Katarzyna Budzyńska
University of Cardinal Stefan Wyszyński
in Warsaw

ARGUMENTATION FROM SEMANTIC AND PRAGMATIC PERSPECTIVE

Introduction

The aim of this article is to describe some of argumentation attributes considered on the ground of logic. I will advocate that argumentation is a kind of reasoning which: (1) is based on specific inference ground, (2) is always accompanied by pragmatic elements occurring with reasoning.

I claim that argumentation has to be studied on two independent levels – objective level of *truth* separated from our knowledge, and subjective level of human *beliefs* on the truth. However, we can be interested in the properties that belong to both of these levels. We have to remember that they act independent from each other, so that argument attributes from one level may not influence attributes from another level at all. That is why I will consistently separate and highlight them in order to insure the clarity to our considerations until I develop exact tools, which will allow me to analyse actual everyday discussions on both levels simultaneously.

In first part of the article I would like to present basic notions necessary to create theory in second chapter. I will compare two levels: (i) objective level with notions of theory, formulae generally valid, semantic entailment and deduction, and (ii) subjective level with the notions corresponding with the previous ones – set of believes, formulae believed as generally valid (*topoi*), pragmatic entailment and reasoning based on *topoi*. In second part of the article I will introduce my definition of argumentation. Then I will analyse such understood discussion from two perspectives: objective – when argumentation allow to find the truth (i.e. leads disputants to true conclusions) and subjective – when argumentation is efficient (i.e. leads to persuade the audience of discussion). To present argumentation characteristics on the objective level I will apply the notice of statistical probability and on the subjective level – the notice of psychological probability.

ISBN 83-7431-015-4 ISSN 0860-150X

I. Basic notions

Our considerations will be related to the language J in which argumentations are formulated. J is a fragment of natural language and a set of sentences includes norms, values, questions, orders and decisions. Let WS_J be a set of J-sentences (both open and closed). In WS_J we will distinguish the set $\mathcal{F}$:

$$\mathcal{F} = \{\varphi : \varphi =: \text{'if } A \text{ then } B\text{', where } A, B \in WS_J \text{ and in } A, B \text{ occur } var_1, \ldots, var_n \text{ as free variables of some syntactic category}\}$$

We will use a symbolic representation as follows:

- $A, B, \alpha, \beta, A_i, B_j, \alpha_k$, where $i, j, k \in N \setminus \{0\}$ – variables representing constituents of the set WS_J,
- φ – variable representing constituents of the set F,
- L – variable representing groups of language users (the group may contain only one person),
- L_P, L_O, L_A – constants representing groups of disputants:
  L_P – proponent of argumentation,
  L_O – opponent of argumentation,
  L_A – audience of argumentation,
- S, H, As – pragmatic predicates:
  S – "assume that",
  H – "allow that",
  As – "assert that", "be sure that",
- B – predicate: "belief (in some degree) that".

1. Theory and the set of beliefs

Objective level: *Theory* T formulated in language J will be following ordered pair:

$$T = \langle \mathcal{A}, \vDash \rangle, \text{ where } \mathcal{A} \neq \emptyset,$$

in which $\vDash$ is the relation of inference on the ground of T-theory. The set A is the set consisting of logical axioms AL and specific axioms AT.

Subjective level: Before we formulate a definition of the set of beliefs, we need to specify the notion of strict believing:

(B1) $LB^*\alpha \iff$ a group of language users L ***strictly believes*** that the sentence α is true or right.

The language users believe that the sentence is right if the sentence expresses a norm, value or decision. When the formula α is a sentence in a logical sense, then it is believed to be true.

Here, the notion of believing is based upon the notion of ***subjective*** (***psychological***) ***probability***. The subjective probability of the sentence α for the group L (symbolized as: $P_{sub(L)}(\alpha)$) is the degree in which the group L believes in truthfulness/rightness of α, and is represented by the value from the interval of $(0, 1)$. So now we can note a following relation:

$$LB^*\alpha \iff P_{sub(L)}(\alpha) > 0,5$$

We will distinguish two degrees of strict sentence believing: assertion ($LAs\alpha \iff P_{sub(L)}(\alpha) = 1$) and hypothetical degree ($LH\alpha \iff 0,5 < P_{sub(L)}(\alpha) < 1$). We will name the sentence α, that group of language users L believes in a strict way that it is true/right, a belief of this group. The supposition ($LS\alpha \iff P_{sub(L)}(\alpha) \leq (0,5)$ will not be considered as strict believing, because it does not generate the set of one's beliefs. We will say that it is a nonstrict acceptance and we will symbolize it as B.

Beliefs are not only related to the group of language users, but also to the moment of time in which we consider one's beliefs [Tokarz, 1993, 157–158]. The statement "$LB^*_t\alpha$" will describe such a situation taking place at the moment t that the group L believes in truthfulness/rightness of α. In cases when we do not highlight time we will understand that it occurs at any moment.

Below are stated axioms that further specify the meaning of predicate B^*:

(B2) $LB^*\alpha \Rightarrow \neg LB^*$ (it is not the case that α)

(B3) LB^*(if α then β) $\Rightarrow (LB^*\alpha \Rightarrow LB^*\beta)$

(B4) α is intuitive logical tautology or intuitive rule of reasoning $\Rightarrow LAs\alpha$

(B5) LB^*(α and β) $\Rightarrow LB^*\alpha \wedge LB^*\beta$

(B6) $LB^*\alpha \Rightarrow \alpha$ or it is not the case that α

(B7) At any moments $t_1 \neq t_2 : (LB^*_{t_1}\alpha \wedge \neg LB^*_{t_2}\alpha) \vee (LB^*_{t_1}\alpha \wedge LB^*_{t_2}\alpha)$[1]

Basing on the axiom (B3) we obtain the rule of consequence in beliefs:

(RKB) LB^*(if α then β); $LB^*\alpha$ therefore $LB^*\beta$

[1] [Marciszewski, 1972, 98–99]. Here we consider human beliefs in an idealistic way – we assume that language users are rational. However, some of assumptions indicated by axioms may be not satisfied in the everyday life.

