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A LINEAR APPROXIMATION METHOD IN PREDICTION OF CHAOTIC TIME SERIES

This paper presents a method to make predictions regarding the chaotic time series, known as a linear approximation method. After embedding a time series in a phase space, it is necessary to replace nonlinear mapping using a local approximation. This allows making a short-term prediction of future behaviour, using information based only on past values. The effectiveness of this method is demonstrated by applying it to the prediction of share prices.

Introduction

The basic problem of scientific investigation is forecasting – How can we predict the future, given the past? The behaviour of periodical structure we can predict in infinity, but chaotic structure is predictable in the short term only. It is connected with a basic property – the sensitivity on initial conditions.

It consists in that very similar initial conditions sometimes give very different structure's behaviour. The reason for this is that we can establish initial conditions with finished exactitude, but miscalculations grow exponentially. (Therefore forecasting such structures is sensible only in short intervals). This means, that when we want to predict the behaviour of such a structure in any moment, we would dispose of the data entrance passed with infinite exactitude as well as execute all calculation with finite accuracy. Otherwise, small mistakes in setting initial values as well as miscalculations (e.g. the mistakes of roundings) grow in an exponential way. This means that the evolution of such systems is very complex and virtually unpredictable in the long-term.

There exist different methods of forecasting a chaotic time series. This article presents a method of the linear approximation applied to a short-term prediction regarding the share prices on the Warsaw Stock Exchange. The

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time series of share prices is a type of deterministic series and can behave chaotically [6, 1997].

A Linear Approximation Method

If we want to model nonlinear systems:

$$x_{i+1} = f(x_i), \quad i = 0, 1, ... \tag{1}$$

we might fit the data to combinations of nonlinear functions. This however is a very complicated method. We can therefore apply a method based on an approximation of behaviour in the midst of any point on an attractor[1] by a unique local function. Then, the evolution on an attractor is represented by the set of such functions. Functions are linear at each point. This means:

$$x_{i+1} = \mathbf{a} + \mathbf{b}x_i, \tag{2}$$

where matrix **b** and vector **a** are defined for every point. A class of the local map creates a global nonlinear map [1, 1993].

Suppose the time series folded from T observations: $x_1, x_2, \ldots, x_T$. We can establish a dimension of embedding m and make the reconstruction of the phase space[2]. In such a space we get the following set of vectors:

$$x_i^m = (x_i, x_{i-1}, \ldots, x_{i-m+1}), \quad i = m, m+1, \ldots, T. \tag{3}$$

We should predict the value of the time series with number $P - x_P$, which is the first component of a point: $x_P^m = (x_P, x_{P-1}, \ldots, x_{P-m+1})$. In the neighbourhood of this point we can estimate the following equation parameters:

$$x_P^m = \mathbf{a} + \mathbf{b}x_{P-1}^m + \varepsilon_P^m, \tag{4}$$

a is mx1 dimensional vector of parameters,
b os mxm dimensional matrix of parameters,
ε_P^m is mx1 dimensional vector of errors.

We assume that the function is linear.

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[1] Attractor is a set to which the system evolves after a long enough time. For a set to be an attractor, trajectories which get close enough to the attractor must remain close even if slightly disturbed.

[2] Phase space is the space in which all possible states of a system are represented, with each possible state of the system corresponding to one unique point in the phase space. Dimension of the phase space depends on quantity of variables necessary to the description of the system. The reconstruction of phase space consists in reproducing the multidimensional attractor based on the one-dimensional time series.

