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HISTORIC AND CONTEMPORARY CONTROVERSIES ON THE CONCEPT OF UTILITY[1]

Abstract. The concept of utility was first introduced in the demand theory, together with the assumption of decreasing marginal utility. Later, postulates of consumer rational preferences and their violations were debated. The classic concept of utility, which was based exclusively on consumer preferences has recently been questioned by Kahneman and his colleagues, who introduced the concept of experienced utility. In the context of decisions under risk, for a long time the model of maximizing the expected utility has been accepted. However, some hardship to explain paradoxes were also debated. An alternetive model, known as the prospect theory, includes basic changes concerning the valuation of outcomes and probabilities by the decision maker. This "psychological" theory has turned out to have several appealing implications for various areas of economics.

1. Demand theory, the assumption of decreasing marginal utility, and postulates of consumer rational preferences

Utility is a term which, for a long, time has been used in connection with the desire to possess goods or services. For economists utility has been a subjective measure of value which differs from the value of goods expressed as a price on the market. For example, Adam Smith in his book "The Wealth of Nations" (1776/1986) states that the value of goods expressed by their price is distinct from their value in the sense of utility. Water, for example, has high utility but a low price. Diamonds, on the contrary, have low utility but a high price. Ricardo put it similarly in his book "Principles of Political

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Economy and Taxation" (1817/1957). For him utility is not measured by the price on the market. However, goods which are exchangeable must be to some extent useful. Useless things are not exchanged.

Classical economists could not explain the full relationship between utility and demand on the market because explanation required analysis of marginal values. Many economists in the mid-19th century begun such analyses (eg. H. H. Gossen, W. S. Jevons, C. Menger, A. Marshall). (Stankiewicz 2000). For them consumer utility was a function which assigns values $u(y)$ for commodity bundles y. The term marginal utility is then an increment of utility as the result of consuming the successive unit of a commodity.

Economists noticed that consumption of each successive unit of a commodity brings us less utility. This is the law of decreasing marginal utility which has been till now one of the fundamentals of economics.
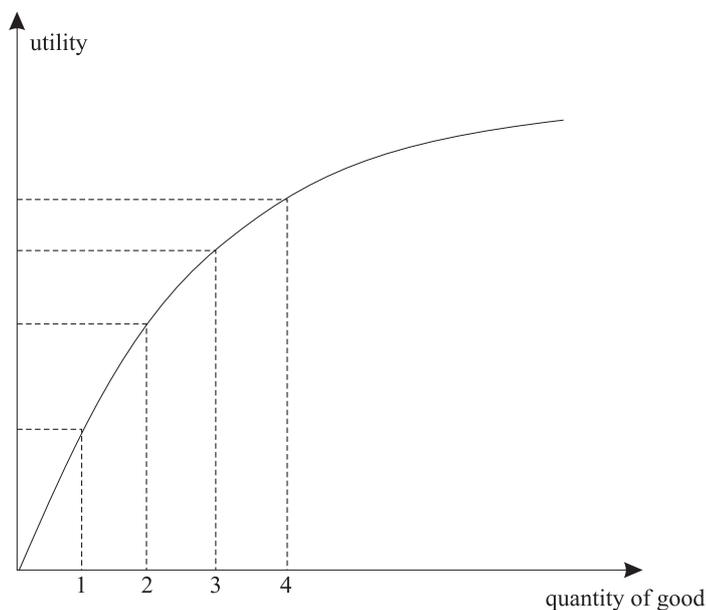

Fig. 1. Consumer utility function for one commodity

Utility then became a measure of consumer satisfaction, a measurable psychological magnitude. Total consumer utility was the sum of the utilities of every good consumed. This approach involved two basic problems: how to measure the utility and whether interpersonal comparisons of utility are possible.

Many economists were aware of these problems. For example Jevons (1871/1970) states that it is difficult to conceive a unit of utility. On the one hand, he regarded every man as distinct and noticed that it was not

possible to find a common denominator of feelings for two individuals. On the other hand, utility in his analysis is treated as a measurable value, and is compared and aggregated. This shows that in spite of many attempts, economists of that time failed to solve the problems of defining utility. The interesting thing is that these problems resulted in difficulties in developing a complete theory of the consumer. The producer theory for economists was easier because profit and other economic values connected with the production were measurable.

The problems connected with utility were not solved until the 1930s. In 1934 Hicks and Allen described how demand theory and utility can be based on consumer preferences alone. This work was later developed and began to dominate in microeconomic analysis. This meant that it was no longer necessary to create any psychological measure of utility. The theory goes as follows.

There is a set X comprising elements x_i, which can be quantities of any goods or bundles of these goods, e.g. a bar of chocolate, a loaf of bread or a ticket to a cinema. We choose from this set two elements and ask consumer which one he prefers. Notice that at this stage the consumer neglects the prices of the goods. By indicating the good he prefers, he sets a preference relation between these two goods. The week preference relation $x_1 \succcurlyeq x_2$ means that x_1 is at least as good as x_2. We can also define the strict preference relation $x_1 \succ x_2$ which means that the consumer prefers x_1 to x_2. The indifference relation $x_1 \sim x_2$ means that the consumer is indifferent as to which commodity to choose.

We assume that customer preferences are rational. This means that they possess the following two properties:

1) Completeness. This assumption means that a preference relation is set for any two alternatives from the set X, that is:

    for all $x_i, x_j \subset X$ at least one of relation holds: 1) $x_i \succcurlyeq x_j$ 2) $x_i \preccurlyeq x_j$

2) Transitivity. This assumption means that there is a stable hierarchy in the choices of elements from the set X.

    for all x_i, x_j, x_k: if $x_i \succcurlyeq x_j$ and $x_j \succcurlyeq x_k$ then: $x_i \succcurlyeq x_k$.

