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NON-WELL-FOUNDEDNESS IN JUDAIC LOGIC

In this paper I consider the historical background of Hebrew Orthodoxy finally formed in Belarusian lands. Further, I try to explicate the Judaic logic (i.e. the logic used by Talmudists for inferring Judaic laws from the Pentateuch). The only logical connective of that logic is the Judaic conjunction “and” which is not idempotent or commutative, but it is associative. I propose Austin’s style of semantics for Judaic logic and explicate also the inference rules used by Talmudists. I show that the Judaic logic is characterized by non-well-foundedness.

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1. Introduction

In this paper we consider the logic used by Talmudists for inferring the Judaic laws from the Torah; we also show that this logic is non-well-founded. Let us remember that a foundational axiom of set theory asserts that the membership relation \in is well-founded (there is no descending sequence for \in). One can deny this axiom in order to postulate a set that has an infinite descending \in -chain, i.e. that is not well-founded. The particular case of such a set is one of the form $A = \{A\}$ with a circular membership relation. The set theory denying the axiom of foundation is considered in [1], [4]. A non-well-founded set can be exemplified as a succession of seasons, unfolding in a cyclic pattern: seasons = (spring, (summer, (fall, (winter, seasons))))).

An interest in non-well-founded phenomena is mainly motivated by modern developments in computer sciences, where many structures and phenomena do have non-well-founded phenomena features: self-applicative programs, self-reference, graph circularity, looping processes, transition systems, paradoxes in natural languages, etc. Strings, streams, and formal se-

ries are potentially infinite, and can only be approximated by partial and progressive knowledge. It is natural to use universes containing adequate non-well-founded phenomena sets as frameworks to develop semantics for these objects or phenomena. Moreover, sometimes it is not relevant to use the classical principles of definition and reasoning by induction to define and reason about these objects. This is how a need for new meta-mathematical logical properties arises.

Non-well-founded sets have also been implicitly used in non-standard (more precisely, non-Archimedean) analysis like infinitesimal and p -adic analysis. The main advantages of non-well-founded sets consist in that we get an extension of standard sets so that the way of setting mathematical objects changes and we have a more general approach to computation without classical induction and recursion.

Notice that denying the foundation axiom in number systems implies setting the non-Archimedean ordering structure. Archimedes' axiom affirms: for any positive real or rational number y , there exists a positive integer n such that $y = 1/n$ or $ny = 1$. The informal meaning of Archimedes' axiom is that anything can be measured by a ruler. The logical calculi with non-Archimedean multiple validity were proposed for the first time in [14], [15].

In the beginning we consider the historical context of Hebrew Orthodoxy formed in the 18 to 19th centuries in Belarus. Further, we explicate some logical ideas of Talmudists and show non-well-founded phenomena in those ideas.

2. Historical Background

The Belarusian Jews (“Litvaks”¹, as they name themselves) have had a long history in this country and they can be named autochthos in Belarus. It is known that they lived here before the 14th century. So, the *privilege* (in Old Belarusian “priveley”) of *Vitaut* (Witowt), the Grand Duke of Litva, was one of the first legal documents, regulating life of the Jewish communities in Belarusian lands. It was granted to the Jews of Brest on June 24th, 1388 in the city of Lutsk. The Jews were proclaimed to be free people and like all noblemen, were subjected to the Grand Duke. The Jews were granted protection of their lives and property, the right of unrestricted mobility, trade, financial activity, etc.

¹ Traditionally, “Litvaks” was the name of Jews especially from the Vilna and Minsk Gubernias.

The other Vitaut privilege, regulating life of the Jewish communities, appeared on June 18th, 1389 and was granted to the Jews of Grodno. According to this document, the synagogue and graveyards were also declared under protection and they were released from any taxation. As we see, exceptional tolerance was shown toward the religion. Jews were given the right to practice their faith and celebrate religious holidays. The accusations of Jews using the blood of Christians for ritualistic purposes were forbidden (because such accusations contradicted with Jewish religious laws). Severe punishment awaited those Jews who violated Jewish religious rights.



Figure 1. The Choral Synagogue in Vilna

Vitaut's privileges were later extended to other communities, where the Jews of other Belarusian large towns (Trakai, Lutsk, Vladimir) were owed privileges too. Notice that in the 14th century Vitaut's privileges, granted to Jews, were the most liberal legal documents which guaranteed them more rights and freedom than documents of any other European country. Vitaut's privileges were later accepted by other Grand Dukes and their main points were included in the First Statute of Litva (1529).

