Alexander S. Karpenko

MODERN STUDY IN PHILOSOPHICAL LOGIC: WORLDWIDE LEVEL AND RUSSIAN SCIENCE

In this paper, we survey the development of philosophical logic in Russia within the framework of worldwide tendencies. Philosophical logic is an extremely wide area of logical studies, requiring philosophical judgement of the basic concepts, used in modern logic, and the outcomes obtained by means of mathematical logic. However, we need to remark that the term 'philosophical logic' is uncertain and has no uniform use. Even if philosophical logic is represented as a special scientific discipline, it is not possible to define its subject, limits of application, and methods. Therefore we consider the background of different directions in philosophical logic and its connection with philosophy of logic, foundations of logic, and computerization of logic.

Head of the Department of Logic,
Institute of Philosophy of the Russian Academy of Sciences, Moscow, Russia
e-mail: karpenko@iph.ras.ru

1. Introduction

The modern development of logic and comparative analysis of two logical journals: 'Journal of Symbolic Logic' and 'Journal of Philosophical Logic', published under aegis of the international Association for Symbolic Logic, started in 1936, shows a rapprochement of topics, methods, and results in logical papers. At the 9th International Congress of Logic, Methodology and Philosophy of Science (Uppsala, Sweden, 1991) G. von Wright stated: "The point of logic is that it has fused in diverse researches of mathematics." Nevertheless, modern researches in logic should be divided into three basic sections:

I. Mathematical (symbolic) logic.
II. Philosophical logic.
III. Non-classical (unconventional) logic.

The third section in its following directions

1. intuitionistic and superintuitionistic logics (including Markov's constructivism),

2. modal and temporal logics,
3. many-valued and fuzzy logics,
4. relevant and paraconsistent logics

unconditionally belongs to philosophical logic by virtue of those especially philosophical premises from which these directions appeared. However, logical schema become so refined, formally developed and mathematized that they do not keep a place to philosophical gamble. But by the end of the 20th century, the question was raised at the junction of all these three sections: "what is a logical system?" and generally "what is a logic?"

In this paper, we shall be basically limited by section (**II**). However, it is important to notice that Russian logicians have scientific results of the highest worldwide level just in non-classical logics.

First of all, we shall characterize how mathematical logic should be understood. In the preface to 'Handbook of Mathematical Logic' [6], J. Barwise writes: "The mathematical logic is traditionally subdivided into four sections: model theory, set theory, recursion theory and proof theory." However, nowadays the state of affairs was changed a little, taking into account the significant role which logic plays in computer science. Hence, we observe an amplification of significance of proof theory and we see that problems of computability and complexities took the first place in recursion theory. It was regarded at the conference of Association for Symbolic Logic in Urbana-Champaign in June, 2000, namely at a special meeting "Perspectives of mathematical logic in the twenty first century." As a result of this meeting, the work of the four authors S. R. Buss, A. S. Kechris, A. Pillay, and A. Shore is published with the same title [20] in correspondence with the four main sections of mathematical logic.

Notice that in this work no directions of non-classical logic are mentioned and references to Russian logicians are extremely rare (it is typical for any western survey on logic). However, there is V. A. Uspensky's survey "Mathematical logic in the former Soviet Union: Brief history and current trends" [149].

A unique history of Russia in the 20th century has also predetermined a unique development of logic in it, in many respects this development is not clear for the western historians of science. In a totalitarian system, the "truth" becomes a subject of only ideological manipulations, and the lie and terror become the highest values. One of the features of the logic development in the former Soviet Union had consisted in the thesis about the union of logicians-mathematicians and logicians-philosophers. It is not surprising: in the Soviet political system, logicians-philosophers searched for a support of logicians-mathematicians who still had possibility for scientific

publications even in foreign issues, as philosophical papers were published in the 20's and 30's years in the unique Soviet philosophical journal "Under the Banner of Marxism", where formal logic was compared to dialectics and dialectical logic. The first (formal logic) was declared bourgeois and the second (dialectic) proletarian; as a result, formal logic was betrayed to the anathema. The disastrous atmosphere of those years for philosophy and logic is circumscribed in Bazhanov's book [9], which unambiguously is named 'The Interrupted Flight' (see also [76]). Even when in 1947 formal logic was returned in the system of the secondary and higher education, its position in this system was not independent. Already in the beginning of the 50's years during the imposed controversy it has been fixed that the highest level of thinking is dialectical logic, the lowest is formal logic. Within this framework all 50's and even 60's years have passed in mutual polemic.

All this should be taken into account, if we want to conduct the comparative analysis of development of logic in our country and abroad.

2. Philosophy of logic or philosophical logic

Some tendencies of development of logic have been revealed at the end of the 20th century [75]. Here we shall be limited to philosophical logic.

Philosophical logic is extremely wide area of logical studies, requiring philosophical judgement of the basic concepts, used in modern logic, and the outcomes obtained by means of mathematical logic. However, we need to remark that the term 'philosophical logic' is uncertain and has no uniform use. In modern logic and philosophy, philosophical logic is understood by various experts variously and in their own way. Even if the philosophical logic is represented as a special scientific discipline, it is not possible to define its subject, limits of application and methods. Moreover, it is not possible to divide strictly the two different directions of researches: philosophical logic and philosophy of logic. Frequently, one is substituted another and occasionally one does not distinguish them.

The term 'philosophical logic' has appeared in the English-speaking logical-philosophical literature and has had a wide application already in the 50–60's years of the 20th century, when in the USSR logicians-philosophers have discussed what level of thinking (either formal or dialectical) they belong to. On the one hand, the crisis in the foundations of mathematics (detecting paradoxes in set theory and A. Tarski and K. Gödel's limitative theorems) has required a deep judgement of the most conceptual means of logic. On the other hand, the appearance and rapid development of

non-classical logic, first of all modal logic, has drawn a wide attention of logicians with the philosophical orientation.

2.1. Philosophy of logic

Firstly, we shall sketch the area of logical studies which have the title 'philosophy of logic'. For logicians-mathematicians, philosophy of logic is a development of set theory and appropriate problems on the method of forming sets and on the nature of number. The detection of paradoxes in set theory and, in particular, Russell's paradox has raised the question about the nature of mathematics. *Logicism* tried to define the basic concepts of mathematics by means of logical terms (G. Frege in 1884 and B. Russell in 1903). It is both technical and philosophical problem: whether it is possible to infer all mathematics from some (or even one of) logical terms? In this sense, the grandiose construction, undertaken by Whitehead and Russell in 'Principia Mathematica', has appeared unsuccessful. And though in their logical-mathematical theory there were no paradoxes, but it has appeared impossible, for example, to infer the existence of infinite sets from only logical axioms. *Intuitionism*, as one more reply to the detection of paradoxes, has set up the problem about a distinction between the finite and infinite, e.g. about a difference of the potential infinity from the actual one. There was the problem of existence and substantiation of proofs; and it is the most important there was the problem of the status of classical logical laws. All this is philosophical problematics. D. Hilbert's formalistic program, in particular, his problem of *finitism*, has also caused a brisk philosophical controversy. One more method to avoid paradoxes in mathematics is *axiomatic set theory.* All these four approaches to **foundations of mathematics** require the deepest philosophical judgement (see [79], [44], [24], and also the book [110], in which one criticizes a philosophical basis of classical programs of foundations of mathematics). Chapter 7 in [57] is devoted to Hilbert's program within the framework of results of Gödel's theorems.