Illustrated bellow is the matrix form:

$$\begin{bmatrix} x_P \\ x_{P-1} \\ \vdots \\ x_{P-m+1} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mm} \end{bmatrix} \cdot \begin{bmatrix} x_{P-1} \\ x_{P-2} \\ \vdots \\ x_{P-m} \end{bmatrix} + \begin{bmatrix} \varepsilon_P \\ \varepsilon_{P-1} \\ \vdots \\ \varepsilon_{P-m+1} \end{bmatrix}. \quad (5)$$

Therefore we should estimate only the first component a_1 of vector **a** and the first row of matrix **b**. We need to solve the following equation:

$$x_P = a_1 + \sum_{j=1}^{m} b_{1j} x_{P-j} + \varepsilon_P. \quad (6)$$

To calculate the parameters we use an approximation by k nearest neighbour x^m_{P-1} point.

The estimation of the parameters relating to equation 6 proceeds as follows [3, 1989]:

- We define components of k points in the m-dimensional reconstructed phase space: $x_{n_{1m}}, x_{n_{2m}}, \dots, x_{n_{km}}$, where $k > m$. They are the nearest, in the sense the of the Euclidean metric, neighbours of the point $(x_{P-1}, x_{P-2}, \dots, x_{P-m})$. We should consider $k \geq 2(m+1)$ the closest neighbours.
- We mark the first components of the nearest neighbours: $x_{n_1}, x_{n_2}, \dots, x_{n_k}$, and then corresponding to them, the following points in time series: $x_{n_1+1}, x_{n_2+1}, \dots, x_{n_k+1}$.
- As a result we form a system of k equations with $m+1$ unknowns:

$$x_{n_i+1} = a_1 + \sum_{j=1}^{m} b_{1i} x_{n_i+1-j} + \varepsilon_{n_i+1}, \quad i = 1, 2, \dots, k. \quad (7)$$

- Parameters a_1, b_{1j} $(j = 1, ..., m)$ are estimated by the least squares method.
- Using equation 6 we predict the value of element x_P:

$$\hat{x_P} = \hat{a_1} + \sum_{j=1}^{m} \hat{b_{1j}} x_{P-j}. \quad (8)$$

Thus for any x_t there is a marked predicted value $\hat{x_t}$. It can precisely determine an absolute error $\varepsilon_t = x_t - \hat{x_t}$. Relative error ψ_t it is the percentage deviation of the obtained value from the real value.

The Share Prices on the Warsaw Stock Exchange Prediction

The method of linear approximation can be used for forecasting share prices. It has been proven was that the time series of share prices are generated by a deterministic system, showing a tendency for chaotic behaviours.

The time series, we studied, consisted from around 2500 observations. The method will be applied to an example time series of share prices for the Żywiec company. We take $t = 2001, \ldots, 2300$.

Firstly, we assume the dimension of embedding $m = 2$, and its nearest neighbours' number $k = 8$ (Fig. 1). For an exact analysis of the method we specified the fragment of time series for $t = 2110, \ldots, 2120$. The last point of this fragment is x_{2120}. The nearest eight neighbours of point x^2_{2119} in reconstructed phase space for $m = 2$ are points: x^2_{2111}, x^2_{2112}, x^2_{2113}, x^2_{2114}, x^2_{2115}, x^2_{2116}, x^2_{2117}, x^2_{2118}. The consequents are: x^2_{2112}, x^2_{2113}, x^2_{2114}, x^2_{2115}, x^2_{2116}, x^2_{2117}, x^2_{2118}, x^2_{2119}. We ascertained the following matrix equation:

$$\begin{bmatrix} x_{2112} \\ x_{2113} \\ x_{2114} \\ x_{2115} \\ x_{2116} \\ x_{2117} \\ x_{2118} \\ x_{2119} \end{bmatrix} = \begin{bmatrix} 1 & x_{2111} & x_{2110} \\ 1 & x_{2112} & x_{2111} \\ 1 & x_{2113} & x_{2112} \\ 1 & x_{2114} & x_{2113} \\ 1 & x_{2115} & x_{2114} \\ 1 & x_{2116} & x_{2115} \\ 1 & x_{2117} & x_{2116} \\ 1 & x_{2118} & x_{2117} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ b_{11} \\ b_{12} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \\ \varepsilon_8 \end{bmatrix}. \tag{9}$$

As a result of the estimation parameters we get the equation:

$$x_{2\hat{1}20} = 150{,}3669467 + 0{,}18326264 \cdot x_{2119} - 0{,}040650558 \cdot x_{2118}. \tag{10}$$

The predicted value comes to 175,23, with an error $\varepsilon_{2120} = -0,73$. The error amounts to $\psi_{2120} = 0,42\%$ of the real value. In most cases the mistakes did not exceed 2%, so this is quite an effective method (Fig. 2).