The above two postulates are not straightforward if we relate them to our everyday decisions. It is not always simple to set the preferences between two alternatives. Completeness assumes that we can make such a decision concerning any two alternatives. The other postulate is even stronger. It says that the preferences revealed in the pairwise choices will never circle and are ordered in a stable and consistent hierarchy.

The above postulates are very important in economics as they allow us to define the utility function[2]. Utility is based on rational preferences revealed by the consumer. Utility function assigns numerical values to elements of X. The commodity which is preferred, has higher value of utility function:

for all $x_i, x_j \subset X$: if $x_i \succcurlyeq x_j$, then $u(x_i) \succ u(x_j)$,

where $u()$ is the utility function.

Notice that the above formula does not specify value of the utility function. In fact there is an infinite number of functions which can represent the same rational preferences of the individual[3]. So what is the utility of a loaf of bread? One function will assign it value 3, other $-\pi$. Every function describing the rational preferences of a consumer is equally good.

Notice that utility based on preferences does not have to be a measure of satisfaction. It is simply based on preferences ignoring the motives of consumer choice.

Among numerous utility functions representing consumer choices we can now take one and check the utility values assigned for the alternatives belonging to the set X. A consumer wants to maximize his total utility within the budget constraint. Then he tries to find such a bundle of goods which will bring him the highest possible value of total utility.

The solution of this problem is a point at which a quotient of marginal utility of all the goods consumed and their prices are equal, that is for every x_i and x_j the following holds (Varian 1992):

$$\frac{\frac{\partial u}{\partial x_i}}{p_i} = \frac{\frac{\partial u}{\partial x_j}}{p_j} \tag{1}$$

where p_i is the price of a commodity on the market.

To understand this equation let us assume that a consumer chooses a point which is not optimal, at which the quotient of marginal utility of x_i to its price is lower than this value calculated for x_j. Let the prices of both goods be equal. The consumer can sell one unit of x_j and buy one unit of x_i. These changes will bring him more utility as long as the marginal utility of x_j is higher than x_i. The important thing is that the optimal bundle

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[2] There are also other assumptions concerning preferences e.g. continuity, monotonicity, or convexity. They imply certain properties of utility function. However, if the set X is finite, then no other assumptions are needed to find a utility function for rational preferences.

[3] In fact, very positive transformation of utility function is also utility function.

of goods does not depend on the choice of utility function. Each function which represents the preferences of our customer will bring us the same results.

The demand of the consumer is based on the analysis of marginal utilities. The demand curve, which represents the demand value for the commodity price, can be set as following. For every price of the commodity we can find how much the price should be lowered to buy one more unit of the good. A decrease in the price of one commodity will force the rational consumer to adjust his optimal bundle of goods so to conform with formula (1). At a new optimal point the marginal utility of a commodity with the price reduced will be lower. According to the law of decreasing marginal utility this means that the consumer will buy more units of this commodity. The demand of every consumer can be summed to create the aggregate demand on the market.
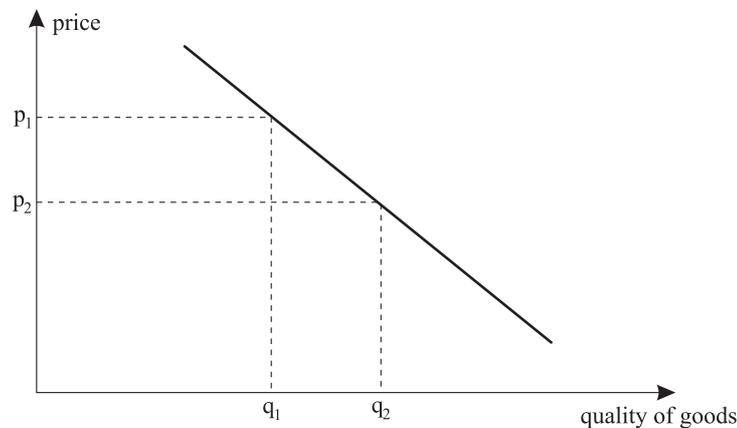
Fig. 2. The demand curve

The demand theory is based on assumptions of consumer rational choices. It turns out that these assumptions are not in reality always true. This especially concerns the transitivity property.

One example of violations of rationality postulates concerns the limited perception of a man. Imagine somebody who wants to paint his house. We show him two similar shades of yellow paint and ask which one he prefers. If the difference between them is very small then the man can be indifferent as to which one to choose. Then we show him the lighter shade of the two previous ones and a new one – a little lighter. Again the two shadows may seem so similar to the consumer that he will not notice the difference.

We can repeat the procedure until we get the colour white. Then we can show it with the darkest of all the previous ones. We expect that the man will distinguish the two paints and probably prefer one of them. In this case the preferences are not transitive.

Violation of rationality assumptions can be caused by the ways in which we present the alternatives. Kahneman and Tversky (1984) describe the following example. Imagine that you want to buy a jacket for \$125 and a calculator for \$15. The salesman tells you that in a store located 20 minutes away you can purchase the calculator with a discount of \$5. The price of the jacket is the same there. 68% of respondents declare that they would be willing to travel to the other store to buy a calculator with the discount. If the \$5 discount in the other store was only on the jacket, only 29% respondents would go to the other store. Now let's imagine that we go to the shop to buy a jacket and a calculator and we learn that because they are of stock we must go to the store 20 minutes away to buy both items. More, we learn that we get \$5 on either item as a compensation. We suppose that we do not care which good will be discounted. The preferences presented in the above situations are not rational. Let's denote:

x_1 – go to the other store to purchase a calculator with \$5 discount

x_2 – go to the other store to purchase a jacket with \$5 discount

x_3 – buy both items in the first store

The first two situations imply that $x_1 \succ x_3$ and $x_3 \succ x_2$ whereas the second situation brings the indifference relations $x_1 \sim x_2$[4].