Since then, the Jewish diaspora of the Great Duchy of Litva grew exponentially. Tolerance, a relatively free way of life, and the ability to practice their own religion was the reason why Jews became deeply root-

ed in Litva, and developed there a rich ethnic culture. Up until now, the religious tradition of Jews from Belarus (Litvaks) is characterized as being more rational and orthodox than other branches of Ashkenazi (European) Jewry. Many well-known Talmudic authorities of the 18th and 19th centuries lived in Belarus, one of whom was Rabbi Elijah ben Solomon (1720–1797), the Vilna Gaon (who lived in Vilna – the cultural center of the Great Litva). His style of Torah and Talmud study shaped the analytical “Litvanian-style” of learning still practiced in most present-day yeshivas (Rabbinic schools).

The first yeshiva in Belarus was started in 1806. It was opened by Rabbi Chaim (the Vilna Gaon’s main disciple) in the Belarusian town of Volozhin. It became the first Rabbinic school in the whole of Eastern Europe. The second yeshiva in Belarus was founded in the Belarusian town of Mir in 1847. Later the yeshiva movement, initiated by Rabbi Chaim from Volozhin, was spread over the whole Orthodox Hebrew world. The followers of the Vilna Gaon and Rabbi Chaim from Volozhin (later known as *Orthodox*) were opposed to the Chassidim, the emotional and anti-rational branch of Ashkenazi Jewry founded by Rabbi Baal Shem Tov in Ukraine.

By the end of the 18th century, the Jewish diaspora of the Belarusian lands was already numerous enough. So, the exact data exists of the number of Jews in the Commonwealth of the Kingdom of Poland and the Grand Duchy of Litva. According to the data of S. Staszic, the number of Jews was about 800,000 (it was approximately 14 per cent of the whole population of the country). According to the data of Solomon Bennet, the Jewish population had more than 2 million people. M. Butrymowicz’s data are now considered as the most objective. By his calculations, the Jews numbered about 900,000. This number was the larger than the number of noblemen (720,000) and bourgeoisie (500,000).

By the end of the 19th century, in the majority of Belarusian cities, not less than 50 per cent of inhabitants were Jews. For example, the Jewish population of Minsk reached 12,976 people in 1847, and then 47,562 people in 1897. At that time, it was more than 52.4 per cent of the whole population of the city. In Vitebsk, the Jewish population reached 34,420 people in 1897 (52.4 per cent of the whole population), in Mogilev 21,539 people (50 per cent), in Pinsk 21,065 people (74.2 per cent), in Bobruisk 20,759 people (60.5 per cent), and in Gomel 20,385 people (54.8 per cent).

Since Vitaut’s privileges, for many years the Jewish population of the Belarusian cities obtained self-governance and autonomy, “kahal”. Note that the Belarusian word ‘kahalam’, meaning ‘together’, originates from the Hebrew word ‘kahal’. Jewish self-governance existed until its abolition in 1844.

Under the Statutes of Litva, Belarusian Jews formed a class of freemen subjected in all criminal cases directly to the jurisdiction of the Grand Duke and his official representatives. The official representative of the Grand Duke was called the Elder (in Hebrew “Gabbay”, in Russian “starosta”), known as the “Jewish judge” (judex Judæorum). The Jewish judge decided all cases between Christians and Jews and all criminal suits in which Jews were concerned; in civil suits, however, he acted only on the application of the interested parties. The Elder represented the communities in all external relations, in securing new privileges, and in the regulation of taxes. The Elder acted in conjunction with the Rabbi, whose jurisdiction included all Jewish affairs with the exception of judicial cases assigned to the Elder.

The Belarusian Jewry had an influence on the protestant anti-trinity movement and the protestant movement of Christian Judaization (in Belarusian “Żydoustwujuszczye”). The Christian followers of this movement directed their attention to the translation of the ancient Judaic literature and Middle Age Jewish-Arabian texts. Their denial of the Trinity led to the denial of Christ’s divine nature. These protestant movements were a source of Belarusian humanism thriving around 1570 in the Grand Duchy of Litva (two centuries before it happened in France). The followers of these movements considered feudal power over people as anti-Christian, since only God’s power is sacred. Anti-trinitarians did not recognize the sacredness of church buildings, icons or statues, which were considered idols. Some of them even denied the necessity of prayer. Fasting, baptism, the ritual tasting of God’s body and drinking of his blood, the cult of saints, the Holy Mother, relics, and the cult of the cross were severely criticized. The church organization was considered to be against the Bible.

Protestant movements were very popular in Belarusian lands. For example, the Belarusian Calvinists were the first Calvinistic community in European countries outside the UK. The protestant brotherhoods were in Vilna, Smargoń, Mir, Aszmiiany, etc. This religious tolerance that originated thanks to Vitaut’s privileges was typical phenomena for Belarus.