Actually, the aforementioned concerns more to philosophy of mathematics, than to philosophy of logic, but the problem of philosophical judgement of applying logic to a solution of various problems of mathematics remains. A convincing example here is K. Gödel's limitative incompleteness theorems (1931) which say that there is no adequate formalism, enveloping all mathematics, and in general this formalism is impossible. Philosophical corollaries of these outcomes are discussed up until now and have drawn a huge attention not only of logicians-professionals, but also of philosophers and methodologists. However, we can refer to papers of experts [32], [111], [137].

To this it is necessary to add also a philosophical controversy concerning the thesis of Church-Turing, asserting that all computers are equivalent among themselves.[1] If we consider a human brain as a computer, then there are no obstacles for a computerization of human logic.

It is interesting that mathematicians were occupied with philosophy of logic who have the deep results which have obtained in mathematics (G. Frege, B. Russell, L. Brower, K. Gödel, W. Quine, R. Carnap, etc.). Quine published in 1940 the book with the title 'Mathematical logic', and in 1970 with the title 'Philosophy of logic' [115] (it is reprinted in 1986) where logic is understood as a systematic study of logical truthes and philosophy of logic becomes a tool for the analysis of the natural language. The book contains the following sections which Quine refers to philosophy of logic: 'Meaning and Truth' (problem of sentences and propositions, sentences as the information, the theory of sense of language expressions, truth, and the semantic consent); 'Grammar' (the recursive setting of grammar, categories, the revision of the purpose of grammar, names and functors, the lexical criterion; time, events and verbs, propositional aims and modality); 'Truth' (definition of truth in Tarski's style, paradoxes in language, connection between semantic and logical paradoxes); 'Logical Truth' (in terms of structure, in terms of model, in terms of substitution, in terms of proof, in terms of grammar); 'Scope of Logic' (the word problem, set theory, the quantification); 'Deviant of logic' (namely, non-classical logics, first of all, many-valued logic, intuitionistic logic, branching quantifiers); 'Foundations of Logical Truth' (place of logic, logic and other sciences).

Thus, Quine has concentrated the work around of the main problem in philosophy of logic: what the truth is? However, only due to the development of mathematical logic, namely in A. Tarski's paper of 1933 [143] the semantic definition of truth for the big group of the formalized languages was given for the first time and the limits of such a definition were simultaneously indicated. To the further discussion concerning Tarski's definition of truth the special issue of the journal "Synthese" 126, Nos 1–2 (2001) is devoted. Problems of truth are considered in [35] in the context of philosophy of logic. The significant part of the book [138] is devoted to Tarski and Kripke's theory of truth and generally envelops a wide circle of problems of modern interest to the truth concept.

There is a special site 'Philosophy: Philosophy of Logic'[2]. Here it is

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[1] *Edit.*: More precisely, all conventional approaches to computability reduced to recursive functions or Turing machines (in Turing's words automated machines) are equivalent.

[2] http://dirt.netscape.com/Society/Philosophy/Philosophy_of_Logic/

possible to find many links concerning philosophy of logic. Also, notice the site of the Logical Sector of Institute of Philosophy of the Russian Academy of Science develops permanently too (http://iph.ras.ru/~logic/).

Among recent monographs on philosophy of logic we remark S. Haak's books [58] and [59]. See also the monograph [117]. We pay attention to the site Factasia [70], created in 1994, where it is possible to find the universal philosophical approach to understanding of logic, to its significance and applications.

As it is noticed in the electronic 'Encyclopedia Britannica' (1994–1999): "The diversification of different logical semantics became central area of researches in philosophy of logic." Problems of logical semantics are considered in books [151] and [136], the first of them became classical. However, the main problem is a development of the uniform semantic approach, enveloping completely various logical systems, and as well as a contemporary analysis of various semantic concepts and their distribution on classes of logics. R. Epstein's monograph [36] (the 2nd edition in 1995) is devoted to this. In it so-called "set assignment semantics" are developed. The same fundamental work [37] is devoted to first-order logics. There is a site devoted to logical semantics with personalias, starting from G. Frege[3]. The existence of infinite classes of logics puts the problem on the semantic basis of logic in a new way. We underline that one of the most popular topics becomes a research of classes of semantics for which various non-classical logics are complete. As a result, model theory is transformed too. If initially it dealt with mutual relation between a formal language and its interpretation in mathematical structures, now logic becomes the tool for study of the most various structures and their classification (see [7]).

Certainly, the area of philosophy of logic is much wider. It includes the theory of the propositional form as sentences about some states of affairs in the world, generally, the doctrine about the logical form (see the monograph [121]), the doctrine about logical and semantic categories, the theory of reference and prediction, the identification of objects, the problem of existence, the doctrine about presupposition, the relation between analytical and synthetic judgments, the problem of scientific law, the informativeness of logical laws, ontological assumptions in logic and many other things. And even such, apparently, only logical problems concern to philosophy of logic: the essence and the general nature of deduction, the logical deducibility between any expressions or sets of expressions, the meaning of logical connec-

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[3] http://www.phil.muni.cz/fil/logika/til/inks

tives, the significance of fundamental theorems, obtained in mathematical logic, and in this connection the careful analysis of such concepts as 'induction', 'computability', 'decidability', 'demonstrability', 'complexity' and besides 'truth'.

2.2. Philosophical logic

Initially, philosophical logic referred to modal logic, i.e. to the logical analysis of such philosophical concepts as 'possibility' and 'necessity'. Historically, these two concepts, especially since Aristotle, drew to themselves a constant attention of philosophers; and due to the development of symbolic logic, there is a unique possibility to explicate modality and their mutual relation by means of exact methods. This also concerns to such philosophical concepts as 'future' and 'past'. In modal logic one started to investigate new kinds of modalities: temporary, modal-temporary (synthesis of modal and temporary operators), physical or causal, deontic, epistemic, etc. Since the edition of Gabbay and Guenthner's 'Handbook of Philosophical Logic' [47] in the 80's years (hereinafter *HPL*), some results of development of philosophical logic are fixed. The 2nd and 3rd volumes content a consideration of various non-classical logics: in the 2nd volume one considers extensions of classical logic $\mathbf{C}_2$, for example, such as modal, temporary, deontic logic, etc., and in the 3rd volume one examines alternatives to classical logic, for example, such as many-valued, intuitionistic, relevant logic, etc. Notice that such a division of non-classical logic is not convincing. The point is not that there are logics which do not concern to one of these subdividings, for example, syllogistics, Leśniewski's logical systems, combinatory logic, infinitary logic, etc. It is a bit unexpected that logics, which originally were under construction as limitation of some classical laws and principles, actually are the extension of classical logic: for example, some many-valued logics as well as modal logics are extensions of $\mathbf{C}_2$. The matter is that the definition of non-classical logic is open yet. Therefore it is not surprising that the more neutral term 'non-standard (unconventional) logic' recently began to be used. There is a site with the short description of 29 non-standard logics and the short references to each of them [140].