Another example is the changing preferences of individuals. A young smoker usually prefers to smoke one cigarette a day to smoking heavily, e.g. 20 cigarettes a day. But once he gets used to smoking he may prefer to smoke more and more. His preferences change to a point at which he prefers to smoke 20 cigarettes to one cigarette a day. If we compare the first and last choices of the smoker we conclude that they are not rational. The changing tastes of the consumer lead to violations of transitivity assumptions. The rational consumer has a stable and well defined preference order. In reality preferences may change every minute, e.g. our decisions may be influenced by emotions. This is important as in reality we do not purchase items only on payday.

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[4] Such preferences are explained by mental accounting of the individuals.

2. The concept of *experienced utility*

The classic concept of utility based exclusively on the consumer's preferences has recently been questioned by Kahneman and his colleagues (see Kahneman, Wakker, and Sarin, 1997). They provide an enlightening example. Let us imagine a man who has two toasters in his kitchen. One of them is working and makes an excellent toast, whereas the other one is out of order and any time it is switched on, it gives an electric shock. Additionally, the owner of both these toasters suffers from advanced amnesia and quickly forgets his experiences. Consequently, when he comes to the kitchen to make a toast he uses, in turn, the working and the broken toaster. Now, if the economist would like to determine the utility of these two toasters for our decision-maker, based on his preferences, he would have to come to the conclusion that the man is indifferent between the toasters, and hence they have equal utility to him. We can conclude after Kahneman *et al.* (1997) that the utility concept prevailing among economists is, at least in some contexts, insufficient.

As Kahneman *et al.* (1997) bring it up, the classic concept of utility based on revealed preferences prevailed among economists due to the assumption that subjective hedonic experience i.e., experienced utility, cannot be measured. However, Kahneman and his collaborators (Fredrickson Kahneman, 1993; Kahneman, Fredrickson, Schreiber, and Redelmeier, 1993), showed that subjective feelings of an individual may be successfully measured. For example, in the experiments by Kahneman, Fredrickson, Schreiber, and Redelmeier (1993), the participants watched video clips of which one half were pleasant and the other one aversive. The participants were asked to evaluate, while watching a given clip, the intensity of the experienced pleasure or discomfort. This measure is called by the authors the *instant utility* of an episode or a *moment-based* measure of experienced utility. We can say that this is a hedonistic or affective experience measure. (The subjectively experienced intensity turns out to be an exponential function of physical values which is similar for various subjective experiences (where the exponent is specific to a given category of sensory experience).

Subsequently, after the presentation of all the clips, the participants were to evaluate *how much pleasure [or discomfort] they generally experienced while watching a given video clip*. The results obtained are considered by the authors as the measure of remembered utility. Thus, *remembered utility* is the total pleasure or displeasure reported retrospectively by the individual pleasure.

It turned out that the *remembered utility* of an episode (pleasant or unpleasant), measured as the retrospective evaluation of overall pleasure or pain, is predicted with high accuracy by the average of the most extreme instant utility recorded during the episode, and the utility recorded at the end of the episode. This was called by the authors 'The Peak and End Rule'. At the same time the retrospective evaluation of overall pleasure or pain was found not to be affected by the duration of the episode (duration neglect effect). 'The Peak and End Rule' violates the time monotonicity principle: adding moments of pain to a given adversive episode can improve the remembered utility of the episode, provided that the additional moments are less painful than the end of the original episode. We can, for instance, imagine this rule being applied by a dentist who, in order to moderate the (unpleasant) experience remembered by the patient, at the end of the appointment gives the patient an extra treatment which is also unpleasant but slightly less so than the actual end of the session.

Kahneman (2000, p. 689) has summarized these different concepts of utility as:

- *Decision utility* is inferred from observed preferences.
- *Total utility* is a *moment-based* measure of experienced utility. It is derived from measurements of moment-utility, statistically aggregated by an objective rule.
- *Remembered utility* is a *memory-base*d measure of experienced utility, which is based on retrospective assessments of episodes or periods of life.

In their experiment Kahneman, *et al.* (1993) showed that remembered utility influences an individual's choice behavior. Namely, we may presume that individuals, faced with choices among different episodes, will prefer those of them which have the greatest remembered utility for them. Since remembered utilities are formed in accordance with 'The Peak and End Rule' the individual's choices should also violate the time monotonicity principle. Thus, we may expect that when making choices among various episodes (either pleasant or unpleasant) experienced previously, individuals will act in conflict with the time monotonicity condition and can go for an episode containing a period of an extra discomfort whenever the additional period ends in a less severe discomfort.

The results of an experiment carried out by Kahneman *et al.* (1993) fully supported the above presumption. Namely, they asked participants to put their hands into very cold water for several seconds. Subsequently, they informed each individual that three more tests of that type were still ahead of them. In the Short trial the first individual kept his hand for 60 seconds in

water of 14 degrees centigrade. In the Long trial an individual kept his hand for 60 seconds in water of 14 degrees centigrade and, subsequently, during the next 30 seconds, the water temperature was gradually increased by 1 degree centigrade, from 14 degrees centigrade up to 15 degrees centigrade. Then, after completion both trials (the order of these trials was randomized for different participants) individuals were given a choice for the third trial. He/she could choose between:

> **Short trial**: to put his/her hand for 60 seconds into water of 14 degrees centigrade;

and

> **Long trial**: to put his/her hand for 60 seconds into water of 14 degrees centigrade + keep his/her hand for the next 30 seconds in water of 15 degrees centigrade.

