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TEMPORAL LOGIC APPROACH TO EXTENSIVE GAMES[1]

A mathematical n-players game model can be represented in a normal or extensive form. A normal form representation of games is ideal to represent situations where players make one choice and move simultaneously. An extensive form provides an explicit description of a strategic interaction by specifying a physical order of play, actions available to players each time they get to choose, and eventual payoffs for each player for any sequence of choices. For these reasons the extensive form provides a richer environment to study interesting questions such as rivalry, repeated interaction, etc.

Obviously every normal form of the game can be represented in an extensive form, but it is more natural and simpler just to write the normal form. It is less obvious that every extensive form of the game can be written in a normal form. This translation is possible if we note that a strategy in an extensive form game is not just a move or sequence of moves, but rather is a complete contingency plan. A strategy for a player i must specify what the player will do at every node or what information set the player has.

Games in extensive form

In this section we will study extensive form games. The basic notion of the extensive form games theory is a *game tree* notion. It contains nodes and branches. Nodes represent decision points where only one player has to make a decision. Branches represent possible choices available for player. A game tree includes all alternative actions that can be taken by all players

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and all possible outcomes. If we construct a game tree, we have to obey the following rules:

- at least one branch leads from each decision node,
- only one branch leads to a decision node.

Example 1

Let us consider the following examples of structures:

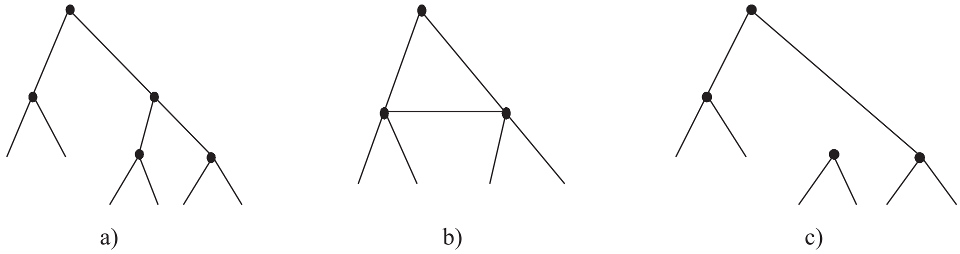

Fig. 1.

In case a) we have a game tree. In case b) tree does not form a game tree, because in the structure of the tree we have a closed path. In case c) we do not have a game tree, because there is not a path between two nodes.

Definition 1

A rooted tree is a pair $\langle T, \longmapsto \rangle$, where T is a set of nodes and $\longmapsto$ is a binary relation on T satisfying the following conditions:

- there is a distinguished node t_0 $(\in T)$ (it is called the *root*), such that no immediate predecessors,
- for every node $t \in T \setminus \{t_0\}$ there exists a unique path from t_0 to t.

A *terminal node* is a node which has no immediate successors. Let $L(T)$ denote the set of terminal nodes.

Example 2

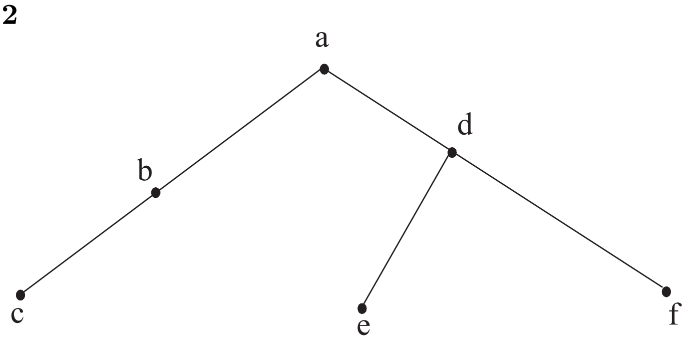

Fig. 2. A rooted tree with the root a and terminal nodes *c, e, f*

Let us remark that in the normal form game all parts of the game other than the strategies are removed (for this reason it is also called the strategic form). For a given game in a normal (strategic) form

		PLAYER II	
		C	D
PLAYER I	A	1,1	5,0
	B	2,3	2,3

Fig. 3.