Let us pay attention that in each of those logics there is an own philosophy of logic and as well as all aforementioned philosophical problems, because the definition of truth-value of formula, the logical deduction, the concept of sentence, and the meaning of logical operations are different for the majority logics. In each philosophical logic there is additional philoso-

phical problematics. For example, in modal logics those are the problem of reference, cross-identification, i.e. identifications of objects in the various possible worlds, and in this connection there is the problem of quantification. In many-valued logics there is a very difficult philosophical problem of interpretation of the set of truth-values, usually expressed by numbers: rational, natural, real. Many philosophical problems are connected to intuitionistic logic, for example, the existence of two heterogeneous and irreducible to each other classes of semantics for it: realizedness and Kripke's models.

Philosophical logic has a language and technical means that are much reacher and more flexible than in symbolic logic; it allows to start the analysis and reconstruction of only philosophical problems and even such fundamental ones as the problem of logical and theological fatalism, determinism and contingencies, asymmetries of time, etc. (See [74]). An *introduction* to philosophical logic is contained in the monographs [159] and [56]. A unique monograph in Russian with the title 'Philosophical Logic' is written by A. Schumann [123], he understands philosophical logic as "general semantics of various logical calculi."

A modern understanding of philosophical logic is reflected in the collected works, basically representing surveys on the most important directions in modern philosophical logic [67]. It contains 46 papers in the following 14th sections:

I.   "Historical development of logic";
II.   "Symbolic logic and usual language";
III.   "Philosophical dimensions of logical paradoxes";
IV.   "Truth and the certain description in the semantic analysis";
V.   "Concepts of logical deduction";
VI.   "Logic, existence, and ontology";
VII.   "Metatheory and orb and limits of logic";
VIII.   "Logical foundations of set theory and mathematics";
IX.   "Modal logic and semantics";
X.   "Intuitionistic, free and many-valued logics";
XI.   "Inductive, fuzzy, and quantum-probability logics";
XII.   "Relevant and paraconsistent logics";
XIII.   "Logic, mechanization and cognitive science";
XIV.   "Mechanization of inference and detection of proofs".

Notice that the last section is especially indicative that it is referred to philosophical logic.

Let us pay attention that both philosophy of logic (see W. Quine and S. Haak's monographs) and philosophical logic study non-standard logics. In the latter case it becomes the obvious tendency exhibited already in the

book of N. Rescher [118] and precisely designated, as it was already spoken, in the first *HPL*, namely referring the increasing class of non-standard logics to area of philosophical logic. This tendency amplifies in the new 18 volumes of *HPL* [48]. In these books one have already refused from the division of non-classical logics into extensions of $\mathbf{C}_2$ and alternatives to it. Another tendency consists in that one considers as philosophical logic all branches which directly do not concern the four sections of mathematical logic: model theory, set theory, recursion theory, and proof theory. Therefore it is not surprising that the paper with the title 'Algebraic logic' [2] is included in the 2nd volume of new *HPL*. The major area of researches in algebraic logic is a definition of necessary and sufficient conditions for construction of algebraic semantics, i.e. for constructions of Lindenbaum's algebra (algebra of formulas). An appropriate classical work is [14]. The fact that it is not possible for any logical calculus to construct Lindenbaum's algebra (for example, for well-known da Costa's paraconsistent logics Cn) became an additional stimulus of developing new semantic methods. Even earlier, the so-called 'valuation semantics' or 'bivalence semantics' (see [28]) has appeared in the beginning of the 70's years (N. da Costa, R. Suszko). If usually a function of value is an algebraic valuation, i.e. a homomorphism of algebra of formulas into an algebra of the same type, then now this limitation is removed and a value is a simple function, which associates one of two bivalent values with each formula, i.e. two-valued valuations are considered as characteristic functions of sets of formulas. There are some methods of the proof that any propositional logic has bivalent semantics.

Coming back to the problematics of algebraic logic, we underline that its means is a good tool for clearing up such a complicated question as mutual relation between various logical systems. In the book of P. Halmos and S. Givant "Logic as algebra" [61] is shown that standard outcomes in logic well correspond with known algebraic theorems. The famous Russian logicians A. Kuznecov predicted such a universal analysis of logic by algebraic methods in the fine article "Algebra of logic" for the Philosophical Encyclopedia [85], but even he could not foresee a wide line of applying algebraic logic. See the monograph "Algebraic methods in philosophical logic" [33], where the basic attention is concentrated on representation theorems as tool for completeness theorems. The same tool is basic for the study of formal phenomenology (!) n V. L. Vasyukov's monograph [154].

The basic present-day conclusion is as follows: *the logical laws are no other than algebraic laws.* All this happens within the framework of unreasonable revival of psychologism in logic in our country. For the last de-

cade the big number of textbooks and handbooks on logic, and even the encyclopedic editions on philosophy containing the items on logic were published, in them one affirms (however, there are some exception, see, for example, [52] and [3]) that logic studies the laws of thinking. At the same time, not only the mathematical development of logic, but also somewhat philosophical development of logic shows that there are no more laws of thinking distinct from laws of algebra (see [27]).

Generally, the concept of philosophical logic is inconsistent. On the one hand, it includes, as it was already spoken, all logical researches which are not only mathematical. On the other hand, the modern development of modal, temporary, intuitionistic, and especially many-valued logics showed that they are no other than the sections of symbolic logic: the same methods of a symbolization and an axiomatization and in many respects the same technical problems and tasks. This also caused the construction of new theories of sets on the basis of non-classical logics, being on the origin only philosophical, namely many-valued, modal, relevant, paraconsistent theories of sets have appeared, in which one tries to deny corollaries implying from Gödel's theorems.

Notice that Zermelo-Fraenkel's system with the axiom of choice, but without the axiom of foundation is specially interesting, see, for example, [114] and [29].

3. Foundations of logic

Now we should pay attention to the main tendency of development of logic at the end of 20th century and the beginning of the 21st century. As well as the problem on foundations of mathematics has risen hundred years ago, so now there is the problem on foundations of logic. The following topics refer to:

(i)   What is an inference?
(ii)   What are logical concepts (operations)?
(iii)   What is a logical system?
(iv)   What is a logic?

In a very authoritative edition on the history and development of logic [80] (the 9th edition is in 1985) we find the following traditional definition of a subject of logic: "science which researches principles of correct or acceptable reasonings." However, such a definition does not solve the problem of exact area of the given subject, i.e. what area of applying logic is? For traditional logic it is *syllogistic* reasonings and there are equally 24 correct

syllogisms. In turn, mathematical logic studies *mathematical* reasonings: "If his researches are devoted first of all to study of mathematical reasonings, the subject of his investigations can be called mathematical logic" [97]. Informal logic studies *informal* reasonings, and philosophical logic, as a result, studies *philosophical* reasonings. In order to avoid similar senselessness, it is necessary to select the nucleus or base concepts with which the given science deals.