Most frequently the participants opted for the long trial, thus going for an episode containing an additional period of discomfort, i.e. choosing the dominated option. Such a result cannot be explained in terms of the classic utility concept. It requires the concept of experienced utility. Kahneman claims that if we employ only the decision-making utility (which is determined on the basis of preferences among episodes) and disregard experienced utility, experiments such as the one described above, as well as other serious research issues, are left unexplained.

In particular, the classic concept of utility does not allow us to test the assumption of rationality of the decision maker. The assumption is testable and the decision-maker may display even the most unusual preferences. Despite this, in many situations, one would just like to know whether and, if so, in what circumstances, human behavior is or is not rational. Does a consumer who, e.g., buys a product in a store at a higher price while he can buy an identical one at a lower price in a neighboring store violate in his/her preferences the postulates of rationality, or does his/her behavior result rather from the fact that, in addition to price, he/she is also guided by store prestige (he/she does not like shopping in a store for the general public)?

Until recently economists have not been interested in laboratory experiments run by psychologists. As Matthew Rabin (1996) put it, "Economists have traditionally contended that field evidence provides more insight than laboratory evidence" (Rabin, 1996, p. 64).

Contrary to this he proposes: "We should attempt to replace some of the current assumptions in economics with assumptions built from the systematic patterns of behavior identified by psychological research. Whatever the advantages and disadvantages of rigorously deriving conclusions from

formally stated assumptions, our conclusions are likely to be more realistic if our assumptions have a better empirical base. Psychological research has accumulated enough general insights for us to convert these insights into tractable assumptions in formal economics" (Rabin, 1996, p. 62).

Indeed, recently a new paradigm has started to evolve in economics. This is known by the labels of *behavioral or experimental economics*. It refers to the attempts by economists to include psychological assumptions and methods in dealing with economic problems.

The laboratory controlled experiment has become a major research tool for developing behavioral economic theories. Most certainly, the concept of experienced utility is among those which are to be studied under this approach. We still have a lot to learn about how people perceive and misperceive their experienced utilities, how they adapt their preferences to changes in their lives (e.g. changes in wealth), etc.

One example may be a question discussed by Kahneman (2000, concerning studies on well-being (happiness). The assessments of this are typically obtained through self-reports. This type of interviews typically shows the so called treadmill effect – changes in life circumstances have small effects on subjective happiness. For example, in a classic study by Brickman, Coates, and Janoff-Bulman (1978), when the authors interviewed lottery winners and a control group, they found almost no difference in rated happiness between these two groups. However, subjective happiness (remembered utility) measured in such a way may be highly distorted and may dramatically differ from objective happiness measured through moment-based technique (moment-based utility). Indeed, as shown by Schwartz and Clore (1983) reported well-being on sunny days was significantly higher than reported on rainy days.

3. Decisions under risk and uncertainty: the principle of maximizing the expected utility

The term utility has also been used for decision analysis under risk and uncertainty. The first major step was Bernoulli attempt to explain why people neglect the expected value principle in their choices. Take, for example, a choice between a sure win of \$10 and participating in a lottery which offers equal chances to win \$5 or \$15. Although the two alternatives have the same expected value of outcomes, people usually choose a sure win.

Another example is the St. Petersburg paradox. In this game a coin is tossed until the first head appears. The player gets a prize of \$$2^k$, where k denotes number of tosses. What is a sure equivalent of this game? The

expected value of winning this game is infinite but in reality people were not willing to pay for the participation in this game more than \$4.

The above examples show that people do not respect the expected value of the game. Barnoulli noticed that the St. Petersburg paradox could be resolved if we assume that players maximize the expected value of logarithm of a price. The sure equivalent of participating in the St. Petersburg paradox game is then \$4. This finding initiated the development of expected utility theory.

In 1947 von Neumann and Morgenstern developed an expected utility theory which was based on axioms concerning a player's behaviour. In this theory the uncertain outcomes are presented as lotteries with known probabilities and outcomes. We assume that players have a proper perception of the lottery. This means that decisions are made based on probabilities and their outcomes only. The sequence in which we present the alternatives should have no influence on the eventual choices. Probability 1 means a sure win. The player should also recognize that compound lotteries can be simplified to lotteries with the same conditions. Take for example the following lotteries. In the first one you win or lose \$5 with equal chances. The second lottery has two stages. In the first stage the lottery offers 75% chance of qualifying for the second stage and 25% chance of ending the game with a win of \$5. In the second stage the probability of \$5 win is 1/3 and probability of losing \$5 is 2/3.

If the player has a correct perception of lotteries, then we can ask him to show his preferences over lotteries belonging to a set X. The expected utility theory requires that these preferences comply with certain assumptions:

1) Completeness and transitivity. These properties are similar to those presented in the first part of this paper; here they should be defined for a set of lotteries X.

2) Continuity. This assumption means, that if there is a preference order over three lotteries, then there exists such α i $\beta \in \,]0,1[$, that:

$$\alpha L_1 + (1-\alpha)L_3 \prec L_2 \prec \beta L_1 + (1-\beta)L_3$$

3) Independence postulate. It says that for all $\alpha \in \,]0,1]$

$$L_1 \prec L_2 \Rightarrow \alpha L_1 + (1-\alpha)L_3 \prec \alpha L_2 + (1-\alpha)L_3$$

The above conditions are sufficient to set a utility function u which describes well consumer behaviour. It has the following property:

$$L_1 \prec L_2 \iff u(L_1) < u(L_2)$$

The player's aim is to maximize his expected utility calculated as Beroulli did it – as an expected value of utilities for every outcome of the lottery.