As a rule, experimental results show that an increase to the 12 the nearest neighbours numbers improves the effectiveness of a prediction (Fig. 3).

As we can see from above, the method of linear approximation gives enough first-rate results in predicting of share prices. However, we must bear in mind, that it relates only to short-term forecasting.

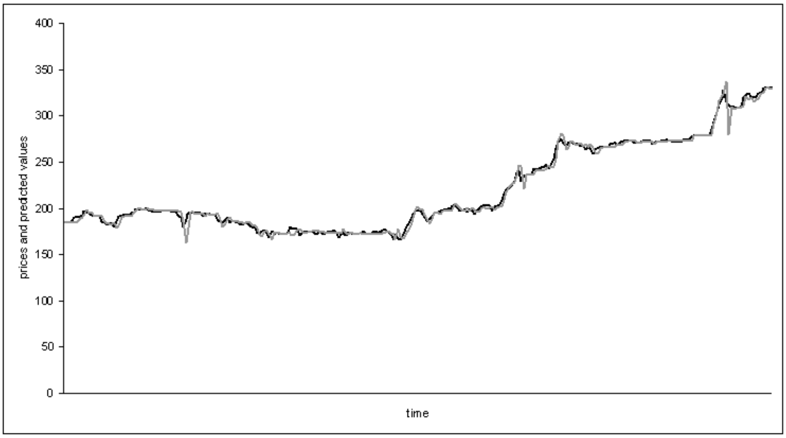

Figure 1. Comparison of the Żywiec's share prices with predicted values for $t = 2001 - 2300, m = 2, k = 8$

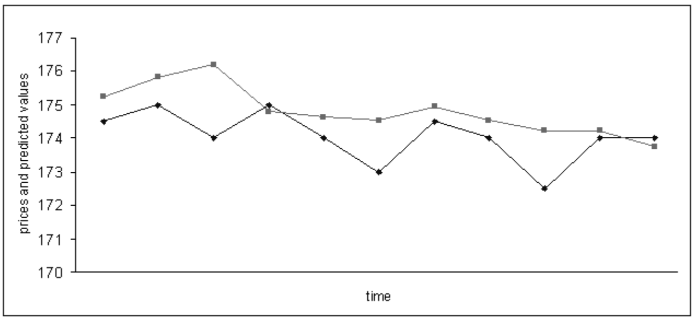

Figure 2. Comparison of the Żywiec share prices with predicted values for $t = 2110 - 2120, m = 2, k = 8$

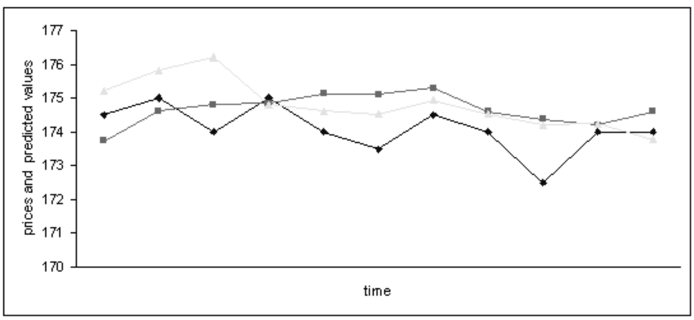

Figure 3. Comparison of the Żywiec share prices with predicted values for $t = 2110 - 2120, m = 2, k = 8$ and $k = 12$

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