we can consider a few extensive forms. Two extensive forms of the normal form showed in Fig. 3 could be formed in the following way:

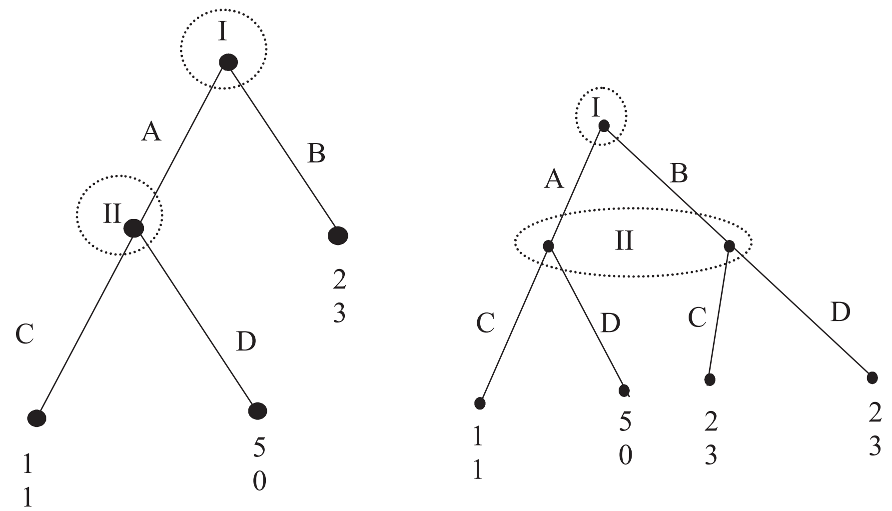

Fig. 4. Two extensive form versions of the same normal form game

If a game is played sequentially, the extensive form allows us to see exactly how the game could be played out.

The knowledge which a player has after every move is essential from a player standpoint. Sometimes in games moves are taken in a sequence and every player observes every event that takes place until that player has to take an action. However, there are games whose rules are such that the players do not obtain the so-called perfect information on the decisions of the other players. This kind of situation usually takes place in the card games when the first move is random.

Now we will explain the differences between perfect information games and imperfect information games. Let us consider the following example:

Example 3

Let us take a game with the following game tree:

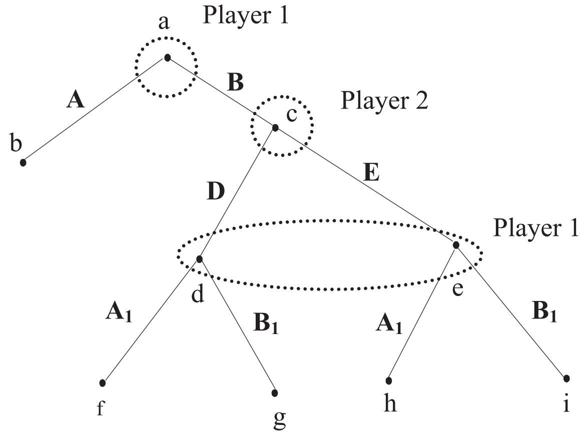

Fig. 5.

A circle (dotted line) denotes information sets. It surrounds (or connects) the nodes that are *indistinguishable* from the decision maker's standpoint. In the above case in the node *d* or the node *e* the player 1 does not know the player 2's decision, and therefore he has imperfect information.

Definition 2

An *information set* is a collection of the decision nodes so that:

1) The same player is mapped to all these nodes,
2) If the play of the game reaches a node in this collection, the player does not know which node has been reached.

A *game of perfect information* is a game in which there is no information set with multiple nodes[2]. If there are multiple nodes information sets in a game, then we have a game of imperfect information.

Now we give some conditions for a finite extensive form with perfect information.

[2] At any node each player knows the entire history of play.

Definition 3

A ***finite extensive form with perfect information*** is a tuple: $\langle T, \longmapsto, N, \xi \rangle$, where:

- $\langle T, \longmapsto \rangle$ is a finite rooted tree,
- $N = \{1, \dots, n\}$ is a set of players,
- $\xi : (T \setminus L(T)) \to N$ is a function that associates with every non-terminal or decision node of the player who moves at that node.