Notice that as in the case of utility presented in the first part of this article, this theory is also based on consumer preferences. Similarly, there is an infinite number of utility functions which describe the same preferences of an individual[5]. All these functions have a similar shape. This shape depends on risk aversion and is specific to each individual.

The shape of the utility function may be set in the following manner. We present the player with two options. The first one is a lottery in which he can either win prize W or end the game with nothing, with equal chances. Another alternative is a sure win of $0.5\,W$.
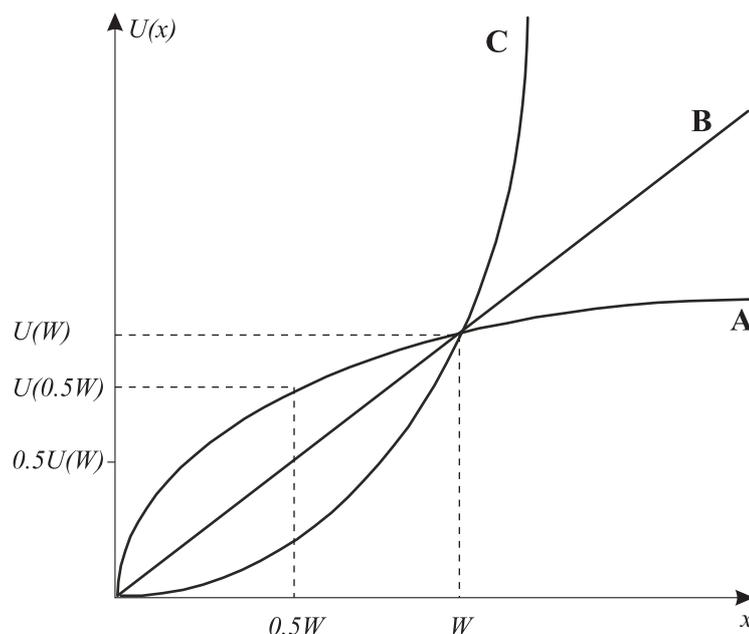

Fig. 3. Examples of utility functions

If both options are equally attractive to the player, then his risk is neutral and his utility function has a shape like the curve B in figure 3. If the player prefers a sure $0.5\,W$, then he is risk-averse and his utility function is concave like curve A. For this player the utility values follow the inequality: $u(0.5W) > 0.5u(W)$. To see how risk-averse our player is we need to find a sure equivalent of the lottery presented. The lower the value of the equivalent, the more concave the utility function is and the higher

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[5] In the von Neumann-Morgenstern theory every linear positive transformation of utility function given for an individual is also the utility function.

the risk-aversion of the player. If the player is risk-seeking, then the shape of utility function is convex like the curve C. The sure equivalent is then higher that $0.5W$. An important thing is that our attitude towards risk can change as value W changes. Thus, to draw the utility function we need to find a sure equivalent for the lottery for every level of W.

The three postulates concerning preferences mean that the consumer behaviour is consistent – described by a utility function of a shape specific to an individual. In practice this assumption is sometimes violated. An example of such a choice could be the well-known Allais paradox, which is a simple comparison of preferences revealed in two situations. In the first one we present the players with two options: a sure win of \$0.5 mln or a lottery in which offers 89% probability of winning \$0.5 mln, and 10% of winning \$2.5 mln. In this lottery there is 1% chance that we end it with nothing. Most respondents prefer the first option.

The second situation is a choice between the following lotteries. The first one offers 11% chance of winning \$0.5 mln and 89% chance to end the game with nothing. The second lottery offers 10% chance of winning \$2.5 mln and 90% chance of ending the game with nothing. Most respondents choose the second option. We can show that the preferences revealed in these two situations are contradictory, in the sense of the expected utility theory – there is no utility function which describes the two choices.

The choices made in the first situation imply that:

$$u(0.5) > (0.1)u(2.5) + (0.89)u(0.5) + 0.01u(0)$$

Adding $(0.89)u(0) - (0.89)u(0.5)$ to both sides we get:

$$0.11u(0.5) + (0.89)u(0) > 0.1u(2.5) + 0.9u(0)$$

The above inequality contradicts the choice made in the second situation[6]. In this case the players violated the independence postulate.

Although the von Neumann-Morgenstern theory sometimes fails to describe people's behaviour, it is often used in economics. One of its classic applications is found in foundations of insurance theory. It provides an answer to the question: why do people buy insurance?

Let's analyse a choice made by a consumer who considers insuring his possessions, valued of W. The probability of losing them equals p. A policy which can protect him from suffering this loss costs Q. In the terms of the expected utility theory, the consumer can take part in a lottery in which he can lose W with a probability p or choose a safe loss of Q. If he decides to

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[6] This paradox can be explained by "regret theory".

buy a policy then his utility would equal: $u(W - Q)$. However if he chooses to take a risk then his utility would be: $(1 - p)u(W) + p\,u(0)$. If a consumer is risk averse then he is ready to pay more for a policy than the expected value of loss:

$$(1 - p)u(W) + p\,u(0) < u(W - Q)$$

For the insurer the premium on the policy, apart from the expected value of losses paid out, must also cover other administrative costs and a profit margin. Risk averse consumers are likely to accept such premium. This fact enables the existence of the insurance sector in the economy.