The language of Belarusian Jews is called Yiddish, its other names are Ivri-Teitch (“Jewish German”) and Mamen-Loshen (“Mother’s language”). For Chassids Yiddish began to play the role as an almost sacred language. Many doctrinal works of Chassids were created in this language.

Yiddish had an appreciable influence on Belarusian. Many Belarusian words with German roots actually originate from Yiddish: ‘zukar’ (sugar), ‘lichtar’ (lamp), ‘vaga’ (weight), ‘ruch’ (in Yidish ‘ruach’, motion), etc. In Belarusian there are even Judaic (Halachic) terms. For example, it is very probable that the Belarusian words ‘svara’, ‘svarytsia’ have the same root

as the Hebrew words ‘sovar’ (‘to assume’) and ‘svara’, ‘svoro’, meaning the Talmudic debate caused by the application of various logical methods of interpretation of the Judaic law.

Some words of the Belarusian language have a curious history of their formation. In the Torah there are some ways of designating the righteous man. One of them is ‘tam’ (‘tom’) or ‘tamim’ (‘tomim’), expressing ‘uprightness’, ‘straightforwardness’ and ‘artlessness’. For the first time it was said that Noah is “a just man, artless [in Hebrew “tam”]” (Genesis 6, 9). Later, before the eternal union with Abraham, God speaks to him: “and be perfect [in Hebrew “tam”]” (Genesis 17, 1). In Yiddish, the word ‘tam’ means ‘artless’, ‘ordinary’ and was used less often as ‘tamevate’. As we see, the word ‘tam’ was sometimes used in Yiddish with the Belarusian suffix. Chassids used the term ‘tmimus’ (‘wholeness’, ‘openness’) for the designation of goodness.

In Belarusian, the root ‘tam’ (‘tsiam’, ‘ciam’) has an opposite value and means ‘cunningness’ and ‘ingenuity’. So, the Belarusian words ‘ciamki’, ‘ciamlivy’ mean ‘cunning’ and ‘ingenious’, and the word ‘ciamkaść’ means ‘ingenuity’ and ‘cleverness’. Let us notice that in Ukrainian translations of Kant’s works, when one tried to differ the terms ‘reason’ (Vernunft) and ‘understanding’ (Verstand), the word ‘reason’ was translated as the Ukrainian-Belarusian ‘rozum’, but the word ‘understanding’ as the Ukrainian-Belarusian ‘ciama’.

By forming words with the roots ‘tam’ and ‘ciam’ as having the initial meaning ‘cunningness’ is probably explained by the fact that success and luck were the context for using the word ‘tam’ with the meaning “perfect, artless Jew”, but outside the Judaic religious practice, due to the given context, this word was already conceived with the opposite value – as “artful, ingenious man”.

Let us note that the Belarusian language had an even more significant influence on Yiddish. A huge part of the Yiddish lexicon of Belarusian Jews has been directly taken from Belarusian, therefore it is possible to say that there existed a special Belarusian dialect of Yiddish.

3. Logical Interpretation of the Pentateuch in Judaism

In Judaism the referential structure of any statement is called the Torah (in Hebrew “soyro”, “toyro”), at the same time the Pentateuch is the legislative foundation of the Torah describing what each statement may mean and how each act may be evaluated. The Torah as Pentateuch has the following two

dimensions: ‘Haggadah’ and ‘Halachah’. The first dimension, *Haggadah*, reflects the historic facts connected to Israel, the second dimension, *Halachah*, expresses the compiled laws which have to be accepted by any Jew. These laws are scattered all over the books of the Pentateuch and explicated only on the basis of the grammatical form: ‘And the Lord spoke/said . . . do/do not . . .’. All these laws are mentioned in a context of historical events. For example, the first commandment (in Hebrew “mitzvo”) given man is as follows: “be fruitful and multiply” (Genesis 1, 28). This commandment was given to Adam and Eve to obey. Thus, Haggadah (in this case it is a history about the creation of Adam and Eve) is to show the historical context of the commandments, therefore Haggadah plays the subordinated role in relation to Halacha.

Haggadah is no more than an addition to Halacha. For example, in the opinion of the Talmudic authority, Rabbi Isaac, the Torah should begin with the verse “This month shall be unto you the beginning of months: it shall be the first month of the year to you” (Exodus 12, 2), as this verse contains the first commandment given to Jews.