Such a nucleus undoubtedly is the concept of 'logical inference (consequence)'. A. Tarski in 1936, as one of creators of modern logic, sketched its essence in the work with the characteristic title "On the concept of logical consequence" (see [114]). However, we can add there the methodological aspects: in what terms it is or what paradigm of the offered answer is. Approaches to the answer concerning an orb of logic, its basic concepts, which are used by the conception of logical inference, may be completely various: model-theoretic, semantic set-theoretic, proof-theoretic, constructive, combinatory, etc. As we shall see, A. Tarski's answer is within the framework of the semantic approach: "A proposition X logically follows from propositions of the class **K** if and only if each model of the class **K** also is a model of a proposition X" [141].

Nowadays Tarski's concept of logical consequence is regarded as debatable. Tarski's work has more philosophical, nontechnical character and allows to interpret it in various conflicting way, for example, there is an opinion that Tarski's definition is incorrect from the point of view of modern mathematical logic [26] or that it should be generally rejected [38] and [39]. An interesting analysis of Tarski's work is proposed in [120], where Sagöillo examines three basic concepts of logical inference, each of them envelops an important part of argument and each of them is accepted by logical community. The conclusion of the author is interesting too that Tarski does not speak, what the logical consequence is, and considers what the logical consequence is similar to G. Ray [116] tried to defend Tarski's conception in his big article (see the reply in [63]) and as well as M. Gómez-Torrente defends this conception [54] and [55].

The basic objections against Tarski's definition of the concept of logical inference are as follows. Anywhere in the given work it is not stipulated that the data domain should vary, as it is in modern logic (see [26]). Logical properties, in particular the general validity of the argument of logical inference, should be independent of a separately selected universal set of reasonings, in which language appears interpreted. Otherwise, many statements about a cardinality of data domain at a special interpretation of language can be expressed only by means of logical constants and, as result,

they should appear logical true. However, Tarski himself considers the idea of the term 'logic' as excluding among logical trues any statements about a cardinality, let even of logical area. Another objection is directed against Tarski's acceptance of the ω-rule (the rule of infinite induction) at formalizing first-order arithmetic. However, actually it was only a version of this rule in the simple theory of types. In connection with these objections it is necessary to make some general notes. Tarski knew very well Gödel's works about the completeness, where the theorem is proved on the basis of truth of statements at all possible interpretations, and as well as about the incompleteness (ω-incompleteness) of first-order arithmetic. In the first case one showed a concurrence of logical consequence in the first-order classical logic (hereinafter by **PC**) with syntactic consequence, in the second case one did not. From Tarski's works it clearly follows that he considers the logical consequence and deductability as various concepts and the first as much wider than the second. The basic intention of Tarski was to define the logical inference, applied for very wide class of languages, so wide that, as we shall see further, there are the problems of already other level relating the item (iv).

For now notice that the concept of logical consequence has taken the central place in logic and therefore the following problem seems to be very important: *What does this mean for the conclusion A to be inferred from premises Σ?* The following criterion is considered conventional: *A* follows from premises Σ if and only if any case, when each premise in Σ is true, is the case, when *A* is true. Pay attention that the famous Russian logician A. A. Markov connects this principle to the definition what logic is: "Logic can be defined as a science about good methods of reasonings. As "good" methods of reasonings it is possible to understand ones, when from true premises we infer a true conclusion" [93]. As a result, the essence of logical inference is a preservation of truth in all cases. There are many ways, when using Tarski's concept of logical consequence, it is possible to represent all laws of **PC** as valid. Thus, we obtain a standard definition of classical logic together with all its logical operations. For instance, the conjunction of two formulas $A \wedge B$ is true at a situation (in a possible world) *w* iff *A* is true in *w* and *B* is true in *w*.

But we have there much more problems. Why we call the obtained logic classical and what does this mean? We consider this problem still. What does it mean, the standard setting of true conditions for logical connectives? Finally, what should we consider as logical operations? The concept of truth is directly connected to the understanding of logical inference, given by Tarski, and all together results in objects which we call 'logical laws': *they*

are deductions preserving the truth. But how we can define the logical law, not having defined what we should consider as logical constants (operations), while we have a natural variability and instability of nonlogical objects of reality. If we consider all objects as logical terms: variables, numbers, etc., then a model-theoretic interpretation of each term should be fixed and, therefore, only one model should exist. It would make the concept of logical truth empty.

In the work "On the concept of logical consequence" Tarski keeps the problem open what should we consider as logical concepts (operations), and what as out-logical? Tarski writes that he does not know any objective basis for strict differentiation of these two groups of terms [141]. It is obvious that this problem did not give rest him and in thirty years he comes back to it in the lecture "What are logical notions?", read in 1966 in London Bedford College, in the same year in the Tbilisi Computer Center, and later in SUNY Buffalo in 1973. The report is published after Tarski's death in 1983 (see [142]). The basic idea consists in that logical notions (concepts, operations) should be invariant in respect to an appropriate group of transformations of reasoning area. Tarski extends an area of applying F. Klein's program, where one proposed a classification of various geometries in accordance with the space transformation, when geometrical concepts are invariant. For example, concepts of Euclid's metric geometry are invariant relatively isometric transformations. In the same way, algebra can be considered as study of concepts, invariant relatively automorphisms of such structures as rings, fields, etc. Then according to Tarski, logical concepts are invariant relatively any one-to-one transformations of the universal set onto itself, i.e. relatively any permutations of reasoning universe (data domain). Implicitly, this idea of an invariant permutability was already contained in various logical-mathematical works (for the first time [100]), in linguistic works (see [77] and [152]), in philosophical works (see [107], [94], and [131]), and as well as in the collected works with the rather actual title "The limits of logic" [126]. Tarski's thesis was a basis with some natural updating for definition of logicality in G. Sher's book [130]: the operation is logical if it is invariant relatively each bijection between areas.

Finally, in the work [96] it is shown that if Tarski's thesis is accepted, then logical operations are defined in the full infinitary language $L_{\infty,\infty}$, (in the same work there is a generalization in Sher's style, i.e. it is given a characterization of logical operations relatively isomorphic invariancy). Recall that the language $L_{\infty,\infty}$ is a language of conventional first-order logic with equality (Frege's language), but admits conjunctions and disjunctions of an arbitrary length and as well as an arbitrary length of sequence of universal

and existential quantifiers. This language is very rich – it contains the whole
first-order logic. The latter allows us to set a quantification on arbitrary
functions, defined on areas of reasoning, as well as a usual quantification
over members from this area. Since sets and relations can be represented
by their characteristic functions, then the second-order logic envelops also
a quantification on arbitrary sets and relations. Not only arithmetic, but
also set theory are included in the second-order logic (natural numbers,
sets, functions, etc. are there logical concepts), as a result, all set-theoretic
problematics, including the continuum hypothesis and many other impor-
tant mathematical statements, are contained in the second-order logic (see
the monograph [91]). Thus, *mathematics is a part of logic.* Depending on
expressive means of new logic, we come to logic of natural numbers, logic
of real numbers, logic of topological spaces, etc.