The next example of application of the expected utility theory is from the domain of money investment. Assume that we have a certain amount of money and two types of assets. The first one is a safe bond with a constant return on investment: r_s. The other is a risky stock which is quoted on the exchange. The average and expected rate of return on this asset is r_r. According to the expected utility theory we say that even if $r_r > r_s$ the risk averse investor can allocate some money in safe bonds as it gives a sure win. In this way we can explain a demand for bonds which often have lower rate of return than stocks.

The weakness of the von Neumann-Morgenstern theory is the assumption of knowledge of all the probabilities offered by the lottery. In reality such situations occur rarely. More often we can only estimate such probabilities, for example, the chance that the fire will destroy our house in the coming year. There is no reason to think that different people will similarly asses this probability. The theory of utility based on subjective probabilities was created by Savage.

According to this theory before we take a decision, we analyse conceivable acts (decision choices) which lead to certain states of nature. The decision-maker has his own preference order over all feasible acts. He prefers the acts which result in higher expected utility of outcomes. To calculate the value of the expected utility the decision-maker uses his subjective assessment of the probabilities of outcomes. Then he compares utility of conceivable acts and sets for them a preference relation. Savage's expected utility function has the following property:

$$a_1 \succcurlyeq a_2 \iff Eu(a_1) \geq Eu(a_2)$$

where $Eu()$ is an expected value of utility;

Savage showed that the utility function exists if preferences comply with certain assumptions. The first one is the completeness of preferences over the set of alternatives. This condition is analogous to the completeness as-

sumption for the von Neumann-Morgenstern theory. The second postulate
is about a sure-thing principle. It says that if we compare two acts, we base
our preference relation on those states of nature which differ those acts. The
third postulate means that the desirability of outcomes does not depend on
the earlier acts and states – those which actually led to a present state. The
fourth condition is about our subjective assessments of probabilities. The
decision maker should say which events are more probable (or equally proba-
ble) for him. This means that there is a complete and transitive probability
relation over all events. The last postulate was a technical monotonicity
condition. Savage also formulated two additional conditions but they are
not necessary to prove the existence of the utility function. The first one
excluded the situation in which a decision-maker is indifferent between any
two acts. The second postulate implied a sort of continuity of the preference
relation and guaranteed an infinite number of all possible states.

As an example of application of Savage's theory let us consider the fol-
lowing example. A decision-maker has to decide whether to invest \$1000
in a venture. If he succeeds, his costs will be covered and additional profit
of \$1000 earned. However, it is possible that the business will not succeed
and the decision-maker will lose all the money invested. According to Sa-
vage's theory, before taking the decision he should first assess the proba-
bility of success: p. The probability of losing money is: $1 - p$. In the next
step the decision-maker should estimate his own utility of possible outco-
mes: $u(-1000)$, $u(1000)$ and $u(0)$. In the last step he chooses what decision
will give him a higher value of expected utility. If:

$$u(0) > pu(-1000) + (1 - p)u(1000)$$

then he will not invest his money in a venture.

Notice that in this case the decision-maker himself estimates the utility
values for the outcomes. This is subjective utility which cannot be used for
interpersonal comparison[7].

We can show that in some situations this theory fails to describe people's
behaviour. A good example is a variation of the Ellsberg paradox (from
Ellsberg 1961). We have two urns, each containing 100 balls. The balls are
either white or black. It is known that the first urn contains 51 black balls
and 49 white ones. The assortment of balls in the second urn is not known.
One ball is picked from each urn; the colour of these balls is not disclosed.
We tell a decision-maker that we can show him one of these balls, either

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[7] As in the von Neumann-Morgenstern theory there is an infinite number of functions
which represent preferences of the consumer. Every positive linear transformation of he
utility function is also an utility function for the individual.

from the first or the second urn. If the ball disclosed is black, he gets a prize of $1000. If the ball is white, he ends the game with nothing. The majority of people want to disclose the ball from the first urn. In terms of Savage's theory it means that their subjective assessment of probability that a black ball can be picked from the second urn is lower than 51%. Then we repeat the decision problem with one modification: a prize of $1000 is given if the disclosed ball is white. It turns out that many people once again ask for a ball from the first urn. This implies that their subjective assessment of probability that white ball is picked from the second urn is lower that 49%. This contradicts the probability assessment revealed in the first step.

4. Prospect theory – critical "psychological" assumptions

There have been several proposed modifications of the utility theory for risky decision making. However, the most fundamental step in the analysis of decisions under risk was proposed in 1979 by two psychologists, Daniel Kahneman and Amos Tversky (1979). Their proposal, called the prospect theory, includes basic changes concerning the valuation of outcomes by the decision maker as well as the way it handles the probabilities. Let us start with the assumptions concerning the valuation of outcomes. They are easily presented with the aid of Figure 4.
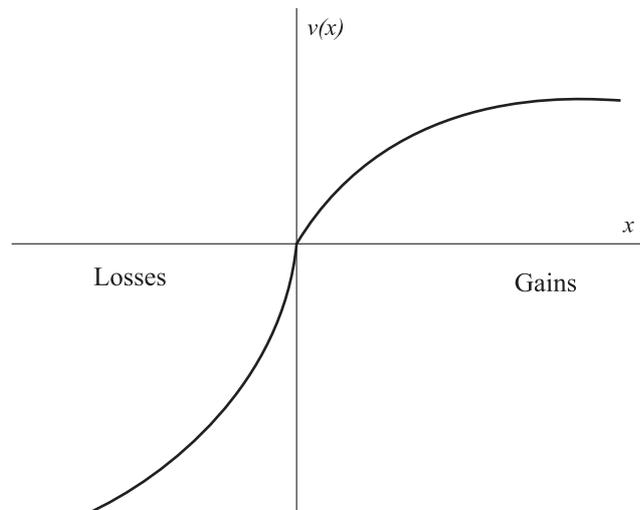