In Christianity, the dimension of Halachah is completely eliminated, and the Bible starts to be considered especially as a historical narration (even about sacred events). All the Lord’s commandments start to have only the form of a recommendation. As a result, a contemplation appears in Christianity. Commandments are not taken literally and are interpreted allegorically with the application of analogies. For example, the Christian commandment “And unto him that smiteth thee on the one cheek offer also the other; and him that taketh away thy cloak forbid not to take thy coat also” (Luke 6, 29) in the most sense, cannot be taken literally.

The Christian contemplation caused the further development of secular culture. Such a contemplation and allegorization is, perhaps, a distinctive feature of Christianity. On the other hand, in Judaism we find the detailed consideration of how the commandments should be obeyed depending on concrete everyday situations. For instance, in the case of the absence of water it is ordered to wash hands by fine sand which can be found in huge quantities in the climatic conditions of the Near East.

In Muslim faith, as well as in Judaism, there are commandments which should necessarily be obeyed. For example, in the same measure it is ordered to keep to one’s own hygiene and in the case of the absence of water it is necessary to carry out the lavabo with fine sand.

“O ye who believe! Approach not prayers with a mind befogged, until ye can understand all that ye say, – nor in a state of ceremonial impurity (Except when travelling on the road), until after washing your whole body.

If ye are ill, or on a journey, or one of you cometh from offices of nature, or ye have been in contact with women, and ye find no water, then take for yourselves clean sand or earth, and rub therewith your faces and hands. For Allah doth blot out sins and forgive again and again” (Sura 4, Women, 43).

The fact that Arabs in the ceremonial purposes are washed sometimes by fine sand is mentioned in a commentary by Rashi (Rabbi Solomon Isaac, the best known commentator of the Torah and Talmud) in the following verse: “Let a little water, I pray you, be fetched, and wash your feet, and rest yourselves under the tree” (Genesis 18, 4). In this verse Abraham invites three people to enter his tent, and before this he suggests they wash their feet. Rashi explains that Abraham has assumed these people were Arabs who worship fine sand on their feet. Abraham very strictly obeys the Judaic laws, e.g., he does not admit an object of idolatry in his house. In Rashi’s opinion, Lot is connected to this commentary. He, who was not such a perfect man as Abraham, does not obey that graven images do not penetrate into his house, therefore in the beginning Lot has suggested strangers to enter the house and only later wash their feet: “tarry all night and wash your feet” (Genesis 19, 2). Foreign religious sacraments are an idolatry for Judaism (in Hebrew “avoydo zero”, the literal translation: “alien work”), therefore the Arabian ceremonial washing with fine sand is a version of idolatry in the Judaic viewpoint.

The example of Rashi’s commentary is a bit astonishing. At first sight, the explanation seems to be completely inappropriate: Arabs as a nation come from Abraham, and more precisely they are Ishmael’s descendants, although by the time of the described events, Ishmael was 13 years old! Besides, the ceremonial washing with fine sand was accepted by Arabs only after Muhammad’s sermon. It is natural that Rashi, living in France in the 11th century and being familiar with the Arabian culture, knew all these historical details. Why has he afforded an obvious historical inaccuracy? The point is that Haggadah, i.e. the historical narration of the Bible, does not play a significant role and the Torah is not a historical story. In this way, the Judaic understanding of the Biblical books is considerably different from the Christian one. In Judaism, the Torah is a compendium of laws for which different events occurring at different times are mapped onto a general plane: *dura lex sed lex*.

For Rashi, therefore, it is very important to have a *logical coordination* of the Torah verses, instead of its historical one. For example, Abraham suggests strangers to wash their feet in the beginning and only later to enter his tent. On the other hand, Lot suggests the same strangers to enter his house in the beginning and only after to wash their feet. The different sequence



Figure 2. The page from a medieval Hebrew book

of actions is obvious. It is our *first premise*. Further, it is known that Lot, though he was Abraham's nephew, was not a perfect man. On the other hand, Abraham was such a man. It is our *second premise*. Notice that due to the given premise we know that Abraham's actions were faultless from the standpoint of Halachah (namely, of the 613 Jewish commandments). Now we need to draw a conclusion from both premisses and that conclusion becomes an explanation of the Biblical verse. For this purpose I should correlate Abraham and Lot's acts with Halachah. The following explanation suits it best here: *fine sand on the feet of strangers could be ritually unclean*, therefore it could profane Abraham's dwelling, so Abraham decided to suggest them to wash their feet. The most widespread example in Rashi's days of ritually unclean fine sand on feet is a result of the ceremonial washing of feet by Arabs (in fact, every sacrament of alien religion is a ceremonial defiling for Judaism). For this reason, Rashi refers to this example.