In connection with these problems S. Feferman's article seems to be very
interesting [40]. In this article Feferman criticizes the thesis of Tarski-Sher
and one of objections is that there is an assimilation of mathematics by
logic. But the main objection consists in that the thesis of Tarski-Sher does
not give any natural explanation, how logical operations behave on data do-
mains of the various cardinality. Therefore Feferman introduces the concept
of operations which are homomorphic invariant on functional-type struc-
tures. Such operations, according to Feferman, are logical and, it is the
most remarkable, they exactly coincide with operations of the first-order
logic without equality. However, here again there is a problem whether the
equality may be considered as a logical operation? See the discussion of
this problem in [115], where Quine is declined to the positive answer, as
a reason (among other things) he says about the deductive completeness
of the first-order logic with equality. As a value of this approach, Feferman
considers that the operations of **PC** are defined in terms of homomorphic
invariant operations of one-place type. Thus, he refers to [78], where the cen-
tral role of one-place predicates in human thinking is shown by the example
of the natural language.

There would be strange, if the exact characterization of **PC** in terms
of its operations appears only in 1999. Actually, already in the 60s years
A. V. Kuznetsov generalized the theorem of functional completeness of pro-
positional logic in the predicative case. Unfortunately, this proof is not pu-
blished still. Much later this theorem was proved in [161], i.e. it is shown that
the certain set of logical operations is adequate both for **PC** and for **PC**(=).
The preference is returned **PC**(=). The author follows from the basic as-
sumption that to be considered as logical operation, its meaning should be
completely contained in axioms and inference rules. Thus, differently from

the semantic approach of Tarski-Sher-Feferman the proof-theoretic approach is used for a characterization of logical operations.

The characterization of logical operations entails the characterization of the logic as a whole. However, the characterization of **PC** can be given in terms of fundamental model-theoretic properties of the theory T in the first-order language. These properties are:

The **theorem of compactness** (for countable languages). *If each finite set of propositions in T has a model, then T has a model.*

The compactness takes place, as only the finite set of premises is used in deductions. This property was revealed by K. Gödel in his paper about the completeness of **PC** (1930). The two other properties of the first-order logic were proved earlier.

I. Löwenheim-Skolem's theorem. *If T has a model, then T has a denumerable model, too.*

II. Löwenheim-Skolem's theorem. *If T has an infinite model, then T has an uncountable model.*

Much later P. Lindström [86] showed that these properties are characteristic for PC in the following sense:

Lindström's theorem. *The first-order logic is the only logic closed in respect to $\wedge$, $\neg$, $\exists$ and satisfying Löwenheim-Skolem's theorems and the theorem of compactness.*

Lindström's paper began paradigmatic for the major researches in logic of the last quarter of the 20th century. In essence, Lindström's theorem defines the first-order logic, more precisely **PC**(=), in terms of its global properties. But a serious limitation on expressive means of the first-order logic follows from these properties. The most simple infinite mathematical structure is built by natural numbers and the most fundamental mathematical concept is the concept of finiteness. However, from the theorem of compactness it follows that central concepts such as a finiteness, denumerability, well-orderedness, etc. cannot be defined in first-order logic. Actually, the finiteness is not distinctive from the infiniteness. In turn, from Löwenheim-Skolem's theorem it follows that the first-order logic does not distinguish the denumerability from the uncountability and, hence, no infinite structure can be described to within isomorphism. Moreover, many linguistic concepts, distinctions and constructions are beyond applications of **PC** (see [101], [88]).

There is a lot of interesting logics, which richer than the first-order logic such as the weak logic of the second order which tries to construct the concept of finiteness in logic in the natural way (it allows to quantify over finite sets); logics with various extra-quantifiers such as 'there exists finitely

many', 'there exists infinitely many', 'majority', etc.; logics with formulas of infinite length; logics of the higher-order (see [150]). However, it doesn't matter how we extend the first-order logic – in any case we lose either the property of compactness, or Löwenheim-Skolem's property, or both and as well as we lose the interpolation property and in most cases deductive completeness. However, G. Boolos [17] protecting the second-order logic, asks: Why the logic should necessarily have the property of compactness? It is interesting that we find a similar question in 1994 on pages of 'The New Encyclopedia Britannica': Why Löwenheim-Skolem's property should correspond to the internal nature of logic? (Vol. 23, p. 250).

The construction of various extensions of **PC**, especially logics with the generalized quantifiers, drew the big attention of linguists, mathematicians, philosophers, cognitivists. A total of development of this direction is reflected in the fundamental work 'Model-theoretic logics' [7], where Barwise comes to the following conclusion: "There is no back way to the point of view that logic is first-order." The authors of monographs [126] and [130] are of the same opinion too.

However, the second-order logic is too complicated. Second-order logics are not recursively enumerable deductive systems. The basic problems arise with logical trues. For example, there are statements which are logically true if and only if the generalized continuum hypothesis holds. All these difficulties and many other are an inevitable corollary of a huge potency of expressive means of second-order languages. Therefore it is no wonder that there are weak versions of second-order logic, and in new *HPL* we find the article 'Systems between first-order and second-order logics' [129]. It can be achieved due to limitative versions of understanding schemes ($\sum_1^1$-formulas and $\prod_1^1$-formulas), to limitations on the axiom of choice and limitations on the principle of induction for arithmetic, etc. Another instance is monadic second-order logic. As a rule, the majority of these languages characterizes the concept of 'finiteness' and allows a categorical characterization of natural numbers. Thus, deductive incompleteness is a characteristic property of these systems.

Probably, one of the most interesting paper belongs here to J. Hintikka [64], the paper with the title 'Revolution in logic?' [65] (and as well as the whole complex of Hintikka's works connected to application of the created by him **IF**-*logic* (Independence Friendly)) is very interesting too. The basic idea of Hintikka consists in comprehension that quantifiers of the standard first-order logic are dependent. The latter means that if we deal with expressions such as "for all x there are some y such that $R(x, y)$", then the choice of y is not independent, and it is determined by the choice

of x, in other words, between x and y there is a functional dependence. The feature of **IF**-logic is its incompleteness that means impossibility to give the list of axioms from which all significant formulas of the first-order **IF**-logic can be obtained only by using formal inference rules. But at the same time it satisfies the properties of compactness and Löwenheim-Skolem (about properties of **IF**-logic see also [122], [34]).

Probably, it is necessary to agree with J. van Benthem and K. Doets [150] that there is no *sacred* logical theory. It is possible to consider it as the answer to A. Tharp's article 'Which logic is the right logic?" [146].

However, the topic of abstract logic and general-theoretical problems of the substantiation of mathematics recede into the background before new tendencies in the logic development of the end of the 20th century. The logic becomes more vital in the computer science, artificial intelligence, and programming. The similar application of logic generates the big number of new logical systems, but already aimed directly to their practical use. In particular, this entails the publication of the collected works (in England and in one year in the USA) with the title 'What is a logical system?' [49].

Generally speaking, the problem is formulated as follows: whether there is the one "true" logic and in the converse case if not, how we can limit our understanding of the logic or, more concrete, of a logical system? There are also other problems: whether there is a real distinction between syntax and semantics from the point of view of applications? And, certainly, there is a problem on traditional properties of logical systems: the completeness, elimination of cut, interpolation property, etc.

Even more problems arises with the extension or reducing of classical propositional logic. It is known that we have an infinite (*uncountable*) number of such logics (logical systems). The first outcome of a similar sort belongs to the Russian logician V. A. Jankov (1968) and concerns a cardinality of the class of extensions of intuitionistic logics. Then this fact hasn't been realized with all problems implying from here. Now the discovery of the continual class of logics is the most ordinary thing.