Given an extensive form, we obtain a perfect information game by adding for every player $i \in N$, for every terminal node $t \in L(T)$ a payoff or utility function $[u(t) = (u_1(t), \dots, u_n(t))]$.

Temporal logic of branching time

The idea of temporal logic of branching time was given by A. N. Prior[3]. One of the main motivations of the construction of temporal logic of branching time was a wish for creation of indeterministic temporal logic. Arguments on determinism were rejected by modification of the structure of time. The basic system of the temporal logic of branching time is Nino Cocchiarella's system called „*CR*"[4]. In the *CR* system a relation of temporal succession is transitive. Because no other conditions are imposed upon the earlier-later relation, then the symmetry of the past and the future[5] is possible in *CR*. In the other systems of temporal logic of branching time there are additional conditions imposed on the earlier-later relation. A left-linearity property of the earlier-later relation is necessary in the K_b[6] system, for example.

An example of the structure of time linear in the past is presented below (Fig. 6).

As we see, the past has no alternatives and it is determined, but the future is open and there are a lot of ways of its realization. Among all possible futures only one is realized. It is called *actual future*.

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[3] A. N. Prior, *Past, Present and Future*, Oxford University Press, 1967.

[4] A. N. Prior, *Past, Present and Future*, Oxford University Press, 1967, Appendix A.

[5] The system *CR* is often considered as a model of ideas in the contemporary physics on real time R. P. McArthur, *Tense logic*, Dordrecht 1976, p. 39.

[6] N. Rescher, A. Urquhart, *Temporal Logic*, Wien, New York, 1971, chapter 4.

Example 4

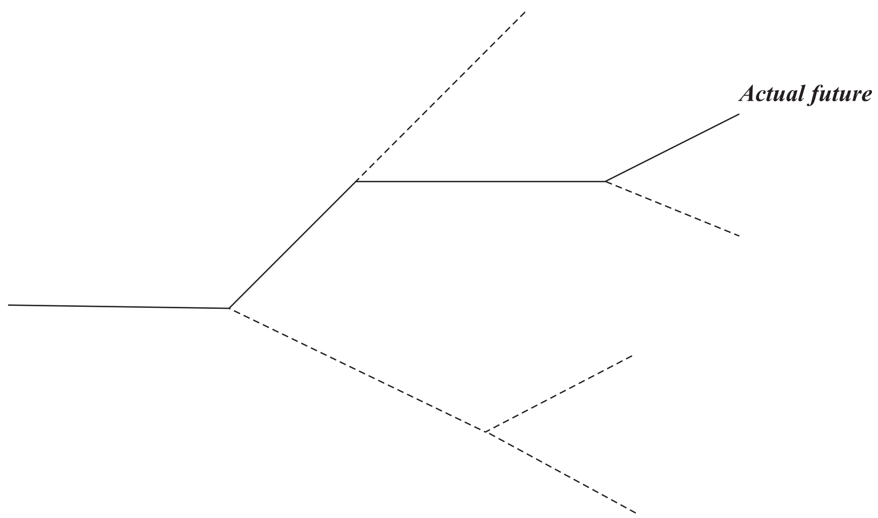

Fig. 6.

Interpretation of extensive games in terms of temporal logic of branching time

The logical foundations of game theory have been a subject of consideration of many scientific papers. Usually these considerations are formulated in terms of modal or epistemic logic. However games are specific processes in which dynamic interactions between the players occur. If we consider a notion of process, we have to consider its temporal context. It seems that a natural way of interpretation of games is its interpretation in terms of temporal logic. Extensive games can be modeled in terms of temporal logic of branching time if we add a notion of agent to the semantics and define a notion of prediction.

Definition 4

A branching-time frame with agents[7] (*BTA-frame* for short) is a tuple $\langle T, \prec, N, \{R_i\}_{i \in N}\rangle$, where:

- T – a set of nodes,

[7] Bonanno G., *Branching time, perfect information games and backward induction*, 1999.