Fig. 4. Prospect theory value function

The first critical assumption is that prospects are evaluated relative to a reference point. Kahneman and Tversky (1979, p. 277) stress that people are more sensitive to how an outcome differs from some reference point than to the absolute level of the outcome itself. They put it:

"An essential feature of the present theory is that the carriers of value are changes in wealth or welfare, rather than final states. This assumption is compatible with basic principles of perception and judgment. Our perceptual apparatus is attuned to the evaluation of changes or differences rather than to the evaluation of absolute magnitudes. When we respond to attributes such as brightness, loudness, or temperature, the past and present context of experience defines an adaptation level, or reference point, and stimuli are perceived in relation to this reference point (Helson (1964)). Thus, an object at a given temperature may be experienced as hot or cold to the touch depending on the temperature to which one has adapted. The same principle applies to nonsensory attributes such as health, prestige, and wealth. The same level of wealth, for example, may imply abject poverty for one person and great riches for another-depending on their current assets."

The second assumption of the prospect theory is that preferences depend on how a problem is framed, in terms of gains or in terms of losses. As implied by the value function in Figure 1, people are risk-aversive in the domain of gains (they prefer a sure gain over a higher but risky prospect) and are risk-seeking in the domain of losses (they prefer a higher risky loss over smaller but sure loss). Tversky and Kahneman (1986) showed in a series of experiments how framing can alternate the decision-maker's choices between two logically equivalent options. One of their examples are the following decision problems.

Problem 1
Assume yourself richer by $300 than you are today. You have to choose between
a sure gain of $100
50% chance to gain $200 and 50% chance to gain nothing

Problem 2
Assume yourself richer by $500 than you are today. You have to choose between
a sure loss of $100
50% chance to lose nothing and 50% chance to lose $200

Majority of respondents preferred sure gain of $100 over fifty-fifty chance of gaining $200 or nothing in Problem 1, and in Problem 2 majority of respondents preferred fifty-fifty chance of loosing nothing or loosing $200 over sure loss of $100. However, the two problems are essentially identical. In both cases the decision maker faces a choice between $400 gain or even

chance of $500 or $300. The observed framing effect results from a change in reference point.

Finally, in accordance with Figure 1, the third assumption of the prospect theory is that people dislike losses significantly more than they like gains – i.e. the hurt of a $100 loss is more noticeable than the benefit of a $100 gain. This predisposition involves loss aversion. Loss aversion has numerous implications. One is the endowment effect which involves that once a person has acquired something, he/she immediately values it more than before he/she possessed it. A simple illustration of this phenomenon is an experiment conducted by Kahneman, Knetsch, and Thaler (1990). In a classroom certain number of mugs (retail value of about $5) were placed in front of some students. Then, these students were given the following instruction: "You now own the object in your possession. You have the option of selling it if a price, which will be determined later, is acceptable to you. For each of the possible prices below indicate whether you wish to: (x) Sell your object and receive this price; (y) Keep your object and take it home with you...." These students indicated their decision for prices ranging from $0.50 to $9.50 in steps of 50 cents. Let us call them sellers. At the same time, another group of students who had not received a mug (the "choosers") were asked to indicate their preference between a mug and various sums of money ranging from $0.50 to $9.50. The authors found that the median value of the mug was $7.12 for the sellers and only $3.12 for the choosers. The difference between these values evidently reflects an endowment effect. Those students who acquired mugs valued them more than those who did not possess them.

Several phenomena in various areas of economics can be explained within the prospect theory. For example, a well known practice in pricing strategy is that price differences for the same product are typically presented as discount from the higher price rather than as premiums over the lower price. Indeed, a tourist looking for an apartment in a tourist site is typically informed that in season one pays a normal price, while out of season one pays a discounted price. Obviously, an opposite convention is equally possible that it is out of season when one pays the normal price, while in season one pays premium. But owners of the apartments know very well that the former convention introduces a more favorable reference point. The potential customer is never on the loosing side.

In the financial market Odean (1998) showed the so-called disposition effect – the tendency to hold losers and to sell winners. For example, imagine an investor who purchased two stocks A and B at $20 per share. Let us say that by the next month, stock A had decreased in price by $10 (i.e. its price has fallen to $10) and stock B had increased in price by $10 (i.e. its

price has risen to \$30). The investor must decide whether to sell or to hold both the stocks for another period. Odean (1998) shows that investors are more likely to keep stock A and to sell stock B. The reason is that the initial purchase price is a natural reference point for the investor. Thus, loss aversion restrains the investor from selling the losing stock.

The prospect theory also differs from expected utility theory in the way it handles the probabilities attached to particular outcomes. The von Neumann-Morgenstern utility theory assumes that decision makers uses probabilities to calculate an expected utility of the risky alternatives. In contrast, prospect theory treats probabilities as "decision weights". The weighting function for probabilities is shown on Figure 2. As can be seen, small probabilities are overestimated while moderate and high probabilities are underestimated.
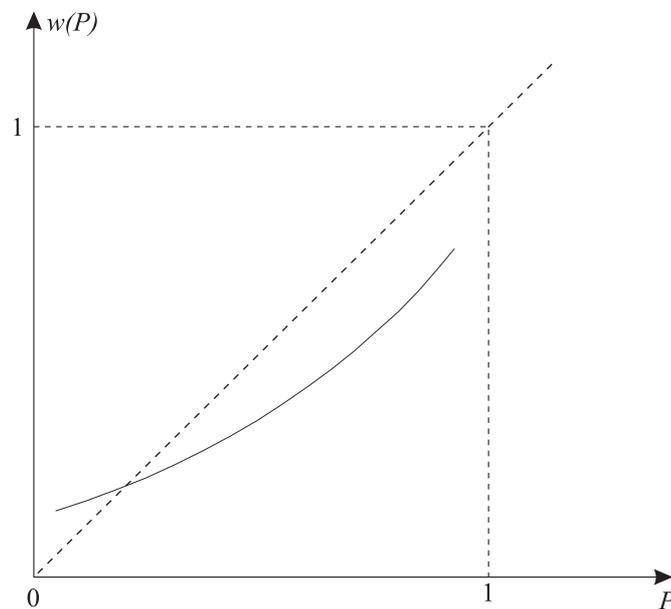

