguments $B_1, \ldots, B_n$", i.e. "A is confirmable by $B_1, \ldots, B_n$"), it is a derivation in which the premisses are compatible with a derivable statement.

By these definitions, the derivation supposed in proof argumentation is a reflexive and transitive relation. Indeed, the standpoint A is provable by the argument A and if the standpoint A is provable by the argument B and B is provable by the argument C, then A is provable by the argument C. The derivation supposed in confirmation argumentation is a reflexive, symmetric and transitive relation. For instance, if the standpoint A is confirmable by the argument B, then the standpoint B is confirmable by the argument A and vice versa. As we see, confirmation argumentation can be viewed as a kind of non-well-founded (cyclic) proof. More precisely, define the binary relation Der of derivation as the least relation satisfying: whenever there is a proof tree containing P in the conclusion and Q among the premises, then $Der(P, Q)$ holds. Further, define Der' to be the reflexive and transitive closure of Der. We say two statements P and Q are *mutually dependent* if both $Der'(P, Q)$ and $Der'(Q, P)$ hold, i.e. Der' is symmetric also.

Usually, confirmation argumentation is regarded as probabilistic inferring, but there is the one more natural way to consider it as a kind of non-well-founded proof, because in the real speech practice the argumentation "if B then A" means "A is confirmable by B" very often. As an example, let us compare the following two proof trees stated at the session of the Association of Moscow Writers:

1. $\dfrac{\textit{Pasternak is an individualist}}{\textit{He is closed into his internal world}}$,
2. $\dfrac{\textit{He is closed into his internal world}}{\textit{He spites at our people, at our business}}$,
3. $\dfrac{\textit{He spites at our people, at our business}}{\textit{Pasternak is an individualist}}$

(a) *Pasternak wanted to receive the Nobel Prize,*

(b) *Many enemies of Soviet people received the Nobel Prize such as the Fascist-like French writer A. Camus,*

(c) *Usually, the Nobel Prize for Literature is awarded for reasons of anti-Soviet policy*

Pasternak is an enemy of Soviet people

The first reasoning belongs to G. Nikolaeva, the second to S. Smirnov. Evidently, Nikolaeva's argumentation (her proof tree) is of the form of cyclic proof because her statements "Pasternak is an individualist", "He is closed into his internal world", "He spites at our people, at our business" are mutually depended, while Smirnov's argumentation is a standard (well-founded) inferring though his derivation is probabilistic.

The majority of proof trees used at the session of the Association of Moscow Writers is cyclic. So, the common root-derivable statement (standpoint) of this session "Pasternak should be sent from our country" means that according to his world outlook, Pasternak is an enemy of Soviet people. The main arguments of Soviet writers that had to prove the root-derivable statement were mutually depended with this standpoint. For instance, G. Nikolayev's argument "The story of Pasternak is the story of treason", V. Soloukhin's argument "That book is the Cold War weapon against Communism", S. Baruzdin's argument "Our people have not known Pasternak as a writer, but they will remember him as a traitor" are compatible with the standpoint, i.e. they just confirm it, but do not prove (in the standard meaning of well-founded proof trees).

Definition 3

An argumentation is cyclic if main arguments and a standpoint are mutually depended.

Notice that the cyclic argumentation has the form of a cyclic (non-well-founded) proof tree, where the root-derivable expression is a standpoint. Obviously, this argumentation is pictured by the graph containing at least one cyclic path.

4. Conclusion

The logical part of argumentation is not reduced to the standard, well-founded proof theory. For example, in the case of excluding Boris Pasternak from the Association of Writers of the USSR the logical part of argumentation contained non-well-founded (cyclic) proofs. This does not mean that it was an exception to the rule. Cyclic argumentation is used very often in human speeches.

References

Aczel, P. (1988), *Non-Well-Founded Sets*, Stanford.

Barwise, J., Etchemendy, J. (1987), *The Liar*, Oxford UP, New York.

Barwise, J., Moss, L. (1996), *Vicious Circles*, Stanford.

Barwise, J., Moss, L. (1992), *Hypersets*, Springer Verlag, New York.

Brotherston, J. (2005), 'Cyclic proofs for first-order logic with inductive definitions' *in* B. Beckert, ed., *TABLEAUX 2005*, volume 3702 of LNAI, Springer Verlag, pp. 78–92.

Brotherston, J. (2006), *Sequent Calculus Proof Systems for Inductive Definitions*. PhD thesis, University of Edinburgh, November.

Brotherston, J., Simpson, A. (2007), Complete sequent calculi for induction and infinite descent, *LICS-22*, IEEE Computer Society, July, 51–60.

Literaturnaya Gazeta [*Literature Newspaper*] (1958), 1 November.

Pasternak, B. (1958), *Doctor Zhivago*, translated by Max Hayward and Manya Harari.

Schumann, A. (2007), 'p-Adic Multiple-Validity and p-Adic Valued Logical Calculi', *Journal of Multiple-Valued Logic and Soft Computing, 13 (1–2)*, 29–60.

Schumann, A. (2008), 'Non-Archimedean Fuzzy and Probability Logic', *Journal of Applied Non-Classical Logics*, 18/1, 29–48.

Sprenger, Ch., and Mads Dam (2003), 'On the structure of inductive reasoning: circular and tree-shaped proofs in the μ-calculus', *FOSSACS 2003*, volume 2620 of LNCS, 425–440.

Mary Dziśko
Department of Philosophy
Belarusian State University of Informatics and Radioelectronics
Minsk, Belarus
m-dzisko@yandex.ru

Andrew Schumann
Department of Philosophy and Science Methodology
Belarusian State University
Minsk, Belarus
Andrew.Schumann@gmail.com