

Remarks on Non-Fregean Logic

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1 Introduction

In 1966 famous Polish logician Roman Suszko read a manuscript written by Professor Bogusław Wolniewicz titled “*Rzeczy i fakty. Wstęp do pierwszej filozofii Wittgensteina (Things and facts: An introduction to Wittgenstein’s first philosophy)*” published in Polish in 1968. This monograph inspired Suszko to create a new logical calculus called by him *Non-Fregean Logic*. To explain what non-Fregean logic is, let me remind you that Gottlob Frege in his essay “Über Sinn und Bedeutung” [2] formulated and, to some extent, justified the view that sentences in the logical sense of the word, have referents. Frege believes that we have to identify the denotations of sentences with their truth values. Let me quote Frege himself:

“... Every declarative sentence concerned with the reference of its words is therefore to be regarded as a proper name, and its reference, if it has one, is either the True or the False. ... If our supposition that the reference of a sentence is truth-value is correct, the latter must remain unchanged if a part of the sentence is replaced by an expression having the same reference. And this in fact the case.

... What else but the truth value could be found, that quite generally to every sentence if the reference of its is relevant, and remains unchanged by substitutions of the kind in question.”

The following principle that:

“all true (and, similarly all false) sentences describe the same state of affairs, that is, they have a common referent”

Roman Suszko has called the semantic version of Fregean axiom. Roman Suszko in his formalization of the fragment of the ontology of Wittgenstein’s “*Tractatus Logico-Philosophicus*” in [7], [8] offers a continuation of Frege’s program without adopting Fregean axiom.

The name non-Fregean logic is justified by the fact that in this logic there are no theorems asserting how many semantic correlates of sentences there can be. In classical logic sentential variables run over two-elements Boolean algebra of truth-values. Because theorems of this logic are the following formulas:

$$\begin{aligned} & (p \vee \neg p), \quad \neg(p \wedge \neg p) \\ & (p \leftrightarrow q) \vee (q \leftrightarrow r) \vee (p \leftrightarrow r) \end{aligned}$$

Non-Fregean logic is extensional, logically two-valued but it is not two-valued ontologically, because it does not follow from the theorems of logic alone how many semantic correlates of sentences there can be. Following Wittgenstein, R. Suszko calls the semantic correlates of sentences “situations”. To say that two sentences describe the same situations Suszko uses a connective identity “ \equiv ”, that is not a truth-functional connective.

2 Non-Fregean Semantic Frameworks

I would like to present the semantic principles which constitute foundations of the non-Fregean sentential calculus. This sentential calculus is the main part of the non-Fregean logic. In my opinion the set of all sentences of any language may be interpreted according to non-Fregean semantics.

To state the principle of non-Fregean interpretation of sentences of any language L we apply the following notations:

(i) Let S be the set of all sentences of language L , and let Cn will be a consequence operation on S . We will use letters: $\alpha, \beta, \gamma, \dots$ as variables running over the set S .

(ii) Let U be an arbitrary non-empty set. The elements of U will represent the situations. Intuitively speaking elements of U will be treated as given by corresponding sentences of L . For simplicity, elements of the set U will be called of situations. We will treat letters: p, q, r, \dots as variables running over the universe of situations U . The sentential variables: p, q, r, \dots are not necessarily in the alphabet of L . However, the sentential variables are needed in the alphabet of the semantic metalanguage of L , in which theorems are formulated about the set of semantic correlates of L .

(iii) Next, let H be an arbitrary relation which has as its domain the set S of sentences of language L and its counter-domain is the set U , i.e., $H \subset S \times U$. If $H(\alpha, p)$ holds we will say that the sentence α refers to the situation p , or α describes the situation p .

(iv) A sentence may be true or false, or it may be necessary or only possible or probable, determined or undetermined, sensible or senseless. In several logical systems, the mentioned properties are logical values of sentences.

Let (V, V_1) be an arbitrary pair of sets such that

$$\emptyset \neq V_1 \subset V \text{ and } V_1 \neq V$$

i.e. V_1 is non-empty set and V_1 is properly included in V . The elements of V will be called logical values, and elements of V_1 the designated logical values.

(v) An arbitrary function v is such that:

$$v : S \rightarrow V$$

and

$$\emptyset \neq v^{-1}(V_1) \neq S.$$

The function v will be called a logical valuation of L . The family of the sets $\{v^{-1}(V - V_1), v^{-1}(V_1)\}$ is called the fundamental partition of the set of sentences of language L determined by the logical valuation v .