It is necessary to notice that all of Rashi's commentaries are the result of a logical processing of the Biblical text, and in most cases his commentaries

are legal conclusions, connecting concrete acts of Biblical characters to Halachah, the laws. The logical portrayal of all the smallest details in the works of Judaic authorities contrasts very sharply with the general contemplation of Christian thinkers.

In Judaism, any details are completely coordinated among themselves from the standpoint of logic. Logic is a unique science which for the whole history of its official existence (more than two thousand years) has not changed in its foundations. Even mathematics has been changed essentially, in the beginning it was based on geometry (ancient Greek mathematics), then on algebra (classical mathematics), and now it is based on mathematical logic. Every scientific knowledge is corruptible and changeable, except for logic. Only this science sets the border for thinking, outside of which there is nothing, only silence. Logic is an eternal science.

One of the well-known Rabbis of present days, Rabbi Adin Even Israel (Steinsaltz) [16], compares logic of the Talmud to mathematical logic, affirming that *the Talmudic logic, different from mathematical logic, gives the description and explanation for all in view of case studies*, i.e. it is constructed on the basis of the analysis of all equiprobable cases whereas mathematical logic eliminates any concrete context.

Many laws described in the Talmud are taken directly from the written Torah (Pentateuch). But there are also laws which were logically inferred. These laws are called the laws of Wise Men. "It would be possible to include laws of Wise Men in the Torah, but the Lord has decided that they should go from us", as Rabbi Moses Chaim Luzzatto said. Thus, while the written Torah (Pentateuch), in the Judaic view, is a result of the Revelation, the oral Torah (the laws of Wise Men) is a logical analysis of any details from the standpoint of the Pentateuch, i.e. it represents the logic which takes into account all equiprobable cases. Logic as the basis of the oral Torah goes from us, Pentateuch, from the Lord.

*In the Pentateuch there are no logical connectives in the sense of conventional (European) logic. The only logical connective in the Pentateuch (that is used very often) has the form of the conjunction "... and ..." (in Hebrew "... ve ..." or "... u ..."), and it is called the **Judaic conjunction** and denoted by $*$.*

This connective is not idempotent and commutative, but it is associative:

- $A * A \neq A$,
- $A * B \neq B * A$,
- $A * (B * C) = (A * B) * C$,

where A, B, C are either names/concepts or propositions/sentences.

The Judaic conjunction differs from the conventional one, because the latter is idempotent, commutative, and associative. For example, in the Judaic view, Lot's event 'enter my dwelling and wash your feet' ($A * B$) is not equivalent to Abraham's event 'wash your feet and enter my dwelling' ($B * A$).

4. Self-reference of Judaic Reasoning

In conventional semantics, referred to as Russellian, facts are referents of sentences: "sentences are used to express propositions, claims about the world, and these claims are true just in case the world is as it is claimed to be" [4]. Sentences and propositions held in Russellian semantics are called *Russellian sentences* and *Russellian propositions*, respectively.

However, given Russellian semantics are limited, because self-referential sentences have no meaning in them. For example, take the Liar sentence: 'This sentence is not true'. It has no meaning according to the Russellian account. Indeed, assume that it is a fact that the Liar proposition is true, then the same fact shows that the Liar proposition is untrue. Since there exists no fact with such a property, the Liar proposition has no truth valuation. Thus, the Liar sentence concludes that Russellian propositions cannot make claims about the whole world.

Another approach to semantics was developed by Austin. According to him, "a legitimate statement A provides two things: a historical (or actual) situation S_A , and a type of situation T_A . The former is just some limited portion of the real world; the speaker refers to it with what Austin [in his paper "Truth"] calls "demonstrative conventions". The latter is, roughly speaking, a property of situations determined from the statement by means of "descriptive conventions" associated with the language. The statement A is true if S_A is of type T_A ; otherwise it is false... While Austin did not use the term "proposition", it seems in the spirit of his account to identify what we will call the Austinian proposition expressed by A with the claim that S_A is of type T_A , and to individuate such a proposition by its two components, the situation referred to and the type of situation it is claimed to be" [4].

Sentences and propositions held in Austian semantics are called *Austian sentences* and *Austian propositions*, respectively. Also, an Austinian statement is a reference to a situation and an utterance of a sentence. The proposition expressed by that statement is a claim that the situation is of a type, and that proposition is true if and only if the claim holds.

In Austinian semantics, the Liar sentence occurs in statements that refer to different situations, and so it expresses a family of propositions indexed by the situations. The Liar shows that each of these propositions is untrue. Therefore, it cannot be a fact in the world that a Liar proposition is true (the Liar sentence refers to the *greatest set of situations* and shows that for each situation of that set, the claim does not hold). Thus, we do not obtain a paradox here like that, which was in Russellian semantics: the Liar proposition has a negative truth valuation.