Basing on the notions specified above, ***the set of L-group beliefs***, symbolized as S_L, is:

$$S_L = \langle B_L, \vDash^L_{pragm} \rangle\, [2], \text{ where } B_L \neq \emptyset,$$

in which B_L is the set of common beliefs of language users L: $B_L = \{\alpha : \alpha \in WS_J \wedge LB^*\alpha\}$, and $\vDash^L_{pragm}$ is the pragmatic relation of inference based on the ground of B_L. The set of beliefs S_L is ordinarily called as someone's outlook or philosophy of life.

The important point to notice is that, in contrast to the set $\mathcal{A}$ in theory T, set B_L can include norms, values and decisions.

2. Formulae generally valid and topoi

Objective level: In set $\mathcal{F}$ we will distinguish three separable subsets each of which consists of:

- Formulae generally valid,
- Formulae generally invalid,
- Formulae ungenerally valid.

Definition 1.1

The open formula $\varphi(var_1, \ldots, var_n) \in \mathcal{F}$ is ***formula generally valid*** on the ground of the theory $T \iff$ '$\forall var_1 \ldots \forall var_n(\varphi)$' is true in any model of theory T.

Definition 1.2

The open formula $\varphi(var_1, \ldots, var_n) \in \mathcal{F}$ is ***formula generally invalid*** on the ground of the theory $T \iff$ '$\forall var_1 \ldots \forall var_n$ (it is not the case that φ)' is true in any model of theory T.

Definition 1.3

The open formula $\varphi(var_1, \ldots, var_n) \in \mathcal{F}$ is ***formula ungenerally valid*** on the ground of the theory $T \iff$ '$\exists var_1 \ldots \exists var_n(\varphi)$' and '$\exists var_1 \ldots \exists var_n$ (it is not the case that φ)' are true in any model of theory T.

Subjective level: With regard to some of the formulae from $\mathcal{F}$, it is either absolutely impossible or only possible for the given language user to establish if they are generally valid, invalid or ungenerally valid. Such a situation occurs when the formula refers to the complex range of reality describing relations e.g. from psychology, ethics or social and economic field.

[2] Compare this notion in: [Marciszewski, 1969, 137], [Tokarz, 1993, 157].

The problem arises when a language user is forced to use such formula in reasoning, because e.g. s/he has to solve some dilemma from the area mentioned above. Since s/he does not know objective attributes of the formula, s/he assigns them subjectively. If the person believes that given formula $\varphi \in F$ is generally valid, we will say that this formula is *topoi* for the person[3]. Such subjective assignments, which not necessarily correspond to objective features of formula, sometimes are justified e.g. when individual is forced to make decisions and take actions.

Definition 1.4

The open formula $\varphi(var_1, \ldots, var_n) \in \mathcal{F}$ is ***topoi*** on the ground of S_L $\iff LB^*(\forall var_1 \ldots \forall var_n \varphi)$

When the formula '$\forall var_1 \ldots \forall var_n \varphi$' is believed by the group L to be right, we will say that φ is ***formula generally right*** for group L.

Objective and subjective level: Besides subjective believing in general validity of *topoi*, some of them are indeed valid up to a various degree. These are the *topoi* the generalization of which are sentences in logical meaning. We will distinguish set $\tau \subseteq \mathcal{F}$ that:

$$\tau = \{\varphi : \varphi \in \mathcal{F} \text{ and '}\forall var_1 \ldots \forall var_n(\varphi)\text{' is sentence in logical meaning}\}$$

Topoi from τ can be represented by the specific value of statistical probability. The conditional φ describes events of the kind $\underline{A}$ and events of the kind $\underline{B}$ anytime when free variables $var_1, \ldots, var_n$ are replaced with closed formulae of proper syntactic category. We will symbolize the open sentence φ that describes occurrence of the event of type $\underline{B}$ caused by occurrence of the events of type $\underline{A}$, as $\varphi(\underline{B}/\underline{A})$. The set of events $\underline{A}$ will be called population and symbolized as $\underline{\mathrm{A}}$. The set of events $\underline{B}$, which occur on condition that any of the events $\underline{A}$ occurred, will be symbolized as $\underline{\mathrm{BA}}$. Relative occurrence of events $\underline{B}$ following $\underline{A}$ occurrence, is the ratio of number of events $\underline{B}$, which occurs in a given population, to the number of the population's elements [Ajdukiewicz, 1965, 292]. To introduce a definition we have to assume that occurrence of $\underline{A}$ approaches infinity and that the limit of such infinity exists.

[3] The notion of *topoi* was broadly studied by Aristotle on the ground of rhetorics [Aristotle, 1990], [Aristotle, 2001]. Even though his perspective was taken here as the starting point, my notion of *topoi* differs in some aspects from created by that philosopher.

Definition 1.5

Let $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}}) \in \tau$, where $\varphi =$: 'if A then B'. For any replacement of free variables with closed formulae in φ, the formula A describes event $\underline{A}$ and formula B describes event $\underline{B}$. Let $n(\underline{\mathrm{A}})$ denote number of $\underline{A}$ which occurred and $n(\underline{\mathrm{BA}})$ – number of $\underline{B}$, which occurred if $\underline{A}$ occurred. ***Statistical probability*** of event $\underline{B}$ caused by event $\underline{A}$ on the ground of theory T is:

$$P_{stat(T)}(\underline{\mathrm{B}},\underline{\mathrm{A}}) = \lim_{n(\underline{\mathrm{A}})\to\infty} \frac{n(\underline{\mathrm{BA}})}{n(\underline{\mathrm{A}})}$$ [4]

For our convenience we will speak in short that $P_{stat(T)}(\underline{\mathrm{B}},\underline{\mathrm{A}})$ is the probability of relation described by the open sentence $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}})$.