Definition 1. (Non-Fregean Semantic Framework)

An arbitrary sequence of the form:

$$(*) \quad \langle (S, Cn), U, (V, V_1), H, v \rangle$$

is named non-Fregean semantic framework for the sentential fragment of language L iff the sequence satisfies the following seven postulates called semantics principles of non-Fregean sentential logic:

(P1) Principle of Correlation:

For each sentence α of L there is at least one situation p , such that α refers to p , symbolically $\forall \alpha \exists p H(\alpha, p)$.

(P2) Principle of Univocality:

Each sentence α of language L refers to at most one situation i.e.

If $H(\alpha, p)$ and $H(\alpha, q)$, then $p = q$.

(P3) Principle of Stability:

For any $\alpha, \beta \in S$, if for any situation $p \in U; (H(\alpha, p) \leftrightarrow H(\beta, p))$, then for each sentence $\gamma \in S$, and any situation $q, (H(\gamma, q) \leftrightarrow H(\gamma[\alpha/\beta], q))$.

(P4) Principle of Logical Bivalence:

The set of logical values V is two elements i.e. $V = \{0, 1\}, 0 \neq 1$.

(P5) Principle of Maximality of Truth:

For any $X \subseteq S$, if $v^{-1}(V_1) = X$,

then $X \neq S$, and for any $\alpha \notin X, Cn(X \cup \{\alpha\}) = S$.

(P6) Principle of Subordination to Fundamental Partition:

For any sentences α, β , if for any situation $p, (H(\alpha, p) \leftrightarrow H(\beta, p))$,

then $v(\alpha) = v(\beta)$.

(P7) Principle of Contextual Differentiation:

If for any sentences γ of the language $L, v(\gamma) = v(\gamma[\alpha/\beta])$,

then for any situation $p \in U, H(\alpha, p) = H(\beta, p)$.

Observation 1.

In view of both principles: (P1) and (P2), we may define the function:

$$h : S \rightarrow U$$

by the formula:

$$p = h(\alpha) \leftrightarrow H(\alpha, p).$$

In this case we say the situation p is the semantic correlate of α or the sentence α refers to the situation p .

Observation 2.

The function h is not an arbitrary function but it must satisfy certain conditions. In particular, if S is an algebra on the set S with respect to connective of language L

(i.e. any connective of the language L is additionally treated as algebraic operation on the set sentences S) then h is a homomorphism from S to the corresponding algebra on the set $h(S) \subseteq U$.

Corollary 1.

In arbitrary semantic framework (*) the logical syntax of language L , imposes upon the set references of the sentences of L the structure of an algebra similar to the algebra of sentences of language L .

The principle (P4) asserts, that the set logical values has two elements. If $v(\alpha) = 1$, then we say that α is true in the considered semantic frame. If $v(\alpha) = 0$ we say that α is false in this frame.

It follows from principles (P4) and (P5) that logical valuations are characteristic functions of complete theories in (S, Cn) . By contraposition from principle (P6) we obtain: if any sentences α, β have different logical values then there is at least one situation which is semantic correlate for only one of these sentences i.e.

$$v(\alpha) \neq v(\beta) \rightarrow \exists p \neg[H(\alpha, p) \leftrightarrow H(\beta, p)].$$

These observations allow us to simplify the notion of a non-Fregean semantic framework namely, henceforth as quadruple:

$$\langle (S, Cn), U, h, v \rangle$$

where:

- (S, Cn) is any sentential calculus,
- U is arbitrary set such that $|U| \geq 2$,
- v is a characteristic function of any arbitrary complete theory in (S, Cn) ,
- $h : S \rightarrow U$ such that for arbitrary sentences: $\alpha, \beta \in S$ following three conditions are satisfied:

- (P3') if $h(\alpha) = h(\beta)$, then $\forall \gamma \in S [h(\gamma) = h(\gamma[\alpha/\beta])]$
- (P5') if $h(\alpha) = h(\beta)$, then $v(\alpha) = v(\beta)$
- (P7') if $h(\alpha) \neq h(\beta)$, then $\exists \gamma \in S (v(\gamma) \neq v(\gamma[\alpha/\beta]))$.

One direct consequence of definition 1, remarks, observations 1, 2, and corollary 1 is the following theorem:

Theorem 1.