Judaic semantics are similar to Austinian ones, they can express self-reference too. Sentences and propositions held in Judaic semantics will be called *Judaic sentences* and *Judaic propositions*, respectively. In Judaism, a legitimate statement A also provides two things: a historical, i.e. Haggadic, (or actual) situation S_A , and a type of situation T_A (Halachic situation). The Judaic proposition expressed by A is the claim that S_A is of type T_A , i.e. S_A corresponds to the Halachic situation T_A . The statement A is true if S_A is of type T_A ; otherwise it is false.

In Judaic semantics, a *model of a Judaic proposition* A is a pair comprising the situation S_A and a type T_A ; $\langle S_A, T_A \rangle$ models the proposition A that situation S_A is of type T_A ($S_A \in T_A$). The Judaic conjunction $\langle *, A * B \rangle$ models the type of situation B such that the type of B includes the type of A . Also, if $\langle *, A * B \rangle$ is a Judaic conjunctive type, then proposition $\langle S_{A*B}, \langle *, A * B \rangle \rangle$ is true if and only if $\langle S_{A*B}, T_{A*B} \rangle$ is true for T_B and $T_A \in T_B$. For example, Lot's sentence '*enter my dwelling and wash your feet*' ($A * B$) is false, because $\langle S_{A*B}, T_{A*B} \rangle$ is not true for T_B and $T_A \notin T_B$, and Abraham's sentence '*wash your feet and enter my dwelling*' ($B * A$) is true, because $\langle S_{B*A}, T_{B*A} \rangle$ is true for T_A and $T_B \in T_A$.

The Judaic exegesis is a kind of collective solipsism: there are no facts in the Russellian meaning (no facts outside the Halachah, the Judaic laws), there exist only contexts of utterance that have reference to Halachic types of situations.

5. Inference Rules of Judaic Logic

In Talmud there are very difficult schemes of logical reasoning. For example, there exists an original analogue of classification of knowledge into two groups: analytical and *a priori* synthetic. In Kant's view, a reasoning in which all concepts have a common genus is called an analytical reasoning, i.e. between them it is easy to find a semantic connection of including. For example, the reasoning '*every man is an animal*' is analytical, because the

concept ‘*animal*’ includes the concept ‘*man*’ (‘*man*’ is a specie of ‘*animal*’). On the other hand, *a priori* synthetic reasoning is regarded as a reasoning in which there exists no semantic connection of including between concepts, but it is possible to find an empirical connection of ordering (e.g., relationship of cause and effect). *A priori* synthetic reasoning can be exemplified as ‘*if there is fire, then there is smoke*’.

Let us remember what Kant said about such a classification. “In all judgments in which the relation of a subject to the predicate is thought (I take into consideration affirmative judgments only, the subsequent application to negative judgments being easily made), this relation is possible in two different ways. Either the predicate to the subject *A*, as something which is (covertly) contained in this concept *A*; or outside the concept *A*, although it does indeed stand in connection with it. In the one case I entitle the judgment analytic, in the other synthetic. Analytic judgments (affirmative) are therefore those in which the connection of the predicate with the subject is thought through identity; those in which this connection is thought without identity should be entitled synthetic (...) If I say, for instance, ‘*All bodies are extended*’, this is an analytic judgment. For I do not require to go beyond the concept which I connect with ‘*body*’ in order to find extension as bound up with it. To meet with this predicate, I have merely to analyze the concept, that is, to become conscious to myself of the manifold which I always think in that concept. The judgment is therefore analytic. But when I say, ‘*All bodies are heavy*’, the predicate is something quite different from anything that I think in the mere concept of body in general; and the addition of such a predicate therefore yields a synthetic judgment.

Judgments of experience, as such, are one and all synthetic. For it would be absurd to found an analytic judgment on experience. Since, in framing the judgment, I must not go outside my concept, there is no need to appeal to the testimony of experience in its support. That a body is extended is a proposition that holds *a priori* and is not empirical. For, before appealing to experience, I have already in the concept of body all the conditions required for my judgment. I have only to extract from it, in accordance with the principle of contradiction, the required predicate, and in so doing can at the same time become conscious of the necessity of the judgment – and that is what experience could never have taught me. On the other hand, though I do not include in the concept of a body in general the predicate ‘*weight*’, none the less this concept indicates an object of experience through one of its parts, and I can add to that part other parts of this same experience, as in this way belonging together with the concept.