The unusual diversity of logical systems is generated, on the one hand, by serious criticism of "basic" and not only basic laws of propositional logic, on the other hand, by almost unlimited extensions of the concept of the logical truth (in essence this process is an inverse to the first), and also by various specifications of the concept of logical inference and by the development of computer science. All this brings us to the most important problem: **What is a logic?**

R. Jeffery's note in the book with the rather remarkable title [69] seems to be a bit interesting that Tarski's definition of logical inference does not

allow us to define, what logic is, because we should take into account cases, included already in the definition of logical inference. We can consider cases as the "probable worlds" and then we have problematics connected with, what the "possible worlds" are? (see the interesting monograph [18]). Moreover, our cases can be considered as situations in Barwise and Perry's sense [8]. Situations can be regarded not only as incomplete parts of the world, but as contradictory, and also as both incomplete and contradictory. As a result, we obtain completely new logics, in essence distinct from classical such as intuitionistic, relevant, paraconsistent, paracomplete logics, etc.

If the essence of logic consists in a preservation of the designated truth-value in all cases, then various logics are obtained by various explications of these cases. So, the approach appeared in logic, called 'logical pluralism' (see [12]). The Internet project 'Logical pluralism' (http://pluralism.pitas.com/) is also created with G. Restall's participation. Actually, the logical pluralism existed in logic before the serious analysis of A. Tarski's understanding of logical deduction, namely it began in the criticism of basic laws of logic started in the beginning of the 20th century by L. Brower, N. A. Vasiljev, and J. Łukasiewicz. Furthermore, the understanding of logic had another tradition rather distinct from Tarski's and starting from G. Frege and B. Russell.

The definition of logic given by Frege is unusually beautiful: "Logic is a science about the most general laws of the existence of true" (see [45]). It seems to be a little surprising that such an understanding of logic held on almost hundred years and after a small modification come in the basis of Quine's aforementioned book 'Philosophical Logic', where a subject of logic, recall, is the "systematic study of logical trues." This almost centenary period of similar comprehension of logical subject was called by M. Dummett 'logicism's dominance'. However, the development of computer science entails the change of the paradigms in logic.

It is necessary to notice that the traditional approach to the understanding what a logic is seems to be very attractive in respect to the possibility to define logic by means of its basic laws. From the modern point of view the 'logical law' means the 'theorem of a formal system'. Without details, what a formal system is and what a proof in it is, we can consider laws of logic as preserving truths. Such an understanding of logical laws was proposed by Aristotle, but here we collide a problem of unusual complexity: What is a true? At the end, we can try to agree what we have to regard as logical laws concerning our understanding of truth. Various concepts of truth, for example, correspondent, coherent or pragmatical, apparently, do not dictate the specificity of this or that logic. But already the case of intuitionistic un-

derstanding, what a true is, speaks about irreconcilability of standpoints. In the first interpretations of intuitionistic logic (before appearance of Kripke's semantics), the concept of truth does not arise at all. However, there is again a problem, how we can define the logical laws, not defining before what logical constants (operations) are. As we saw, the extension of the classical first-order logic entails that the set of logical trues is not, certainly, axiomatizable.

Exactly in hundred years after the appearance of G. Frege's well-known work 'Concept Calculus' (1879) (see [45]), in which predicates, negation, conditional, and quantifiers are introduced as the basis of logic, and also the idea of formal system is introduced, in which demonstrating should be carried out by means of obviously formulated syntactic rules, – after hundred years of the triumphal development of logic as the independent science calling the worship, surprise, and occasionally bitter dismissal and even revenge for its former adherents and the mystical fear for the majority of others, suddenly there is J. Hacking's article under the title 'What is a logic?' [60]. Hacking highly evaluates G. Gentzen's introduction of structural rules, because the operation with them allows us to express the aspects of logical systems which have no direct relation to logical constants. This important discovery is made by G. Gentzen in 1935 (see [51]). The presentation and development of logic by the way of sequent calculus, where the principles of deduction are set by the rules, permitting to pass from one statements about the deducibility to others, allowed Hacking to define logic as science about deduction. (Pay attention that we find such an understanding of logic in V. A. Smirnov's book [133] reprinted in [135].) Therefore there are some reasons why Hacking's article is in the beginning of the collected works under the title 'What is a logical system?' [49].

J. Lambek's paper under the title 'What is a deductive system' [49] was published the same year, Lambek considers the following five styles of deductive systems: (1) Hilbert's style (deduction of the form $f :\to B$, for a formula B; (2) Lover's style ($f : A \to B$, for formulas A and B); (3) Gentzen's intuitionistic style ($f : A_1 - A_m \to B$); (4) Gentzen's classical style ($f : A_1 \dots A_m \to B_1 - B_n$), and (5) Schütte's style ($f :\to B_1 \dots B_n$). Lambek prefers Gentzen's style by virtue of introduction of structural rules. Notice that Lambek pay attention on equalities between deductions. In this connection recall that in G. Mints book [99] (now he works at the Stanford university) the deductive system **HCC** of Hilbert's type contains a definition of equivalence relation for deductions. It converts **HCC** into the closed category: formulas are objects, and equivalence classes of deductions are morphisms.

The similar approach to the proof theory became especially actual under influence of the category theory and computer science.

4. Computerization of logic

V. Carnielli in the review on [49] puts forward the basic supposition: "There are no proofs, there is no logic" [21]. The proof theory recently draws to itself much more attention (see, for example, the two-volume book [124]). At the same time, there is the site 'Proof theory on the eve of the year 2000', created by S. Feferman [41], where 10 problems are formulated and the well-known logicians, working in the given area, are proposing some solutions.

However, in the last quarter of the 20th century the proof theory has undergone significant modifications and directly began to be applied in computer science. We mean here an automatic scan of proofs. In our country the research in this area was carried out since the 60s years in Ju. M. Maslov's group. The book [25] became the classical monograph devoted to the automatic proof, translated into Russian, in this book the method of resolutions for the first-order logic develops (see also the monograph [50]).

Within two last decades many theoretical ideas of the automatic proof have been embodied in computer programs, so-called provers. These programs carry out the search of deductions in various logical calculi. So, in the middle of the 80s years in the Aragonne National Laboratory in the USA the resolutive prover OTTER was created for the first-order logic, its description is in [95]. Up until now its creators work at the development of the program and improve of speed in its separate parts. In 90s SCOTT appeared (see the report [132]) – the program of the Australian Project of Automatic Proof, the set including OTTER and permitting to use semantic limitations and therefore essentially to reduce an operating time of the program during construction of deductions.

The Russian logicians, employees of the Philosophical Faculty of the Moscow State University and employees of the Institute of Philosophy of the Russian Academy of Science, have written their interactive prover DEDUCTIO which is described in detail in [135]. The distinctive advantage of DEDUCTIO consists in the wide area of its possible usage: the axiomatic deduction, natural, analytic-tableaux. There is the site with the bibliography on provers, created in Canada (1997–2001), containing more than 3000 references [4].