- $\prec$ – a binary relation on T (the precedence relation) satisfying the following conditions:
  A1) if $t_1 \prec t_2$, then $t_2 \not\prec t_1$ (antisymmetry)
  A2) if $t_1 \prec t_2$ and $t_2 \prec t_3$, then $t_1 \prec t_3$ (transitivity)
  A3) if $t_1 \prec t_3$ and $t_2 \prec t_3$ then $t_1 = t_2$ or $t_1 \prec t_2$ or $t_2 \prec t_1$ (left linearity),
- $N = \{1, \ldots, n\}$ is a finite set of agents,
- for any $i \in N$, R_i is a binary relation on T, such that: if $t_1 R_i t_2$, then $t_1 \prec t_2$ (R_i is subrelation of $\prec$).

Properties A1-A3 constitute the definition of branching time. In particular, the left linearity property limits a class of frames to the frames, where at any node the future has alternatives and the past is unique.

The interpretation of $t_1 R_i t_2$ is as follows: at node t_1 agent i can make a decision, which leads from t_1 to t_2. It is possible, that for some agent i and for some node t, the set $R_i(t) \stackrel{\text{def}}{=} \{t' \in T : t R_i t'\}$ is empty. In this case agent i does not have any actions available at node t.

Definition 5

For a given a BTA-frame *prediction* is a binary relation $\prec_P$ on T, satisfying the following conditions:

P1) if $t_1 \prec_P t_2$, then $t_1 \prec t_2$ ($\prec_P$ is subrelation of $\prec$),
P2) if $t_1 \prec_P t_2$ and $t_2 \prec_P t_3$, then $t_1 \prec_P t_3$ (transitivity),
P3) if $t \prec t_1$ for some t_1, then $t \prec_P t_2$ for some t_2,
P4) if $t_1 \prec t_2$, $t_2 \prec t_3$ and $t_1 \prec_P t_3$, then $t_1 \prec_P t_2$ and $t_2 \prec_P t_3$.

The condition P1 shows that the relation of prediction is subrelation of $\prec$. The conceivable future is a subset of the set of all future nodes. Let us remark, that we do not assume that the conceivable future is unique for a given node. We do not require $t' = t''$, $t' \prec_P t''$, $t'' \prec_P t'$ in case $t \prec_P t'$ and $t \prec_P t''$. Moreover, we do not assume that the predictable future for a given node is a proper subset of the conceivable future. P2 (transitivity) is a natural condition for a notion of prediction[8].

Every $t \in T$ should be thought of as a complete description of the world. Sets of nodes represent propositions. We introduce a formal notation and a notion a model for a correct interpretation.

Let us consider a language of propositional logic with the following specific operators: $G, H, G_p, H_p, \rhd_i$.

[8] More detailed discussion on these conditions we find in G. Bonanno, *Branching time, perfect information games and backward induction*, 1999.

The interpretation of the operators is as follows:

$G\alpha$ – it is going to be the case in every conceivable future that α,
$H\alpha$ – it has always been the case that α,
$G_p\alpha$ – it is going to be the case in every predictable future that α,
$H_p\alpha$ – it has always been the case at every past node at which the current node was predicted that α,
$\rhd_i \alpha$ – no matter what action agent i takes, it will be the case that α.

The operators G and H are the standard operators of the temporal logic of branching time. The operators G_p and H_p are specific operators introduced by G. Bonanno[9]. The operators $\rhd_i$ (for any $i \in N$) are action's operators.

Formal language

Alphabet:

- a countable set of propositional letters S,
- connectives: $\neg, \Rightarrow$,
- temporal operators: G, H, G_p, H_p,
- action operators $\rhd_i$,
- parentheses:), (.

The set of the sentences is defined as follows:

Definition 6

The set of the sentences is the smallest set $\mathbb{Z}$, such that:

- $S \subseteq \mathbb{Z}$,
- if $\alpha, \beta \in \mathbb{Z}$ then $\neg\alpha, (\alpha \Rightarrow \beta), G\alpha, H\alpha, G_p\alpha, H_p\alpha, \rhd_i \alpha \in \mathbb{Z}$.