The quadruple:

$$(**) \quad \langle (S, Cn), U, h, v \rangle$$

is non-Fregean semantics framework iff the following conditions are satisfied:

- (1) (S, Cn) is arbitrary sentential calculus
- (2) U is arbitrary set of at least two elements
- (3) v is characteristic function of some complete theory in (S, Cn)
- (4) h is function

$$h : S \rightarrow U$$

such that for arbitrary sentences: $\alpha, \beta \in S$, condition

$$h(\alpha) = h(\beta) \leftrightarrow \forall \gamma \in S [v(\gamma) = v(\gamma[\alpha/\beta])]$$

is satisfied.

The proof of this theorem is in [5].

According to theorem 1 any two sentences: α, β language L have the same semantic correlate in the non-Fregean semantic framework (***) if and only if they are mutually interchangeable in any sentential context γ without changing logical value this context.

In any semantic framework (***) we may define relation \approx_v on the set S in the following way: for any: $\alpha, \beta \in S$:

$$\alpha \approx_v \beta \Leftrightarrow \forall \gamma \in S [v(\gamma) = v(\gamma[\alpha/\beta])].$$

Thus

$$\alpha \approx_v \beta \Leftrightarrow h(\alpha) = h(\beta).$$

Definition 2. (Suszko [8], [9])

The functor “ \equiv ” of the language L is the identity connective of sentential calculus (S, Cn) iff the following rules:

- (r₀) $\vdash \alpha_1 \equiv \alpha_2$, when α_1, α_2 differ at most in bound variables,
- (r₁) $\alpha \equiv \beta \vdash \gamma \equiv \gamma[\alpha/\beta]$
- (r₂) $\alpha \equiv \beta, \gamma[p/\alpha] \vdash \gamma[p/\beta]$

are the rules of the consequence operation Cn .

The direct consequence this definition is that: in classical logic, the truth- functional connective of equivalence is also the identity connective.

Theorem 2.

If (S, Cn) is the language with identity connective “ \equiv ” then for any non-Fregean semantic framework $\langle (S, Cn), U, h, v \rangle$ the following condition is satisfied:

for any sentences $\alpha, \beta \in S$, $h(\alpha) = h(\beta)$ if and only if $v(\alpha \equiv \beta) = 1$.

Theorem 3.

If (S, Cn) is the language with connective “ \equiv ” such that, for any non-Fregean semantic framework $\langle (S, Cn), U, h, v \rangle$ the following condition is satisfied:

for any sentences $\alpha, \beta \in S$, $h(\alpha) = h(\beta)$ if and only if $v(\alpha \equiv \beta) = 1$, then the connective “ \equiv ” is identity connective the calculus (S, Cn) i.e. the rules: (r₀), (r₁), (r₂) are rules of consequence Cn .

The proof those theorems 2 and 3 may be find in [5].

3 Non-Fregean Logics

Following L.Wittgenstein, R. Suszko calls the semantic correlates of sentences ”situations”. To speak in formal way about the structure of universe of situations and universe of objects, Suszko introduced to literature a family of logic languages which he called W -languages (in honor of L. Wittgenstein). In the alphabet of these languages, there are:

1. two kinds of variables, sentential variables: p, q, r, \dots and nominal variables: x, y, z, \dots ,
2. truth-functional connectives: \neg (negation), \wedge (conjunction), \vee (disjunction), \rightarrow (implication), \leftrightarrow (equivalence),
3. predicate-letters: P_1, P_2, \dots, P_n ,
4. function-symbols: F_1, F_2, \dots, F_m ,
5. symbols identity: identity connective and identity predicate which are both symbolized by the sign “ \equiv ”,
6. quantifiers: \forall, \exists binding both kinds of variables.

Each of the quantifiers can bind both a sentential variable or a nominal one, depending on which variable follows directly after it. Similarly the context uniquely determines whether we have to do with the identity connective or the identity predicate since the expression $x \equiv p$ is not a formula of the language discussed. A detailed description of the syntax of the W -kind languages has been presented in the papers [1] and [3].

Given an W -language L , let S be the set of all sentential formulas, and N be the set of nominal formulas of language L . Sentential formula α is a sentence iff α has no free variables. By S^+ will be denoted the set of all sentences of language L . We consider two consequence operation on S , Cn and Cn^* , analogous as Suszko considered in [11] for open W -language. The constructions presented here extend what has been done in [11] for SCI -language.