By considering definition (1.5) we obtain:

(1.1) Let $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}}) =$: 'if A then B', where for any replacement of free variables with closed formulae in φ, the antecedent A describes event $\underline{A}$ and the consequent B describes event $\underline{B}$. Then on the ground of theory T:
- (i) *Topoi* $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}}) \in \tau$ is generally valid $(T) \iff P_{stat(T)}(\underline{\mathrm{B}}/\underline{\mathrm{A}}) = 1$
- (ii) *Topoi* $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}}) \in \tau$ is generally invalid $(T) \iff P_{stat(T)}(\underline{\mathrm{B}}/\underline{\mathrm{A}}) = 0$
- (iii) *Topoi* $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}}) \in \tau$ is probable $(T) \iff 0 < P_{stat(T)}(\underline{\mathrm{B}}/\underline{\mathrm{A}}) < 1$

If the value of statistical probability of the relation described by probable *topoi* is close to 1 (approaching value 1), we will say that such *topoi* is highly probable [Ajdukiewicz, 1965, 336], [Luszniewicz, 1994, 23]. Such *topoi* describe statistical relations establishing statistical laws of the theory T [Szaniawski, 1994, 18–28]. Highly probable *topoi* refer to the complex reality, in which events $\underline{B}$ are influenced not only by events $\underline{A}$, but also by accidental, unexpected ones [Luszniewicz, 1994, 21].

Definition 1.6

$\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}}) \in \tau$ is ***highly probable*** $(T) \iff P\{1-P_{stat(T)}(\underline{\mathrm{B}},\underline{\mathrm{A}}) < \varepsilon\} = 1$, where ε is any small number, $\varepsilon > 0$ and P is classical probability.

3. Pragmatic entailment based on *topoi*

Let $Roz(J)$ be the set of reasonings formulated in language J, S_J – the set of closed sentences of the set WS_J and Fin – the set of finite sets.

Definition 1.7

$(\beta_k) \in Roz(J) \iff \{\beta_1, \beta_2, \ldots, \beta_k\} \in Fin$, $\{\beta_1, \beta_2, \ldots, \beta_k\} \subset S_J$ and β_k is obtained from the previous sentences in the sequence $\beta_1, \beta_2, \ldots, \beta_{k-1}$ basing on certain inference ground.

[4] [Wilczyński, 1980, 39], [Szaniawski, 1994, 27], [Mortimer, 1982, 44]

In further considerations the sentences: $\beta_1, \beta_2, \ldots, \beta_{k-1}$, will be symbolized interchangeably as: $P_1, P_2, \ldots, P_n$, and the last sentence β_k as: W. We will write: P, when we indicate the set of premises $P_1, \ldots, P_n$.

I will introduce inference ground that is different from semantic entailment (which is considered on **objective level**)[5]: pragmatic entailment based on *topoi* (which has to be considered on **subjective level**). Let "$\{A_1, \ldots, A_n\} \vDash^{L,\varphi}_{pragm} \beta$" mean that the set of sentences $\{A_1, \ldots, A_n\}$ basing on *topoi* φ pragmatically entails sentence β on the ground of S_L, i.e. on the ground of the set of L-group beliefs.

Definition 1.8

Let $S_L = \langle B_L, \vDash^{L}_{pragm} \rangle$ and let $\varphi \in \mathcal{F}$ be *topoi* for the group L. $\{A_1, \ldots, A_n\} \vDash^{L,\varphi}_{pragm} \beta \iff$ conditional 'if A_1 and ... and A_n then β' is obtained from *topoi* φ on the ground of set S_L.

Topoi φ stated in the definition above will be called a ground of pragmatic entailment.

Each inference makes the sentence β inherit some features from the elements of the set $X = \{A_1, \ldots, A_n\}$. In the semantic entailment a sentence attribute of being true in any model of theory T is inherited, and in the pragmatic entailment – a sentence attribute of being believed as truth/right by the group L is inherited.

Let B^*_L be predicate of being strictly believed as true/right sentence by the group L. When the sentences from the set X entails, according to the group L, the sentence β and simultaneously the group L believes all the elements of X, then the group L will believe sentence β. ***The rule of inheritance of believing*** in truthfulness/rightness for pragmatic entailment can be stated as follows:

(1.2) $X \vDash^{L,\varphi}_{pragm} \beta \Rightarrow$ if $B^*_L(X)$ then $B^*_L(\beta)$.

In the next paragraph we will formulate ***the rule of inheritance of truthfulness***.

[5] Following Ajdukiewicz perspective I understand semantic entailment in broad sense, i.e. it may be founded not only upon logical general schemes, but also on generally valid ones, taken from other scientific theories [Ajdukiewicz, 1965, 99].

4. Pragmatic reasoning based on *topoi*

Subjective level: Let $Roz_{\varphi}(J)$ be the set of reasonings in language J based on *topoi* φ.

Definition 1.9

Let $\varphi \in \mathcal{F}$ be *topoi* on the ground of the set of beliefs S_L:

$$(\beta_k) \in Roz_{\varphi}(J) \iff (\beta_k) \in Roz(J) \text{ and } \exists(L \neq \emptyset)[\{\beta_1, \ldots, \beta_{k-1}\} \vDash^{L,\varphi}_{pragm} \beta_k]$$

The ground of pragmatical entailment (*topoi* φ) will also be called a ground of reasoning.

Following (1.2) and definitions (1.8) and (1.9): if $P_{sub(L)}(P) > 0{,}5$ and $P_{sub(L)}(\forall var_1 \ldots \forall var_n \varphi) > 0{,}5$ then $P_{sub(L)}(W) > 0{,}5$[6]. Hence in reasoning $Roz_{\varphi}(J)$ believing is inherited by its conclusion from the premises.

Objective and subjective level: Besides subjective believing, in reasonings based on *topoi* $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}}) \in \tau$, objective statistical probability can be assigned to the conclusion. This is the probability, which indicates how often the conclusion, derived from true premises and believed *topoi* φ, is guaranteed to be true in any model of theory T.