Thus it is evident: 1. that through analytic judgments our knowledge is not in any way extended, and that the concept which I already have is merely set forth and made intelligible to me; 2. that in synthetic judgments I must have besides the concept of the subject something else (X), upon which the understanding may rely, if it is to know that a predicate, not contained in this concept, nevertheless belongs to it. In the case of empirical judgments, judgments of experience, there is no difficulty whatsoever in meeting this demand. This X is the complete experience of the object which I think through the concept A – a concept which forms only one part of this experience” [10].

For the two kinds of reasoning (analytical and *a priori* synthetic) the following two groups of inference rules are applied in Talmud.

Judaic inference rules for analytical reasoning are as follows:

- the rule “*particular after universal*” (in Hebrew “klol ufrot”),
- the rule “*universal after particular*” (in Hebrew “prot uklol”),
- the rule “*particular after universal after particular*” (in Hebrew “prot uklol ufrot”),
- the rule “*universal after particular after universal*” (in Hebrew “klol ufrot uklol”).

Judaic inference rules for *a priori* synthetic reasoning are as follows:

- the rule “*restricting after extending*” (in Hebrew “ribuy umiut”),
- the rule “*extending after restricting*” (in Hebrew “miut u ribuy”),
- the rule “*extending after restricting after extending*” (in Hebrew “rivuy miut ribuy”).

In these rules we can find four semantic sorts: particularity, universality, extension, restriction.

“*Particularity*” (“frot”) and “*universality*” (“klol”) are understood in the standard (Aristotle’s) way.

“*Extension*” (“ribuy”) is applied in case the following words occur in a reasoning: ‘together’ (in Hebrew ‘et’, ‘es’), ‘also’ (in Hebrew ‘gam’) or ‘however’ (in Hebrew ‘af’). For example, the word ‘together’ (‘es’) occurs twice in the following verse: “in the beginning God created the heaven (in Hebrew “*es ashomaim*”) and the earth (in Hebrew “*es oorez*”)” (Genesis 1, 1). This means that for the understanding of this verse it is necessary to extend the concept/statement that was used; as a result, this verse is understood that the heaven and the earth had been created immediately with everything that is contained in them (stars, trees, grass, etc.).

“*Restriction*” (“miut”) is used if there are the following words in a reasoning: ‘but’ (in Hebrew ‘ach’), ‘only’ (in Hebrew ‘rak’) or ‘of’ (in Hebrew

‘min’). For example, the occurrence of the particle ‘of’ (‘min’) in the verse “... any man... bring your offering *of* the cattle” (Leviticus 1, 2) says that not any cattle may be sacrificed, the exceptions are animals that are idols for somebody.

Now consider the above mentioned rules more precisely.

1. The rule “*particular after universal*”: if the particular name/proposition occurs just after the universal name/proposition, then we accept the particular name/proposition. For example, an offering unto the Lord should be brought of “animals, of the cattle, of the flock” (Leviticus 1, 2). The name ‘animals’ is universal, the name ‘cattle’ or ‘flock’ is particular. Then this does not mean that an offering may be brought of any animals, but only of the cattle and the flock. This rule is formalized as follows:

$$\frac{A * B, B \subset A \text{ (i.e. } A \text{ is universal, } B \text{ is particular)}}{B}. \quad (\text{Jud.r.I})$$

2. The rule “*universal after particular*”. According to that if the universal name/proposition occurs just after the particular name/proposition, then we accept the universal name/proposition. For example, “do not sacrifice (...) any bullock and sheep, wherein is blemish, any defect” (Deuteronomy 17, 1). A ‘blemished one’ is particular, ‘one with any defect’ is universal. Hence, it is forbidden to sacrifice the bullock and sheep with any defect.

$$\frac{A * B, A \subset B \text{ (i.e. } A \text{ is particular, } B \text{ is universal)}}{B}. \quad (\text{Jud.r.II})$$

3. The rule “*particular after universal after particular*”: if the particular name/proposition occurs just after the universal name/proposition that occurs just after the particular name/proposition, then we accept the latest particular name/proposition.

$$\frac{(A * B) * C, A \subset B, C \subset B \text{ (i.e. } A \text{ is part., } B \text{ is univ., } C \text{ is part.)}}{C}. \quad (\text{Jud.r.III})$$

4. The rule “*universal after particular after universal*”: if the universal name/proposition occurs just after the particular name/proposition that occurs just after the universal name/proposition, then we accept the latest universal name/proposition. In the mathematical form:

$$\frac{(A * B) * C, B \subset A, B \subset C \text{ (i.e. } A \text{ is univ., } B \text{ is part., } C \text{ is univ.)}}{C}. \quad (\text{Jud.r.IV})$$

The rules (*Jud.r.I*)–(*Jud.r.IV*) assume the fixed genera inside a reasoning.