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[4] http://www.ora.on.ca/biblio/biblio-prover-html

Applying logic in computer science became so wide that it is possible to speak about the main phenomenon in the development of logic of the end of the 20th century. So, the term 'computing logic' and later 'computer logic' appeared in the 70s years.

The creation of artificial intelligence is a special theme. The American Association Artificial Intelligence, issuing the journal 'Artificial Intelligence' and organizing annual international conferences, symposiums and summer schools (http://www.aaai.org/) is started in 1979. In books [148] and [30], various non-standard logics are proposed for artificial intelligence. The two books (but with different titles) in French are devoted to the logical approach to an artificial intelligence, published in 1988 and 1989 (see their translations in Russian: [144] and [145]). About the logical-and-philosophical approach to artificial intelligence see the collected works [147]. Pay attention to multi-volume handbooks [1] and [46].

The creation of artificial intelligence (hereinafter *AI*) passed from obsession to the plane of serious discussions and became a fundamental problem: whether the logic can really become the basis of *AI*? Here it is necessary to mean that logical deduction is a discrete process, while the human thinking isn't.

There are supporters of the 'strong' conception of *AI* (mechanists), asserting that the human brain (reason) can be precisely simulated by a discrete (digital) computer or a Turing machine. The most known criticism against mechanists belongs to J. Lucas [89], his philosophical article was repeatedly reprinted. Lucas uses basically Gödel's theorems of incompleteness, asserting the existence of absolutely insolvable arithmetic propositions. According to Lucas, this essentially limits a computing sphere of computers. J. Webb in the book [158] appeals to the efficiency of Gödel's result, concluding that Gödel for the first time has shown that from the statement "I can find limitations in any computer" it undoubtedly follows that "I am not the computer." The known physicist R. Penrose is of the same opinion [108] (the book is translated in Russian in 2003), who, among other things, including physical arguments, is also based on the insolvability of the decision problem for mass problems, i.e. on the absence of a uniform algorithm for solution of mathematical problems (it is proved by A. Turing in 1937, and later by A. Church in 1941). Lucas and Penrose give reasons that there are human procedures (computing methods) which cannot be simulated by a Turing machine. But if the abilities of the human reason exceeds any computer, then the reason somehow comprehends trues unavailable to the computer. The same opinion belongs to Gödel in his unpublished works (see [156], [157], and also the third volume [53]). How-

ever, the problem consists in finding obvious examples of similar computing processes.

There is the big literature subjected Lucas and Penrose's viewpoint to criticism (see, for example, [105] and also D. Hofstadter's book [66], involved the significant attention and translated into Russian in 2001). After three decades Lucas [90] strengthens, or tries to strengthen, his standpoint; on the other hand, Penrose devotes more than 200 pages to the replies to the critics and as well as devotes them to the invention of rather intriguing arguments in the new book [109]. It is interesting that both mechanists and anti-mechanists understand and accept the power and universality of Gödel's limitative theorems. But there is a little bit paradoxical impression that for the first this means a limitation of human computing abilities, and for the second otherwise: computing abilities of the person are much more difficult than ones of the computer and the human reason also operates with abstract objects (see [83]). Let us especially pay attention to the recent work of the famous logicians S. Shapiro [128], where arguments of contending parties are in detail analyzed. Here it is marked that an extension of the human computing abilities implies that the human reason becomes not only *infallible*, but also *omniscient*. Recall that the fierce discussion in the Middle Ages concerning compatibility of Christian dogmas about God's omniscience and the human free wills proceeds up until now in theological studies, therefore it is clear that a lot of other philosophical problems should be decided in parallel. At last, let us refer to P. Benacerraf's interesting reasoning in the article 'God, the Devil and Gödel' [13]. If the idealized versions of human beings are Turing machines, then they are not capable to execute Socrates' statement: "Learn itself." If the ideal person is a Turing machine, then he cannot know what kind of Turing machines he belongs to (according to Church-Turing's thesis, all Turing machines are equivalent).[5] Hence, there

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[5] *Edit.*: More precisely, Turing's automated machines are equivalent approaches to computability. Notice that the conventional name of Turing machines actually refers, in Turing's words, to automatic machines, or *a*-machines. He also proposed other models of computation: *c*-machines (choice machines) and *u*-machines (unorganized machines). Turing argued for the claim (Turing's thesis) that whenever there is an effective method for obtaining the values of a mathematical function, the function can be computed by a Turing *a*-machine. At the same time, Church formulated the following thesis: a function of positive integers is effectively calculable only if it is recursive. If attention is restricted to functions of positive integers then Church's thesis and Turing's thesis are equivalent. It is important to distinguish between the Turing-Church thesis and the different proposition that whatever can be calculated by a machine can be calculated by a Turing machine. The two propositions are sometimes confused. Gandy termed the second proposition 'Thesis M': whatever can be calculated by a machine is Turing-machine-computable (see *Gandy, R. Church's Thesis and Principles for Mechanisms, [in:] Barwise, J., Keisler, H. J., Kunen, K. (eds). The Kleene Symposium. Amsterdam: North-Holland, 1980*).

is a classical problem about limits of human knowledge and, certainly, about limits of logic.

There is a strong demarcation line between reasonings of artificial intelligence and reasonings of human intelligence. Most likely, it is impossible to overcome this line, but the development of logic, which basic function is an *approximating* of various methods of human reasonings, takes place on the infinite path of overcoming this line. A limiting case of approximating (and while the most effective and fruitful) just also is the formalized deductive method implemented in computer programs.

There exist, certainly, other methods of approximating which also develop: hypothetic-deductive method, induction and abduction, formalizing of probable reasonings (see [42]). Recently non-monotonic logics develop, too. Non-monotonic reasonings, differently from classical, intuitionistic, classical-modal, etc., allow to operate adequately with the incomplete and changed information. The international school-seminar on non-monotonic reasonings [6] proceeds since 1994. Notice the big survey [19] and monographs [5] and [15]. Let us also pay attention to different argumentation theories (notice only the work [43] as the most approximate to the formal-logical modelling).

However, the future belongs to a computerization of logic and to its applications in computer science. We yet do not know completely what it is possible to wait from new computers for: quantum, neural, etc. Pay attention to rather remarkable fact. In 1960 the Nobel winner E. P. Wigner wrote the article about the difficultly explained efficiency of mathematics in natural sciences, following Galilei's words that "The book of the nature is written in the language of mathematics." Something similar corresponds to the attitude of logic to computer science. Now the concepts and methods of logic take one of the central places in computer science and it can even be called calculus of computer science. The article of the six American logicians [62], published in the beginning of the new century, is devoted just to this theme.

In D. Gabbay's preface to each volume of new *HPL* it is fairly noticed that the previous *HPL* became the Bible for the logical community. The basic intention of the new issue is that an exceptional value of logic in computer science, in the development of the formalized (computing) languages such as combinatory logic and λ-calculi and in artificial intelligence is shown in the most complete measure. Gabbay predicts that the day will be

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6 http://www.holds.medg.lcs.mit.edu/nm/

coming, when the scientist in the field of computer science will wake up with comprehension that his professional sort of activity belongs to the formal (symbolic) philosophy.