Let us consider the theory T. When the event $\underline{B}$ occurs on condition that the event $\underline{A}$ occurred, then the consequent B and the antecedent A of conditional obtained from φ (B and A describe $\underline{B}$ and $\underline{A}$), both will be true in any model of T. As long as the occurrence of the event $\underline{A}$ is not followed by the event $\underline{B}$, the sentence obtained from *topoi* $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}})$ will be false in any model of the theory T (A will be true and B will be false). While the occurrence of the events $\underline{B}$ is caused by the event $\underline{A}$, the consequent of conditional obtained from φ will always be true at the time, when true is antecedent of this sentence. In the reasoning based on *topoi* $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}})$ the sentence obtained from the consequent of φ is the conclusion of this reasoning and the sentences obtained from the antecedent of φ are its premises.

We will now formulate ***the rule of inheritance of truthfulness***. It determines how often the conclusion inherits truthfulness from the premises in reasonings based on specific *topoi*.

(1.3) Let P be the set of premises and W be the conclusion of the reasoning based on *topoi* $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}})$ on the ground of T. Let ζ be specified value from interval of (0,1). Let $\underline{\mathrm{W}}_1$ be such an event when W is true, and $\underline{\mathrm{P}}_1$ – such an event when each premise of the set P is true. A statistical

[6] Compare with [Ajdukiewicz, 1966, 194].

probability of obtaining true conclusion in the reasoning $Roz_\varphi(J)$ on the ground of T from true premises, is:

$$\text{If } P_{stat(T)}(\underline{\text{B}}/\underline{\text{A}}) = \zeta \text{ then } P_{stat(T)}(\underline{\text{W}}_1/\underline{\text{P}}_1) = \zeta.$$

Following (1.3) and (1.1) we can specify some relations between "generality" of *topoi* φ and "deductiveness" of reasoning based on this *topoi*:

(i) If *topoi* φ is generally valid in T then the reasoning $Roz_\varphi(J)$ is deductive in T.

(ii) If *topoi* φ is generally invalid in T then the conclusion of reasoning $Roz_\varphi(J)$ never inherits in T truthfulness from its premises.

(iii) If *topoi* φ is probable in T then $Roz_\varphi(J)$ from true premises sometimes leads in T to false conclusions and sometimes to true ones.

(iv) If *topoi* φ is highly probable in T then $Roz_\varphi(J)$ almost always leads in T from true premises to true conclusions.

Here a question arises: why do we use such schemes in our thinking, which are only probable and not general? This is due to the complexity that characterizes the reality fields to which most of everyday reasonings is referred. In such cases, to achieve the highest possible probability, the reasoning person should choose highly probable *topoi*, which describe statistical relations. On the other hand, independently from statistical probability established by φ, there is subjective level of human beliefs. Thus as group L accepts scheme φ as *topoi* then each person from group L believes that any conclusion of reasoning based on this *topoi* φ and true premises will also be true (following the rule of inheritance of believing (1.2)).

Hence in case of any reasoning based on *topoi* (I classify argumentation as this type of reasoning), one has to consider two levels: (1) objective level referring to statistical probability of obtaining the true conclusion from *topoi* and true premises, and (2) subjective level referring to psychological probability of obtaining believed conclusion, as the consequence of the person believing in truthfulness of premises and *topoi*.

A Polish researcher Teresa Hołówka claims that everyday subjective generalizations are the result of incomplete perception of complex reality. It makes people create simplified representations of this reality. In a representation like that the objects are classified and various relations are determined among sets created in this way [Hołówka, 1998]. Basing on any generalization it is possible to formulate the *topoi* that can be applied as the foundation of reasoning. This is why people are able to act in such complexity. Despite of common lack of scientific knowledge concerning statistical probability, the generalizations are seldom "built" groundlessly. So here we can put the question: if the scientific (statistical) methods cannot determine the value of objective probability of the relations described by *topoi* then how it is possible for people to distinguish which *topoi* are highly probable and which lead to false conclusion too often? It is not a purpose of this article to solve such a problem, nevertheless I will suggest a possible answer. It seems that people have though imperfect but still quite effective methods to determine statistical probability. Otherwise if one could assign only subjective probability to *topoi* of any objective (statistical) probability then reasonings based on some of these schemes would lead to believe false or even absurd conclusions (following the rule of inheritance of believing for pragmatic entailment (1.2)). This, as a result, would lead the person to wrong decisions and inefficient actions. However, people who reason on the ground of various *topoi* often make right decisions and effective measures. Thus, we may agree that in many circumstances individuals know at least approximate statistical probability of the relations described by *topoi*, and especially they are able to recognize the highly probable schemes. This knowledge may originate from the generation's wisdom and from "evolutional" adapting processes [Aristotle, 1996, 1143b]. Observations frequently made can be generalized into laws that in majority are statistical, but all laws are treated the same as general. Because each *topoi* is believed as generally valid then it is formulated in such a way as if it, indeed, was general. That is why one says: "<u>Every</u>thing happens because of God's will", "<u>Every</u> man is jealous about their wives", "<u>Every</u> mother loves her children". Afterward, statistical laws are verified effectively during the life of later generations. The schemes that lead to false conclusions too often are eliminated – what can be compared to the "evolutional" adapting processes. It is possible that such processes eliminate not only inefficient patterns, but, in some sense, also individuals that tend to use such schemes. Since the person applies low probable *topoi* in her/his thinking, s/he acts inefficiently, what in effect – makes her/him badly adapted to life.

II. New definition of argumentation

In argumentation the moment of persuasion is substantial. If the sequence of sentences is to be an argumentation, the existence of ***parties to a dispute*** is necessary. We will distinguish three groups of disputants: proponent L_P which is the group of language users that persuades to his/their thesis, opponent L_O which is the group that rejects the arguments of proponent, and audience L_A which is persuaded to believe in truthfulness/rightness of proponent thesis. The necessity of such division is easy to observe in some cases from field of social discourse e.g. in law and politics arguments. In law disputes interchanging parts of proponent and opponent are played by a lawyer and a prosecutor. The audience is a judge/jury. The goal of the proponent is to persuade him/them to the thesis. The audience is not active in discussion i.e. its role is restricted to listening and bringing final verdict on speakers' opinions. The proponent does not intend to persuade his opponent. One can even say that their holding of the positions on the opposite sides from the beginning to the end of law-argumentation is essential for court trials. They aim only to convince passive side of argumentation – the jury or judge. Such types of discussion indicate that opponent and audience are different parts of argumentation even though those two sides may be represented by the same group. In literature this distinction is emphasized when considering *argumentum ad auditores* [Pszczołowski, 1974, 258].