We adopt the following definitions:

Definition 7

$$\begin{aligned}
(\alpha \vee \beta) &\equiv (\neg\alpha \Rightarrow \beta),\\
(\alpha \wedge \beta) &\equiv \neg(\alpha \Rightarrow \neg\beta),\\
(\alpha \Leftrightarrow \beta) &\equiv \neg[(\alpha \Rightarrow \beta) \Rightarrow \neg(\beta \Rightarrow \alpha)],\\
F\alpha &\equiv \neg G \neg\alpha,\\
P\alpha &\equiv \neg H \neg\alpha,\\
F_p\alpha &\equiv \neg G_p \neg\alpha,\\
P_p\alpha &\equiv \neg H_p \neg\alpha.
\end{aligned}$$

[9] Bonanno G., *Branching time, perfect information games and backward induction*, 1999.

By adding a function $V : S \to 2^T$ to a given BTA-frame we obtain a model $\mathfrak{M}$ based on this frame. Validation for formulas is as follows:

Definition 8

a) $\mathfrak{M}, t \models \alpha \quad \equiv t \in V(\alpha), \text{ if } \alpha \in X,$

b) $\mathfrak{M}, t \models \neg\alpha \quad \equiv \text{not } \mathfrak{M}, t \models \alpha,$

c) $\mathfrak{M}, t \models (\alpha \Rightarrow \beta) \equiv \text{if } \mathfrak{M}, t \models \alpha, \text{ then } \mathfrak{M}, t \models \beta,$

d) $\mathfrak{M}, t \models G\alpha \quad \equiv$ for every t' such that $t \prec t'$ holds $\mathfrak{M}, t' \models \alpha$,

e) $\mathfrak{M}, t \models H\alpha \quad \equiv$ for every t' such that $t' \prec t$ holds $\mathfrak{M}, t' \models \alpha$,

f) $\mathfrak{M}, t \models G_p\alpha \quad \equiv$ for every t' such that $t \prec_P t'$ holds $\mathfrak{M}, t' \models \alpha$,

g) $\mathfrak{M}, t \models H_p\alpha \quad \equiv$ for every t' such that $t' \prec_P t$ holds $\mathfrak{M}, t' \models \alpha$,

h) $\mathfrak{M}, t \models \rhd_i \alpha \quad \equiv$ for every t' such that tR_it' holds $\mathfrak{M}, t' \models \alpha$.

Theorem 1

A finite extensive form with perfect information is a special case of a BTA frame[10].

Game model

The theorem 1 shows that we can consider a finite extensive form with perfect information as a special case of a BTA frame. To view a perfect information game as a model we need so, that the set of sentences include sentences of the form $(u_i = q)$, where $i \in N$, and $q \in Q$. The interpretation of the sentences of the form $(u_i = q)$ is: *player i's payoff is.* The sentence of the form $(q_1 \leq q_2)$ we interpret as: *the payoff* q_1 *is less than or equal to the payoff* q_2.

Definition 9

Let $\mathfrak{I}$ be the BTA frame corresponding to a given perfect information game. A game model $\mathfrak{M}$ is a model based on $\mathfrak{I}$ obtained by adding to $\mathfrak{I}$ a valuation $V : s \to 2^T$ satisfying the following conditions:

- if p $(\in S)$ is the sentence of the form $(q_1 \leq q_2)$, then $V(p) = T$, if $(q_1 \leq q_2)$, and $V(p) = \emptyset$ in the otherwise,
- if p $(\in S)$ is the sentence of the form $(u_i = q_2)$, then $V(p) = \{t \in L(T) : u_i(t) = q\}$.

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[10] G. Bonanno, *Branching time, perfect information games and backward induction*, 1999.