Operation Cn is generated by the Modus Ponens rule and the schemas of logical axioms, but operation Cn^* is generated by the rules: Modus Ponens, and the rule of generalization for universal quantifier and logical axioms too. In logical literature operation Cn is called structural, but Cn^* is named invariant consequence operation. To describe the consequence Cn and Cn^* in W -languages the following notations are introduced:

Letters: $v, w, v_1, w_1, v_2, w_2, \dots$ will be metalanguage variables denoted depending on the context, either sentential variables or the nominal variables. By the letters: $\alpha, \beta, \gamma, \dots$ will be denote any sentential formulas, by the letters: ξ, ζ, τ, \dots we denote any nominal formulas, and finally $\varphi, \phi, \psi, \dots$ denote sentential formulas or nominal ones, depending on the context. Symbols $\alpha[v/\varphi]$ denote result substitution in formula $\alpha(v)$ for free variable v the expression φ . The result of prefixing the formula α by any finite number of universal quantifiers, i.e. $\forall_{v_1} \forall_{v_2} \dots \forall_{v_n} \alpha$, where $n \geq 0$ is called generalization of the formula α . For any set of sentential formulas X , by $Gen(X)$ will be denoted the set of all generalizations of formulas in the set X . The formulas of the form $\varphi \equiv \psi$, are called equations.

3.1 Structural non-Fregean consequence operation

The structural consequence Cn in the language L was described and investigated by S. L. Bloom in [1]. The structural version non-Fregean logic in W -language L is introduced by accepting logical axioms and the only inference rule Modus Ponens. The logical axioms are those formulas which are generalizations of any formula of the following sorts:

A1. Axiom Schemata for truth-functional connective:

- (S1) $\alpha \rightarrow (\beta \rightarrow \alpha)$
- (S2) $[\alpha \rightarrow (\beta \rightarrow \gamma)] \rightarrow [(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)]$
- (S3) $\neg\alpha \rightarrow (\alpha \rightarrow \beta)$
- (S4) $(\neg\alpha \rightarrow \alpha) \rightarrow \alpha$
- (S5) $(\alpha \rightarrow \beta) \rightarrow [(\beta \rightarrow \alpha) \rightarrow (\alpha \leftrightarrow \beta)]$
- (S6) $(\alpha \leftrightarrow \beta) \rightarrow [\alpha \rightarrow \beta]$
- (S7) $(\alpha \leftrightarrow \beta) \rightarrow (\beta \rightarrow \alpha)$
- (S8) $(\alpha \vee \beta) \leftrightarrow (\neg\alpha \rightarrow \beta)$
- (S9) $(\alpha \wedge \beta) \leftrightarrow \neg(\alpha \rightarrow \neg\beta)$

A2. Axiom Schemata for quantifiers:

- (S10) $\forall_v \alpha \rightarrow \alpha[v/\varphi]$
- (S11) $\alpha \rightarrow \forall_v \alpha$ (if v is not free in α)
- (S12) $\forall_v (\alpha \rightarrow \beta) \rightarrow (\forall_v \alpha \rightarrow \forall_v \beta)$
- (S13) $\exists_v \alpha \leftrightarrow \neg \forall_v \neg \alpha$

A3. Axiom Schemata for identity connective and predicate:

A3.1. Congruence axioms. All formulas of the form:

- (S14) $\varphi \equiv \phi$ (when φ, ϕ vary in at most bound variables)
- (S15) for every functor Ψ we accept the invariance axiom:
 $\varphi_1 \equiv \phi_1 \wedge \varphi_2 \equiv \phi_2 \wedge \dots \varphi_n \equiv \phi_n \rightarrow \Psi(\varphi_1, \varphi_2, \dots, \varphi_n) \equiv \Psi(\phi_1, \phi_2, \dots, \phi_n)$
- (S16) $\forall_v (\alpha \equiv \beta) \rightarrow (Q_v \alpha \equiv Q_v \beta)$, where $Q = \forall, \exists$.

A3.2. Axiom schema for identity:

- (S17) $(\varphi \equiv \psi) \rightarrow (\alpha[v/\varphi] \rightarrow \alpha[v/\psi])$

The set logical axioms **LA** is the sum of three sets: **A1**, **A2**, **A3** i.e.

$$\mathbf{LA} = \mathbf{A1} \cup \mathbf{A2} \cup \mathbf{A3}.$$

A subset $X \subseteq S$ is called Cn -theory iff X contains **LA** and X is closed under rules MP. A set of all the sentential formulas which are derivable from any set X and from logical axioms in any finite number of steps through the application of MP rule is called theory and is denoted by $Cn(X)$. Theory T is called invariant if $Gen(T) \subset T$. A formula α is called logical theorem of non-Fregean logic iff $\alpha \in Cn(\emptyset)$.