In reference to above statements, following situations may appear in discussion:

- $L_P = L_O = L_A$ when a person tries to convince himself/herself [Perelman, 1984, 147],
- $L_O = L_A$ when an opponent is persuaded,
- $L_O \neq L_A$ when a proponent does not intend to convince his opponent, but audience, which does not participate directly in dialogue.

Someone's beliefs can be influenced in many different ways. For instance, persuasion may aim at audience emotions like in *argumentum ad baculum, ad crumenam or ad misericordiam.* I will not consider those cases as argumentation, which in turn I will understand as the sequence of sentences among which there are thesis and arguments justifying it. Let $Arg(J)$ be the set of simple argumentations in language J.

Definition 2.1

$(\beta_k) \in Arg(J) \iff \exists\varphi[(\beta_k) \in Roz_\varphi(J)]$ and $\exists(L_P \neq \emptyset)\exists(L_O \neq \emptyset)$ $\exists(L_A \neq \emptyset)$ [proponent L_P presents arguments $\beta_1, \beta_2, \ldots, \beta_{k-1}$ against the opponent L_O to convince audience L_A to believe in truthfulness/rightness of thesis β_k], where:

(i) L_P B β_k,

(ii) $\neg L_O$ B β_k,

(iii) L_P presents $\beta_1, \beta_2, \ldots, \beta_{k-1}$ that:

- L_P B (β_1 and … and β_{k-1}) and
- $\{\beta_1, \beta_2, \ldots, \beta_{k-1}\} \vDash^{Lp,\varphi}_{pragm} \beta_k$,

(iv) the objective of L_P is that: $L_A B^* \beta_k$.

The set of argumentation is classified in literature according to many different criteria. These classifications aim to organize a very complicated scope of various persuasion methods. Most frequently indicated are honest and dishonest arguments (or in other words: rhetorical and eristic). Following Aristotle, I will specify the rhetorical argumentation as the one fulfilling three conditions: *logos*, *ethos* and *pathos*[7].

Definition 2.2

(β_k) is ***rhetorical argumentation*** based on $\varphi \iff (\beta_k) \in Arg(J)$ and when (β_k) fulfils following conditions:

(i) *logos*: φ is generally valid or highly probable *topoi*,

(ii) *ethos*: $P_{sub(Lp)}(\forall var_1 \ldots \forall var_n \varphi) > 0{,}5$ and $P_{sub(Lp)}(\beta_1$ and $\ldots$ and $\beta_k) > 0{,}5$,

(iii) *pathos*: argumentation (β_k) is built according to rules of stylistics.

The discussion satisfies the condition *logos* when the reasoning is deductive or leads to the false conclusion very seldom. The schemes highly probable may be selected in honest arguments only when general *topoi* are not available. This way the probability of obtaining true conclusion, i.e. $P_{stat(T)}(\underline{\mathrm{W}}_1, \underline{\mathrm{P}}_1)$, is the highest one can reach. Otherwise less probable *topoi* would lead to false conclusion too often and argumentation would become unreliable.

The conditions (i) and (iii) of the definition (2.1) require that proponent only expresses the elements of (β_k), but not necessarily strictly believes in truthfulness/rightness of these sentences, since it is sufficient that s/he does believe them in a nonstrict way. However, if the discussion is to be honest, it has to fulfill condition *ethos*, which means that proponent has to strictly believe sentences in (β_k). Otherwise, as the set of her/his beliefs does not contain these formulae $(\neg L_P B^* \alpha \iff \alpha \notin \mathrm{B}_{Lp})$, s/he convinces others to believe in something using premises that s/he does not believe her/himself.

Argumentation that does not fulfill at least one of conditions (i)–(iii) in definition (2.2) will be called ***eristic argumentation***. If argument does not meet condition *logos* then the inference foundation is invalid or low probable *topoi*. A proponent is either unaware that statistical probability of the described relation is too low or s/he deliberately uses "catchy", but low probable *topoi*. In second case the argumentation is unsatisfactory for not

[7] According to Aristotle, when considering any argumentation one should examine relation of semantic entailment between premises and its conclusion (*logos*), credibility of proponent (*ethos*) and mood of the audience that is influenced by stylistic methods used during the persuasion (*pathos*) [Aristotle, 2001, 1356a], [Nieznański, 2000, 118].

only *logos*, but also *ethos*. The condition *ethos* is not fulfilled when L_P just assumes (believes in supposition degree) premises, conclusion or the inference ground of given argumentation. S/he may just take into account if the audience strictly believes premises and *topoi* regardless of what proponent's beliefs really are. Anyway, it will be warranted to her/him that the audience will strictly believe the proponent's thesis (according to the rule of inheritance of believing). However believing the proponent's thesis is essential both in rhetorical and eristic arguments, it is achieved in the first type of persuasion by the fulfillment of all the conditions: *logos*, *ethos* and *pathos*, whereas in the second type – by whichever way.

Now I wish to present argumentation on two independent levels. In the first paragraph I will consider its attributes on objective level and I will attempt to determine when a discussion leads to true conclusions. In the second paragraph I intend to study argument features on subjective level. I will investigate when a discussion leads a proponent to achieve her/his main goal i.e. to persuade the audience. In the last paragraph I will describe the arguments most common in everyday life.

1. "Deductiveness" of inference schemes in argumentation

Argumentation is the reasoning based on *topoi* i.e. on the scheme, which is believed as generally valid/right. This is why *topoi* are formulated as general sentences. As a result, the pragmatic entailment seems to be the semantic entailment (the foundation of reasoning is believed as general even though actually it is not). And some of the language users may share the impression that a given argumentation is deductive.