We have the following conclusions: if $\mathfrak{M}$ is a game model, then $\forall_{t \in T}\ \mathfrak{M}, t \models (q_1 \leq q_2)$ if q_1 is less than or equal to the q_2 and $\mathfrak{M}, t \models \neg(q_1 \leq q_2)$ otherwise. In the model $\mathfrak{M}$, at node t holds $\mathfrak{M}, t \models (u_i = q)$, if t is a terminal node, such that $u_i(t) = q$. In the model $\mathfrak{M}$, at node t holds $\mathfrak{M}, t \models \neg(u_i = q)$ in a case, if t is either a decision node or terminal node such that $u_i(t) \neq q$.

In our formal language we can consider the truth of specific formulas in a model. These formulas describe some specific properties of games. Le us consider the following formula, for example:

I1) $F_p(u_i = q) \Rightarrow \rhd_i [((u_i = r) \vee F_p(u_i = r)) \Rightarrow (r \leq q)]$

The interpretation of this formula is as follows: if it is predictable that player i's payoff will be q then, no matter what decision the player i makes, if his payoff is r, or it is predictable that it will be r, then r is not greater than q.

Let us consider another formula:

I2) $$\left[F_p(u_i = q) \wedge \left(F_p(u_i = s) \Rightarrow (q \leq s)\right)\right] \Rightarrow$$
$$\Rightarrow \rhd_i \left\{\left[(u_i = r) \vee \left(F_p(u_i = r) \wedge \left(F_p(u_i = s) \Rightarrow (r \leq s)\right)\right)\right] \Rightarrow (r \leq q)\right\}$$

The interpretation of it is: if, according to the prediction, player i's payoff will be at least q, then, no matter what decision player i makes, if his payoff is r, or is predicted to be at least r, then r is not greater than q.

Both formulas I1 and I2 characterize the backward induction algorithm[11] for generic games in terms of temporal logic of branching time[12].

The backward induction algorithm can be used to solve finite extensive games with perfect information. What about infinite extensive games? Let us consider temporal logic based on intuitionistic propositional logic.

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[11] *Backward induction algorithm* is as follows: Suppose the initial decision node is K steps removed from the terminal nodes i.e. the maximum number decision nodes between the initial node and any terminal nodes is K.
Step 1: At any final decision node, every decision–making player chooses a move that maximizes her payoff.
Step 2: At a penultimate decision node, every decision-making player anticipates step 1 and chooses a payoff maximizing move.
...
Step k: At decision nodes k steps removed from a terminal node, every decision-making player anticipates step $k = 1$, for each $k = 3, \ldots, K$ and chooses a payoff maximizing move.

[12] Proof of this fact we find in the G. Bonanno, *Branching time, perfect information games and backward induction*, 1999.

Intuitionistic temporal logic

Alphabet:
- set of propositional letters: Ψ,
- intuitionistic unary connective: $\neg$,
- intuitionistic binary connectives: $\wedge$, $\vee$, $\Rightarrow$, $\iff$,
- temporal operators: G, H, F, P,
- parentheses:), (.

Notation

I	– a non-empty set of indexes of state of knowledge,
$T_i \quad (i \in I)$	– a non-empty set of moments of time in a state of knowledge indexed by i,
$R_i \quad (\subseteq T_i \times T_i)$	– a binary relation on T_i,
$\mathcal{T}_i \quad (= \langle T_i, R_i \rangle)$	– a time in a state of knowledge indexed by i,
$\mathcal{T} = \bigcup_{i \in I} \mathcal{T}_i$	– a set of all moments of time,
$R \quad (= \bigcup_{i \in I} R_i)$	– a binary relation on the set of all moments of time,
$V_i \quad (\subseteq T_i \times 2^{\Psi})$	– a function mapping to elements t $(\in T_i)$ subsets of the set of propositional letters,
$\wp = \{V_i :\in I\}$	– a class of function V_i,
$m_i \quad (= \langle T_i, R_i, V_i \rangle)$	– a state of knowledge indexed by i,

$\mathfrak{M}_{(\mathcal{T},\wp)} = \{\langle T_i, R_i, V_i \rangle : V_i \in \wp,\ i \in I\}$, then $\mathfrak{M}_{(\mathcal{T},\wp)} = \{m_i : i \in I\}$. $\mathfrak{M}_{(\mathcal{T},\wp)}$ is a model based on time $\mathcal{T}$ and class of functions $\wp$.