In [1] and [3] it is proved that the structural non-Fregean logic has following metatheorems property:

1. The deduction theorem.

For any set X of formulas of L , and any formulas α, β of L ,

$$\beta \in Cn(X \cup \{\alpha\}) \Leftrightarrow (\alpha \rightarrow \beta) \in Cn(X)$$

This comes from the fact that theorems of non-Fregean logic are all formulas of the form:

$$\begin{aligned} &\alpha \rightarrow \alpha, \\ &\alpha \rightarrow (\beta \rightarrow \alpha), \\ &(\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \end{aligned}$$

and because we have that the Modus Ponens rule MP is the only rule for proving theorems.

2. The set of logical theorems $Cn(\emptyset)$ is invariant theory.

3. If $[\alpha(v)] \in Cn(\{\alpha_1, \alpha_2, \dots, \alpha_n\})$,

where the variable v are not free in $\alpha_1, \alpha_2, \dots, \alpha_n$,

then $[\forall_v \alpha(v)] \in Cn(\{\alpha_1, \alpha_2, \dots, \alpha_n\})$.

This follows from the deduction theorem and from the fact that $Cn(\emptyset)$ is an invariant theory.

3.2 Invariant non-Fregean consequence operation

In L invariant non-Fregean consequence operation Cn^* is generated by the rules: Modus Ponens of the scheme:

MP. $\alpha, \alpha \rightarrow \beta \vdash \beta$,

and generalization rule of the scheme:

Gen. $\alpha \vdash \forall_v \alpha$,

and by the logical axioms divided into three groups:

A1. Axioms for the truth-functional connectives:

- (1) $p \rightarrow (q \rightarrow p)$
- (2) $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$
- (3) $\neg p \rightarrow (p \rightarrow q)$
- (4) $(\neg p \rightarrow p) \rightarrow p$
- (5) $(p \rightarrow q) \rightarrow [(q \rightarrow p) \rightarrow (p \leftrightarrow q)]$
- (6) $(p \leftrightarrow q) \rightarrow (p \rightarrow q)$
- (7) $(p \leftrightarrow q) \rightarrow (q \rightarrow p)$
- (8) $(p \vee q) \leftrightarrow (\neg p \rightarrow q)$
- (9) $(p \wedge q) \leftrightarrow \neg(p \rightarrow \neg q)$

A2. Axiom Schemata for quantifiers:

- (10) $\forall_v \alpha \rightarrow \alpha[v/\varphi]$
- (11) $\alpha \rightarrow \forall_v \alpha$ (if v is not free in α)
- (12) $\forall_v (\alpha \rightarrow \beta) \rightarrow (\forall_v \alpha \rightarrow \forall_v \beta)$
- (13) $\exists_v \alpha \leftrightarrow \neg \forall_v \neg \alpha$

A3. Identity axioms:

A3.1. Congruence axioms. All formulas of the form:

(14) $\varphi \equiv \phi$ (where φ, ϕ differ at most bound variables)

(15) for every functor Ψ we accept the invariance axiom:

$$v_1 \equiv w_1 \wedge v_2 \equiv w_2 \wedge \dots \wedge v_n \equiv w_n \rightarrow \Psi(v_1, v_2, \dots, v_n) \equiv \Psi(w_1, w_2, \dots, w_n)$$

(16) $\forall_v (\alpha \equiv \beta) \rightarrow (Q_v \alpha \equiv Q_v \beta)$, where $Q = \forall, \exists$

A3.2. Special identity axiom:

(17) $(p \equiv q) \rightarrow (p \rightarrow q)$.

The set logical axioms LA^* of invariant non-Fregean logic Cn^* , is the sum of three sets: A1, A2, A3 i.e.

$$LA^* = A1 \cup A2 \cup A3.$$

A subset $X \subseteq S$ is called Cn^* -theory iff X contains LA^* and X is closed under rules MP i Gen.