Regardless of subjective human knowledge concerning "generality" of schemes, the formulae from set τ are objectively represented by the specific degree of this "generality". When *topoi* is a generally valid scheme then argumentation is deductive. In turn, if the discussion is based upon *topoi* generally invalid then the statistical probability of relation described by *topoi* equals 0, i.e. $P_{stat(T)}(\underline{\mathrm{B}},\underline{\mathrm{A}}) = 0$. Following (1.3), we obtain that: $P_{stat(T)}(\underline{\mathrm{W}}_1, \underline{\mathrm{P}}_1) = 0$. Thus, in case of each argumentation based on such *topoi*, its conclusion will not inherit truthfulness from the premises.

In everyday life the most frequent arguments are those which are founded upon probable *topoi*. For such schemes $\varphi(\underline{\mathrm{B}}/\underline{\mathrm{A}})$ we have: $0 < P_{stat(T)}(\underline{\mathrm{B}},\underline{\mathrm{A}}) < 1$. Hence from (1.3) we obtain: $0 < P_{stat(T)}(\underline{\mathrm{W}}_1, \underline{\mathrm{P}}_1) < 1$. If the inference ground is *topoi* highly probable even though the argumentation is not deductive and does not always lead to true conclusions, it happens almost always.

Theorem 1.

Let (β_k) be argumentation based on *topoi* φ and let $\nu(\alpha)$ represent a logical value of the sentence α. If $P_{stat(T)}(\varphi) = \zeta$ and $\nu(\beta_1) = 1$ and ... and $\nu(\beta_{k-1}) = 1$, then $P_{stat(T)}(\nu(\beta_k) = 1) = \zeta$.

The theorem (1) is the consequence of the rule of truthfulness inheritance formulated in (1.3). When argumentation is based upon *topoi* φ and true premises: $\beta_1, \ldots, \beta_{k-1}$, then statistical probability of obtaining true conclusion in this argumentation is the same as probability of relation described by scheme φ on which we "built" our argumentation. Thus, when we argue, the higher the statistical probability of relation described by *topoi* is, the higher the warranty of obtaining true conclusions is. If our **goal in discussion is objective**, i.e. we aim to "find the truth", we should select *topoi* with the highest statistical probability that is available for the subject under dispute.

There are a number of points to observe:

– As the audience does not know the degree of "deductiveness" of argumentation, the proponent may deliberately use *topoi* generally invalid or low probable taking advantage of their lack of knowledge. This kind of persuasion is called in literature *argumentum ad ignorantiam*.
– To satisfy the condition *logos*, the proponent aims to approach minimal probability of obtaining the false conclusion.
– Many arguments are based on *topoi*, which are not elements of set τ. These schemes have a following form: *if A then B*, where at least one sentence obtained from A or B expresses the norm, value or decision. Such *topoi* can be believed as formula generally right on the ground of set of L-groups beliefs. In those arguments the conclusion inherits the attribute of strict believing in rightness from its premises.

Summarizing – argumentation is a reasoning in which from true premises it is guaranteed to obtain:

(i) always true conclusions (these argumentations are deductive),
(ii) sometimes true, sometimes false conclusions,
(iii) never true conclusions or
(iv) conclusions that cannot be considered as true/false – this is when argumentation is based on *topoi* believed as generally right.

What is characteristic of the actual persuasion is that the most frequent argumentations are the reasonings of type (ii) and (iv), since in everyday life we discuss about what is uncertain or what concerns values. It differs arguments from many other reasonings, especially the ones from scientific theories.

2. The efficiency of argumentation

The second substantial difference between arguments and other reasonings is the degree to which language users participate in them. The influence of such "participation" like e.g. disagreement over the conclusion by given individuals, is not substantial in deduction at all. The only sufficient condition here for obtaining true conclusion is truthfulness of premises and semantic entailment, no matter who believes it or not[8].

We observe an opposite situation in persuasion. The reasoning is not an argumentation, when one of the following is missing: proponent (it is indicated by (i) and (iii) of definition (2.1)), opponent (ii) and audience (iv). We say that argumentation is efficient when a proponent reaches her/his goal and persuades the audience in her/his favor.

Definition 2.3

(β_k) is ***efficient simple argumentation*** for the audience $L_A \iff (\beta_k) \in Arg(J) \wedge L_A B^* \beta_k$.

To achieve efficiency of argumentation, it is neither necessary nor sufficient that (a) premises are true and (b) premises semantically entail conclusion (like in deduction), but that (1) premises are believed by audience and (2) premises pragmatically (according to audience) entail conclusion. The first two conditions (a and b) are not sufficient when the individuals do not know that the *topoi* is generally valid, so they will not be aware that argumentation based on it is deductive. As a result, it may happen that the audience will not believe the thesis that is really true. "The formal correctness" of argumentation is not the necessary condition of its efficiency either, because the audience may believe the conclusion of argumentation that is not deductive (moreover, a thesis can be false). That is why we quite often observe in everyday life that people believe as true/right the conclusions, which actually are false/wrong. So the efficiency of discussion depends more on the participation of language users than on truthfulness and its "deductiveness" [Perelman, 1984, 147], [Korolko, 1990, 40].

Theorem 2.

If $P_{sub(LA)}(\forall var_1 \ldots \forall var_n \varphi) >$ 0,5 and $P_{sub(LA)}(\beta_1$ and ... and $\beta_{k-1}) >$ 0,5, then the argumentation (β_k) based on *topoi* φ is efficient for the audience L_A.

[8] Perelman, for instance, says that truth is impersonal [Perelman, 1984, 147].

It is easy to show that theorem (2) is the consequence of definition (2.3) and the rule of inheritance of believing (1.2). To achieve efficiency of simple discussion with regard to the given audience, it is sufficient that the audience strictly believes inference ground φ as generally valid (i.e. as *topoi*) and premises as true/right. When following (1.2), we obtain that the group will believe the thesis of the presented argumentation and this finally, according to the definition (2.3), will mean that argumentation will become efficient. If our **goal in discussion is subjective**, i.e. we aim to persuade audience, then we should select as a scheme φ and premises the sentences (describing subject under dispute) with the highest psychological probability for the audience.