Between elements of a model $\mathfrak{M}_{(\mathcal{T},\wp)}$ we introduce a relation $\leq$ $(\subseteq \mathfrak{M}_{(\mathcal{T},\wp)} \times \mathfrak{M}_{(\mathcal{T},\wp)})$.

Definition 10

For any $i, j \in I$:

$$m_i \leq m_j \quad = \quad \big(T_i \subseteq T_j \text{ and } R_i \subseteq R_j \text{ and } \forall_{t \in T_i} V_i(t) \subseteq V_j(t)\big).$$

„$m_i \leq m_j$" means that the state of knowledge m_j is not smaller than the state of knowledge m_j.

Let us consider the example which shows various ways to obtain new states of knowledge. A state of knowledge may not be enlarged to a less one in several cases. One of them is when in the new state of knowledge we are able to describe events which were not known in the smaller one. Another case is when we obtain a new knowledge about the structure of time. One more case is when in a new state of knowledge the relation between time points is changed.

Example 5

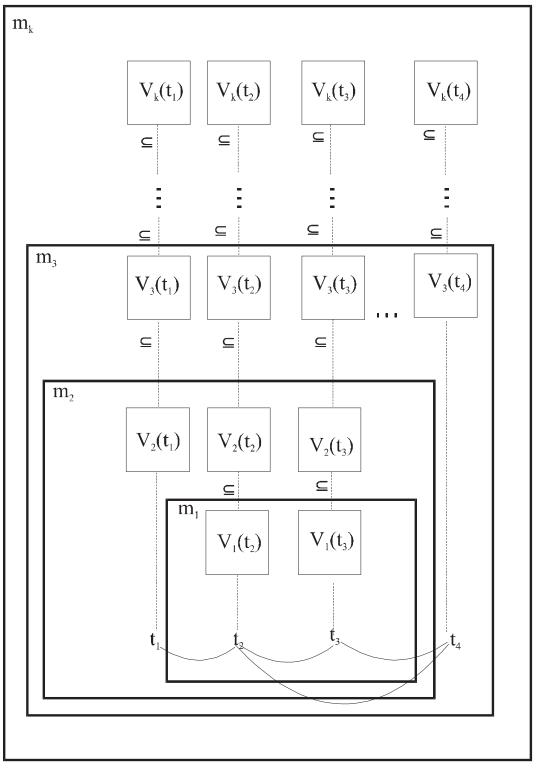

Fig. 7.

Remark

m_i^* (where $i \in I$) means any m_j ($\in \mathfrak{M}_{(\mathcal{T},\wp)}$) such that $m_i \leq m_j$.

Definition 11[13]

For a model $\mathfrak{M}_{(\mathcal{T},\wp)}$, state of knowledge m_i ($= \langle T_i, R_i, V_i \rangle$), element t ($\in T_i$), a formula α $\mathfrak{M}_{(\mathcal{T},\wp)} \models \alpha[t, m_i]$, is defined by the following conditions:

[13] An axiomatization of the minimal intuitionistic temporal logic was given by Surowik D. in the *Tense Logic Without The Principle of The Excluded Middle*, Topics in Logic, Informatics and Philosophy of Science, Białystok, 1999.