The invariant non-Fregean logic satisfies the following properties:

1. For any set $X \subseteq S$, the theory $Cn^*(X)$ is the least invariant theory containing set X .
2. Deduction property: For invariant consequence the following deduction theorem may be proved:

For any set $X \subseteq S$ and for any sentence $\alpha \in S^+$ and any formulas $\beta \in S$:

$$\beta \in Cn^*(X \cup \{\alpha\}) \Leftrightarrow (\alpha \rightarrow \beta) \in Cn^*(X).$$

It may be interesting to consider operation consequence Cn^+ given by only one rule: Modus Ponens and the set axiom:

A1. Axioms for the truth-functional connectives:

- (1) $\forall_p \forall_q [p \rightarrow (q \rightarrow p)]$
- (2) $\forall_p \forall_q \forall_r [p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$
- (3) $\forall_p \forall_q [\neg p \rightarrow (p \rightarrow q)]$
- (4) $\forall_p [(\neg p \rightarrow p) \rightarrow p]$
- (5) $\forall_p \forall_q [(p \rightarrow q) \rightarrow [(q \rightarrow p) \rightarrow (p \leftrightarrow q)]]$
- (6) $\forall_p \forall_q [(p \leftrightarrow q) \rightarrow (p \rightarrow q)]$
- (7) $\forall_p \forall_q [(p \leftrightarrow q) \rightarrow (q \rightarrow p)]$
- (8) $\forall_p \forall_q [(p \vee q) \leftrightarrow (\neg p \rightarrow q)]$
- (9) $\forall_p \forall_q [(p \wedge q) \leftrightarrow \neg(p \rightarrow \neg q)]$

A2. Axiom Schemata for quantifiers:

- (10) $\forall_v \alpha \rightarrow \alpha[v/\varphi]$
- (11) $\alpha \rightarrow \forall_v \alpha$ (if v is not free in α)
- (12) $\forall_v (\alpha \rightarrow \beta) \rightarrow (\forall_v \alpha \rightarrow \forall_v \beta)$
- (13) $\exists_v \alpha \leftrightarrow \neg \forall_v \neg \alpha$

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- (16) $\forall_v (\alpha \equiv \beta) \rightarrow (Q_v \alpha \equiv Q_v \beta)$, where $Q = \forall, \exists$.

A3.2. Special identity axiom:

- (17) $\forall_p \forall_q [(p \equiv q) \rightarrow (p \rightarrow q)]$.

A subset $X \subseteq S$ is called Cn^{+-} theory iff X contains

$$LA^+ = A1^+ \cup Gen.(A2) \cup Gen.(A3.1) \cup (A3.2)$$

and X is closed under rules MP.

4 Semantic Remarks

The intended interpretation of W -languages is such that nominal variables range over the universe of objects while the sentential variables run over the universe of situations. All other symbols in these languages, except the sentential and nominal variables, are interpreted as symbols of some functions both defined over the universe of situations and the universe of objects. The identity connective corresponds to the identity relation between situations, and the identity predicate corresponds to the identity relation between objects. It is obvious that the language of the ordinary predicate calculus with identity is a part of the W -language excluding sentential variables, but the most frequently used sentential languages are the part of the W -language without nominal variables and the identity predicate.

If a language does not contain sentential variables then according to Suszko it does not fit for a full formalization of a theory of situations, but at most for that of a theory of events considered as reified equivalents of situations (see Suszko [10]). In contemporary science the majority of studied theories are those which are expressed in languages without sentential variables. The appearance of the Fregean sentential semantics, Leśniewski's prothotetics, Wittgenstein's *Tractatus* and the Non-Fregean Logic has been an important step in the development of logic-philosophical reflection, because these theories require a language with sentential variables. According to Suszko, situations are primitive with respect to events, for the latter are objects abstracted from the former. In Suszko [10] it is proved that:

- (i) certain theories of situations are mutually translatable into theories of events,
- (ii) certain algebras of situations are isomorphic with algebras of events.

Suszko posed the question:

*“... What, therefore, is the cause of the fact that our thought and the natural language to a certain degree discriminate sentence variables, and particularly, general and existential sentences about situations?
..., Why, therefore, should we prefer the theory of events to that of situations?”*

And in the same paper Suszko answer:

“It is probably the symptom of some deep, historically determined attribute our thought and of natural language, the examination and explication of which will certainly be long and arduous”.

These features of our thinking induce an account of the world rather as the universe of objects having certain properties and connected by certain relations, not - as the totality of facts obtaining a logical space as in Wittgenstein's *Tractatus*.

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