The simple argumentation is ***inefficient*** when audience finds faults in this argumentation[9]. When $L_O \neq L_A$ then the audience finds the fault either on their own or influenced by the opponent who indicates e.g. falseness of specific premise.

In argumentation that contains $P_1, \ldots, P_n$, W and is based on φ, the audience L_A does not believe W when:

(i) $\neg L_A B^* P_i$, where $i \in \{1, \ldots, n\}$, because:
- $L_A B^*$(it is not the case that P_i)
- $L_A B^*$(P_i is not well–founded)

(ii) $\neg(\{P_1, P_2, \ldots, P_n\} \vDash^{LA,\varphi}_{pragm} W)$, because:
- $L_A B^*$(it is not the case that $\forall var_1 \ldots \forall var_m \varphi$)
- $L_A B^*$('$\forall var_1 \ldots \forall var_m \varphi$' is not well-founded)

3. Complex argumentations

The simple argumentation is inefficient if the audience puts forward at least one of the counterplea mentioned above. However, in everyday life the persuasion may be continued. When the next reasoning is presented, the argumentation becomes complex.

In complex discussion a first simple argumentation may be followed by the next one when: either the proponent continues persuasion or the opponent presents her/his own argumentation, in which $\neg\beta_k$ is a conclusion (if a conclusion of previous argumentation was β_k).

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[9] In literature it is called counterplea for the argumentation [Nieznański, 2000, 117], [Łuszczewska-Romahnowa, 1966, 164].

Thus ***the complex arguments*** can have following forms:

- arguments with invariable proponent or
- arguments with variable proponent.

A. Complex argumentations with invariable proponent

To achieve efficiency, the proponent may present a second simple argumentation in which: (1) the conclusion is the scheme φ or the premise, that audience did not believe in first argumentation, or (2) the conclusion stays unchanged, but the new argumentation is based upon other scheme or other premises.

Example 2.1

Let us assume that at the moment t_1, in which the first simple argumentation is presented, the audience believes premises of reasoning. However, it does not believe inference ground as generally valid by claiming that it is insufficiently founded:

(Arg1) $\{P_1, \ldots, P_n\} \vDash^{Lp,\varphi_1}_{pragm} W$,

and $L_A B^*(P_1$ and ... and $P_n)$ and $L_A B^*_{t_1}$(‘$\forall var_1 \ldots \forall var_a \varphi_1$’ is ill-founded).

Because $L_A B^*_{t_1}$(‘$\forall var_1 \ldots \forall var_a \varphi_1$’ is ill-founded) then $\neg L_A B^*_{t_1} W$.
Thus, at the moment t_2 a proponent presents premises $Q_1, \ldots, Q_k$, that according to him entail as conclusion the sentence: $\forall var_1 \ldots \forall var_a \varphi_1$. S/he selects now as the inference ground a new scheme: $\varphi_2(var_1, \ldots, var_b)$, in which $var_1, \ldots, var_b$ are free variables of some syntactic category.

(Arg2) $\{Q_1, \ldots, Q_k\} \vDash^{Lp,\varphi_2}_{pragm} \forall var_1 \ldots \forall var_a \varphi_1$.

Let us assume also that:

$$L_A B^*_{t_2}(Q_1 \text{ and ... and } Q_k) \text{ and } \{Q_1, \ldots, Q_k\} \vDash^{LA,\varphi_2,t_2}_{pragm} \forall var_1 \ldots \forall var_a \varphi_1.$$

Thus following the rule of inheritance of believing (1.2), we obtain: $L_A B^*_{t_2}(\forall var_1 \ldots \forall var_a \varphi_1)$, therefore: $\{P_1, \ldots, P_n\} \vDash^{LA,\varphi_1,t_2}_{pragm} W$. And because: $L_A B^*(P_1$ and ... and $P_n)$ then: $L_A B^*_{t_2} W$.
Hence finally at the moment t_2, the complex argumentation (Arg) (that contains (Arg1) and (Arg2)) becomes efficient, because $L_A B^*_{t_2} W$.

Example 2.2

Let us assume now that the first simple argumentation is the same as in the above example. However, this time to persuade the audience at the moment t_2, the proponent presents the second argumentation based upon the other sentence: $\varphi_3(var_1, \ldots, var_c)$, where $var_1, \ldots, var_c$ are free variables of some syntactic category:

(Arg3) $\{R_1, \ldots, R_m\} \vDash^{Lp,\varphi 3}_{pragm} W$.

Let us also assume that

$$L_A B^*_{t_2}(R_1 \text{ and ... and } R_m) \text{ and } \{R_1, \ldots, R_m\} \vDash^{LA,\varphi 3,t_2}_{pragm} W.$$

Following the rule of belief inheritance (1.2) we obtain: $L_A B^*_{t_2} W$.
Thus, if the audience believes premises and inference ground of (Arg3) then the complex argumentation becomes effective in t_2, even though (Arg1), that led to the same conclusion as (Arg3), was inefficient.

B. Complex argumentations with variable proponent

Argumentation with variable proponent always includes at least one counterargumentation. Let us consider the following example:

Example 2.3
Let t_1 be the final moment of (Arg):

(Arg) $\{P_1, \ldots, P_n\} \vDash^{Lp1,\varphi_1}_{pragm} W_1$,

and $L_A B^*_{t_1} W_1$.

From the definition (2.1) we know that: $\neg L_O B^*(W_1)$. If the opponent does not want the audience to keep believing the proponent's thesis W_1 then s/he may present a counterargumentation with conclusion W_2 which is: W_2 = 'it is not the case that W_1'. The following simple argumentation with the inference foundation: $\varphi_2(var_1, \ldots, var_b)$, is now presented:

(KArg) $\{Q_1, \ldots, Q_m\} \vDash^{Lp2,\varphi_2}_{pragm} W_2$,

Hence, the opponent and the proponent of previous argumentation "changed their parts with each other". It should be noted that: $L_{P2} = L_{O1}$, $L_{O2} = L_{P1}$, $L_{A2} = L_{A1}$, where L_{P2}, L_{O2} and L_{A2} are participants of counterargumentation (KArg) and L_{P1}, L_{O1} and L_{A1} are participants of (Arg).