Fig. 5. Prospect theory weighting function

A simple illustration of the tendency to overweight small probabilities is the pair of problems below (cf. Kahneman and Tversky, 1979):

Problem 1.
You have to choose between:
A 1 in 1000 chance of winning \$5000
A sure gain of \$5.

Problem 2.
You have to choose between:
A 1 in 1000 chance of losing $5000
A sure loss of $5.

The majority of respondents who were presented with these problems preferred the risky option in the first problem and the sure loss in the second problem. Both preferences can be explained in terms of a tendency to overweight small probabilities.

The preference for the risky option in lotteries containing small probabilities of a large gain also explains why people purchase lottery tickets. The preference for the sure loss in the second problem can explain why people buy insurance. However, it is worth of noticing that in accordance with the prospect theory generally people should be reluctant to insure themselves. This is a direct inference from the assumption that people are prone to take risks in the domain of losses. Indeed, insurance can be seen as a choice between two options:

(1) to pay an insurance premium (and this way to lose some money for sure)

(2) to risk (a much) higher loss (e.g. in the case of flood) or to lose nothing (e.g. in the case of no flood).

Thus, generally, people should choose the risky option, i.e. not to insure themselves.

These ambiguous predictions of prospect theory concerning buying insurance seem to have support in real life. Kunreuther quotes a statement by George Bernstein, the former head of the Federal Insurance Administration: "Most property owners simply do not buy insurance voluntarily, regardless of the amount of the equity they have at stake. It was not until banks and other landing institutions united in requiring fire insurance from their mortgagers that most people got around purchasing it. It was also many years after its introduction that now popular homeowners' insurance caught on. At one time, insurance could not give away crime insurance, and we just need to look at our automobile insurance laws to recognize that unless we force this insurance down the throughout of the drivers, many, many thousands of people would be unprotected on the highways. People do not buy insurance voluntarily unless there is pressure on them from one source or another." (Kunreuther, 1978, p. 34).

This claim is supported by several systematic research. For instance, Brown and Hoyt (2000) report that a large portion of the property at risk from flooding in the U.S. and Germany remains uninsured. Similarly, Pynn

and Ljung (1999) found that in spite of the National Weather Service's predictions that a record flood could be produced by snow melting and the Federal Emergency Management Agency's advertisements in the media, only a minority of property owners in that area purchased insurance policies against flooding. Similarly, Tyszka et al. (2002) found that purchasing insurance against flood in the regions at risk of flooding in Poland was until the year 1997 very rare. On the other hand, immediately after the flood of 1997, when people realized that a flood is possible, the number of households insured significantly increased. However, after four further years the willingness to buy insurance in these regions started to decrease again.
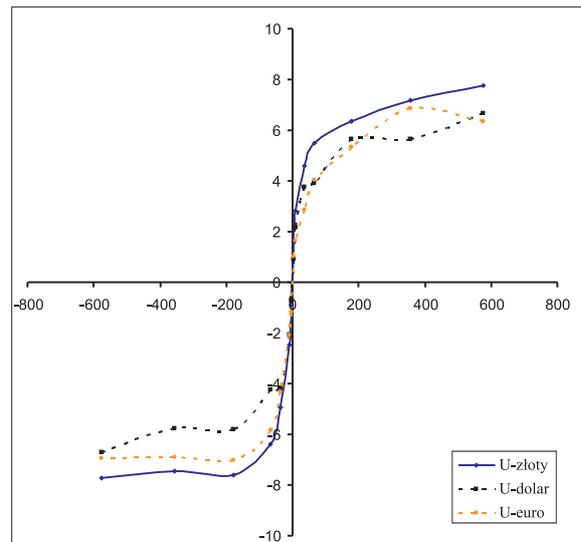

Fig. 6. Mean assessments of the importance of different gains and losses

While presenting the prospect theory, we indicated its implications in various areas of economics. To finish, let us look at its manifestation in the area of currency perception. Tyszka and Pszybyszewski (2003) asked their respondents to imagine that they gain or lose a certain amount of money or an object of a specified financial value. For different groups of respondents the values of gains and losses were expressed in different denominations: the Polish Zloty (PLN), USD and EURO. Participants were asked to express their feelings about the gains or losses, and mark them on a continuous scale ranging from "insignificant gain/loss" to "significant gain/loss". To participants who dealt with foreign currency (USD and EURO) the currency exchange rate was given so that they were able to perform a direct price comparison between foreign and PLN prices.

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Figure 6 shows a direct support for the prospect theory: the value function (measured here by mean assessments of the importance of different gains and losses) was convex and relatively steep in the domain of losses. In contrast, the value function for gains was concave and less steep. At the same time, it can be also seen that both gains and losses expressed in greater nominal values were perceived by the subjects as higher. This is the so-called money illusion effect described first by Shafir, Diamond & Tversky (1997). It consists of a tendency to use the nominal value as an anchor, ignoring the real value of the money, when people evaluate the value of goods. In our case this tendency resulted in different perceptions of the same gains and losses depending on the currency they were expressed in.

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