5. The researches financed by the Russian Foundation for Basic Research

The basic researches in the field of philosophical logic are fulfilled by the Russian scientists beyond the framework of the Russian Foundation for Basic Research (RFBR), which works only 17 years. Nevertheless, it is already possible to examine some tendencies.

First of all, there is a possibility of republishing works of well-known Russian logicians within the framework of appropriate research projects. For the first time the most important works are collected in the separate issues with the detailed introductions, extensive comments and the complete bibliography. It is (in the chronological order) the grant of the RFBR N 97-06-80360 (the chief A. A. Anisov). Here the comments have been prepared for V. A. Smirnov's monograph 'Formal inference and logical calculi'. In comments there is the comparative analysis of V. A. Smirnov's ideas and results in the field of the inference theory with works of representatives of other schools. The work is completed by the publishing grant of the RFBR N 99-06-87071 [134]. The comments were written by V. M. Popov, P. I. Bystrov, A. V. Smirnov and V. I. Shalak. V. A. Smirnov's articles, directly related to the inference theory, are also included in the issue. The works [102], [103], supported by grants N 97-06-80211 and N 00-06-80142 (the chief N. M. Nagornyj) are directed to the solution of the important problems of the comparative analysis of concrete results of one of the best-known present-day math-philosophical programs – 'Markov's constructivism' – with other conceptual programs such as 'Cantor's set-theoretic program', 'Brouwer's intuitionism' and 'Hilbert's proof theory'. Researches on the given project were some kind of summarizing to preparation of he two-volume issuing of A. A. Markov's 'Selected Works'. The publication of the two-volume book [92] is supported by the grant of the RFBR N 00-01-14195. M. N. Nagornyj was an editor, the introduction and comments also belong to him. The preparation of a commented publication of A. G. Dragalin's works on logic and philosophy of mathematics is supported by the grant of the RFBR N 00-06-80122 (the chief E. G. Dragalina-Chernaja), too. The two-volume issuing [31] and [82] is supported by the grant of the RFBR N 01-06-87068. G. E. Mints was the

editor-in-chief and he also prepared comments, N. N. Nepejvoda wrote the introduction. In the second volume, A. N. Kolmogorov and A. G. Dragalin's textbook on logic is reprinted.

The other portion of grants of the RFBR supported researches in the field of non-classical logics. It was an international project INTASS–RFBR 95-8365 (the chief A. S. Karpenko). The project was aimed to solve problems of application of paraconsistent logic to philosophy, artificial intelligence and computability. Only the Russian participants published more than 40 articles. Consider some works published in the international journals. In [73] it is shown that a combination of two three-valued isomorphs of classical logic, which are contained in Bochvar's three-valued logic of senselessness, entails a paraconsistent logic (Sette's logic) and its dual 'weak intuitionistic logic'. In [113] it is established that the implicative logic with inverse negation which is defined in the pure implicative language, is paraconsistent. In [153] we can find the category approach to paraconsistent logics. Participants of the project have taken part in the 1st international congress on paraconsistent logics (Gent, 1997) [106] and in the international conference, devoted 50th years of publishing S. Jaśkowski's paper, in which the system paraconsistent logic is considered for the first time (Toruń, 1998) [139]. (Though actually the first system of similar logic has been considered by A. N. Kolmogorov in 1925 [81].) The special issue of one of the first electronic scientific journals in Russia 'Logical Studies' (A. S. Karpenko is the editor-in-chief) [87] is devoted to the problematics of paraconsistent logics, too.

The project of the RFBR N 00-06-80037 (the chief A. V. Chagrov) is directed on solution of a fundamental problem of the modern logical science connected to finding-out of ratio between non-classical logical systems, oriented on the description of information processes, and the comparative analysis of their expressive means. The old algorithmic problem of the matrixness and finite approximability of the normal modal logics, set by matrix logics, is solved in [22]. In [119] the problem of the complicated description of modal logics and their superintuitionistic fragments is solved by the limitation of the number of used variables. In [23] one considered problems of including the basic propositional logic and its extensions to modal logics, in particular, including formal propositional logic into Gödel-Löb's modal logic of demonstrability. The new relational semantics for extensions of the basic logic was constructed during this research. In [23] one built the propositional logic **LAP** with modified semantics of generalized, according to E. K. Vojshvillo, states and one defines the operations, including the classical propositional logic into **LAP**. In [72] it is shown that Kleene's regular

operations are inexpressible in Yuriev's three-valued logic $\mathbf{Y}_3$, intended for formalizing the formal neuron, and this logic is essentially non-monotonic.

The works [10], [155] and [84] are supported by the grant of the RFBR N 97-06-80191 (the chief S. O. Kuznecov). The problem of simplification of formulation of DSM-method of automatic generation of hypotheses, based on formalizing rules of J. S. Mill's inductive logic, has been solved. The language of the stratified logical relational programs was used for this purpose. Another approach assumes to use the language of the naive set theory. One more result: the general effective method of axiomatization of algebra classes, corresponded to *J*-definable *J*-compact logics (many-valued logics), is described; the completeness theorem of appropriate calculi concerning these algebra classes is proved.

The project of the RFBR N 98-06-80177 (the chief A. S. Karpenko) is devoted to the special class of many-valued logics, namely to Łukasiewicz's finite-valued logics $Ł_n$, arisen from the problematics of logical fatalism. Within this project the corollaries of V. K. Finn's theorems about the number-theoretical nature of logics $Ł_n$ are proved: n is a prime number if and only if the set of functions of logic $Ł_n$ is a precomplete class (see [16]; later, this theorem twice rediscovered abroad). The obtained results have been summarized in the monograph [71], its publication was supported by the grant of the RFBR N 00-06-87014. The solution of the basic problem consists in the following: it is given a characterization of various classes of natural numbers (prime numbers, degrees of prime numbers, odd numbers, even numbers) by means of logical matrices. One of corollaries was a discovery of the law of generation of classes of the prime numbers, supplied with an appropriate computer program.

The interesting researches, published in [105], [104], were supported by the grant of the RFBR N 98-06-90205 (the chief N. N. Nepejvoda). In the first of them it is shown that the applied theories, based on superintuitionistic logics with Carnap's constructive rule, are classical, i.e. we can infer there the law of excluded middle. In the second work one researches the phenomenon of the non-formalizable, for the first time drawn attention after Gödel's theorem of incompleteness of formal theories.

Finally, notice the project supported by the RFBR N 00-06-80149 (the chief V. A. Bazhanov), in that the evolution of university logic in Russia in the period from the 19th to the middle of the 20th centuries was considered on the basis of the analysis of the different sources. The publication of the book [10] and the detailed work about I. E. Orlov [11], who is the founder of relevant logic, became a result of this research.

6. Conclusion

Let us pay attention to the obvious tendency that at the last time more professional math-logicians (mathematicians) began to study philosophical logic. The technical means of non-classical logic is more and more improved and becomes complicated. It is clear, as the future of logic, including philosophical logic in its modern understanding, is connected to computer science. In a word, we can see a mathematization and algebraization of philosophical logic. The half of the projects supported by the Russian Foundation for Basic Research and regarded by us belongs just to math-logicians.

7. Acknowledgements

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