Let us assume now that (KArg) is efficient. Thus at its final moment t_2: $L_A B^*_{t_2}$(it is not the case that W_1). Following the axiom (B2) and rule (RKB) we obtain: $\neg L_A B^*_{t_2}$(it is not the case that it is not the case that W_1). Assuming that: $\neg\neg\alpha \Rightarrow \alpha$ is intuitive tautology and following (B4) and (RKB) we obtain that: $\neg L_A B^*_{t_2} W_1$.

In consequence at the moment t_2, argument (Arg) is inefficient, because counterargumentation (KArg) becomes efficient.

C. Efficiency of complex argumentations

Basing on the above examples we will now formulate the definition of efficiency of complex discussions:

Definition 2.4

Let complex argumentation $\mathrm{Arg}_{\mathrm{cplx}}$ be presented in time-period, where t_1 is the beginning and t_j is the end of this period $(1 < j, j \in N)$. Let W_{Lp} be the conclusion of simple argumentation in which L_P is proponent and W_{Lp} is not a premise or an inference ground of any other simple argumentation in $\mathrm{Arg}_{\mathrm{cplx}}$.

Complex argumentation Arg_{cplx} ***is efficient*** for the proponent L_P and audience $L_A \iff L_A B^*_{t_j} W_{Lp}$.

In the example (2.1) the complex discussion is efficient for the proponent L_P and the audience L_A, because even though $\neg L_A B^*_{t_1} W$, but $L_A B^*_{t_2} W$, and it was t_2 that was the final moment of complex argument. We do not consider the efficiency with regard to the conclusion of simple argumentation (Arg2): $\forall var_1 \ldots \forall var_a \varphi_1$, because it was the inference ground of other argumentation i.e. (Arg1). In the example (2.2) complex argumentation is efficient for L_P and L_A, because $L_A B^*_{t_2} W$, although in this case too first simple argumentation was inefficient and the audience did not believe the thesis W in the beginning, i.e. $\neg L_A B^*_{t_1} W$. In the example (2.3) complex argumentation is efficient for the proponent L_{P2} and audience L_A, because in the end (in t_2) the audience believed the conclusion of argumentation in which L_{P2} was proponent, i.e. $L_A B^*_{t_2} W_2$. While W_1 was believed by the audience, efficiency could not be compared with this sentence, because t_1 was not the final moment of complex discussion. And in moment t_2 we have: $\neg L_A B^*_{t_2} W_1$. Thus, however, the first simple argumentation was efficient for proponent L_{P1}, the whole discussion was "won" by the proponent of counterargumentation, i.e. L_{P2}.

The argument efficiency is related to the audience's set of beliefs, which in turn is related to time. In example (2.1) in the beginning the audience did not believe φ_1 as generally valid, i.e. did not believe the inference ground of the first simple argumentation (Arg1): $\neg L_A B^*_{t_1}(\forall var_1 \ldots \forall var_a \varphi_1)$. And because: $B_{L,t} = \{\alpha : LB^*_t \alpha\}$ then: '$\forall var_1 \ldots \forall var_a \varphi_1$' $\notin B_{LA,t_1}$. Thus, following the definition (1.8) on the ground of the audience's set of beliefs at the moment t_1, the premises P_1 and ... and P_n do not pragmatically entail the conclusion W: $\neg(\{P_1, \ldots, P_n\} \vDash^{LA,\varphi_1,t_1}_{pragm} W)$. However, in (Arg2) the conclusion '$\forall var_1 \ldots \forall var_a \varphi_1$' was pragmatically entailed on the ground of S_{LA,t_1}. In this way at the moment t_2 the formula '$\forall var_1 \ldots \forall var_a \varphi_1$' was added to the set of audience's beliefs i.e.: $S_{LA,t_2} = \langle B_{LA,t_1} \cup \{\text{'}\forall var_1 \ldots \forall var_a \varphi_1\text{'}\}, \vDash^{LA,t_2}_{pragm} \rangle$. So at the moment t_2 it was possible to derive the sentence W on the ground of the set of L_A-beliefs, i.e. if '$\forall var_1 \ldots \forall var_a \varphi_1$' $\in B_{LA,t_2}$, then $\{P_1, \ldots, P_n\} \vDash^{LA,\varphi_1,t_2}_{pragm} W$. And because ($P_1$ and ... and P_n) $\in B_{LA,t_2}$, then $W \in B_{LA,t_2}$.

In the above examples we assumed the simplification that the audience believes the conclusion in the second step (in the second reasoning). In everyday persuasion the discussion may be much more complex. The statements formulated above can be, of course, generalized on any long sequence of simple argumentations that we can observe in day-to-day life.

Once we become interested in the issue of social discourse, we ought to consider the two levels. As long as our main concern is a victory in dispute, we stay on the subjective level. In order to convince the audience to believe our thesis, we have to select for our argumentation such premises and inference ground, which are believed by this audience (i.e. with the highest available psychological probability for this group). While we aim to "find the truth" by discussing with someone, we are on objective level. Our only concern then should be to provide our argumentation with premises that are true and inference scheme, which is general or at least highly probable (i.e. with the highest available statistical probability). In such case we may neglect disputants beliefs, in particular if selected scheme is *topoi* for our audience or it is not. However, as our purpose is both convincing and cognition, we have to connect these levels. Thus, it is necessary that we select: (1) scheme **subjectively** believed by the audience as *topoi*, which **objectively** is generally valid/high probable, and (2) premises **subjectively** believed by the audience and which **objectively** are true. Since we are aware of the presence of these two independent levels in discussion, it makes us understand better the principles governing the argumentation and furthermore helps us to achieve both goals of high importance in everyday persuasion.

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