5. The rule “*restricting after extending*”: if the restricted concept/sentence occurs just after the extended concept/sentence, then we accept the restricted concept/sentence.

$$\frac{A * B, A \text{ is extended, } B \text{ is restricted}}{B}. \quad (\textit{Jud.r.V})$$

6. The rule “*extending after restricting*”: if the extended concept/sentence occurs just after the restricted concept/sentence, then we accept the extended concept/sentence.

$$\frac{A * B, A \text{ is restricted, } B \text{ is extended}}{B}. \quad (\textit{Jud.r.VI})$$

7. The rule “*extending after restricting after extending*”: if the extended concept/sentence occurs just after the restricted concept/sentence, then we accept the latest extended concept/sentence. For instance, take the reasoning “thou shalt bestow that money for whatsoever thy soul lusteth after, for oxen, for sheep, for wine, for strong drink, for whatsoever thy soul desireth” (Deuteronomy 14, 26). The concept ‘whatsoever thy soul lusteth after’ is extended, the concept ‘oxen, sheep, wine, strong drink’ is restricted, and the concept ‘whatsoever thy soul desireth’ is extended again. Then we accept the latest concept taking into account the examples ‘oxen, sheep, wine, strong drink’.

$$\frac{(A * B) * C, A \text{ is ext., } B \text{ is rest., } C \text{ is ext.}}{C}. \quad (\textit{Jud.r.VII})$$

The rules (*Jud.r.V*)–(*Jud.r.VII*) have no fixed genus inside a reasoning.

The difference of the rule “*restricting after extending*” (“*extending after restricting*”) from the rule “*particular after universal*” (“*universal after particular*”) is that in the first case there is no genus relation inside the statement. For example, in the reasoning “thou shalt bestow that money for whatsoever thy soul lusteth after, for oxen, for sheep...” (Deuteronomy 14, 26) the concepts ‘whatsoever thy soul lusteth after’ and ‘oxen’ have no common genus.

The other inference rules are as follows:

1. the rule “*the same restricting after restricting*” (in Hebrew “*eyn miut achar miut eylo lerabeys*”): if the same restricted concept/sentence occurs just after the restricted concept/sentence, then we introduce the extended concept/sentence. For instance, “. . . the pit was empty, there was no water in it” (Genesis 37, 24). We see that the Torah repeats the

same thing twice. Therefore we have to extend the statement: ‘there is no water, but there are snakes and scorpions’. The rule in the mathematical form is:

$$\frac{A * A, A \text{ is restricted}}{B, B \text{ is extended}}. \quad (\text{Jud.r.VIII})$$

2. the rule “*the same extending after extending*” (in Hebrew “eyn ribuy achar ribuy eylo lerabeys”): if the same extended concept/sentence occurs just after the extended concept/sentence, then we introduce the restricted concept/sentence.

$$\frac{A * A, A \text{ is extended}}{B, B \text{ is restricted}}. \quad (\text{Jud.r.IX})$$

3. the rule “*universal after universal*” (in Hebrew “shney klolim hasmichim zeh lozeh hatel prot beyneyhem veduneym klol ufrot uklol”): if the universal name/proposition occurs just after the universal name/proposition, then we introduce the particular name/proposition between them and use rule (*Jud.r.IV*).

$$\frac{A * B, A \text{ is universal}, B \text{ is universal}}{(A * C) * B, A \text{ is univ.}, C \text{ is part.}, B \text{ is univ.}}. \quad (\text{Jud.r.X})$$

In Judaic logic, the inference rules (*Jud.r.I*)–(*Jud.r.X*) are used for simplifying the Torah statements. Thus, the statements of the Pentateuch are axioms of Judaic logic. This proof system is non-well-founded, because it contains cyclic proofs like:

$$\frac{A * B, A \text{ is restricted}, B \text{ is restricted}}{A * B, A \text{ is restricted}, B \text{ is restricted}}. \quad (\text{Jud.r.XI})$$

The inference rule (*Jud.r.XI*) is called in Hebrew “miut achar miut”. As an example, consider the verse “and they said, Hath the Lord *however* [in Hebrew “harak”] spoken *only* [in Hebrew “ach”] by Moses?” (Numbers 12, 2). As we see, two restrictions occur in the verse. Therefore it is inferred that not only Moses, but Aaron and Miriam (the two restrictions) had prophecy too.

6. Conclusions

Judaic logic is built on the basis of the only logical connective * (“...and...”) which is called the Judaic conjunction. It is a kind of non-well-founded logic,

because Judaic semantics assume that self-referentiality and Judaic proof systems have cyclic proofs.

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