a) $\mathfrak{M}_{(\mathcal{T},\wp)} \models \alpha[t,m_i] \quad \equiv \alpha \in V_i(t)$, if $\alpha \in \Psi$,

b) $\mathfrak{M}_{(\mathcal{T},\wp)} \models \neg\alpha[t,m_i] \quad \equiv \forall_{m_i^* \in \mathfrak{M}_{(\mathcal{T},\wp)}} \mathfrak{M}_{(\mathcal{T},\wp)} \not\models \alpha[t,m_i^*]$,

c) $\mathfrak{M}_{(\mathcal{T},\wp)} \models (\alpha \vee \beta)[t,m_i] \quad \equiv \mathfrak{M}_{(\mathcal{T},\wp)} \models \alpha[t,m_i]$ or $\mathfrak{M}_{(\mathcal{T},\wp)} \models \beta[t,m_i]$,

d) $\mathfrak{M}_{(\mathcal{T},\wp)} \models (\alpha \wedge \beta)[t,m_i] \quad \equiv \mathfrak{M}_{(\mathcal{T},\wp)} \models \alpha[t,m_i]$ and $\mathfrak{M}_{(\mathcal{T},\wp)} \models \beta[t,m_i]$,

e) $\mathfrak{M}_{(\mathcal{T},\wp)} \models (\alpha \Rightarrow \beta)[t,m_i] \quad \equiv \forall_{m_i^* \in \mathfrak{M}_{(\mathcal{T},\wp)}} \big(\mathfrak{M}_{(\mathcal{T},\wp)} \not\models \alpha[t,m_i^*]$ or $\mathfrak{M}_{(\mathcal{T},\wp)} \models \beta[t,m_i^*]\big)$,

f) $\mathfrak{M}_{(\mathcal{T},\wp)} \models F\alpha[t,m_i] \quad \equiv \exists_{t_1 \in T_i}$ (such that tR_it_1 and $\mathfrak{M}_{(\mathcal{T},\wp)} \models \alpha[t_1,m_i]\big)$,

g) $\mathfrak{M}_{(\mathcal{T},\wp)} \models G\alpha[t,m_i] \quad \equiv \forall_{m_i^* \in \mathfrak{M}_{(\mathcal{T},\wp)}} \forall_{t_1 \in T_i^*}$ (if $tR_i^*t_1$, then $\mathfrak{M}_{(\mathcal{T},\wp)} \models \alpha[t_1,m_i^*]\big)$,

h) $\mathfrak{M}_{(\mathcal{T},\wp)} \models P\alpha[t,m_i] \quad \equiv \exists_{t_1 \in T_i}$ (such that t_1R_it and $\mathfrak{M}_{(\mathcal{T},\wp)} \models \alpha[t_1,m_i]\big)$,

i) $\mathfrak{M}_{(\mathcal{T},\wp)} \models H\alpha[t,m_i] \quad \equiv \forall_{m_i^* \in \mathfrak{M}_{(\mathcal{T},\wp)}} \forall_{t_1 \in T_i^*}$ (if $t_1R_i^*t$, then $\mathfrak{M}_{(\mathcal{T},\wp)} \models \alpha[t_1,m_i^*]\big)$.

In the language of the intuitionistic temporal logic the argument on determinism based on the principle of excluded middle is formulated as follows:

$$F\alpha \vee F\neg\alpha$$

The formula $F\alpha \vee F\neg\alpha$ is not a tautology of intuitionistic temporal logic. A countermodel for formula $Fp \vee F\neg p$ is showed on Fig. 8.

As we can see, the formula Fp is not true in our model in the state of knowledge m_1 at moment t, because in the state m_1 there is not a moment of time t' later than t, so that the sentence p is true at t'. The formula $F\neg p$ is not true in the state of knowledge m_1 at moment t either. The necessary condition for the truth of the sentence $F\neg p$ is: the sentence p is false at every moment of time later than t in every state of knowledge not smaller than the state of knowledge m_1. As we can see, our model does not satisfy this condition.

Infinite branching time structures are proper for semantic considerations in the temporal logic of branching time based on intuitionistic logic. We can consider an intuitionistic temporal logic of branching time if we add the following formulas to the axioms of the minimal intuitionistic temporal logic:

B1) $FF\alpha \Rightarrow F\alpha$,

B2) $G\alpha \Rightarrow GG\alpha$,

B3) $(P\alpha \wedge P\beta) \Rightarrow [P(\alpha \wedge \beta) \vee P(P\alpha \wedge \beta) \vee P(\alpha \wedge P\beta)]$.

Example 6[14]

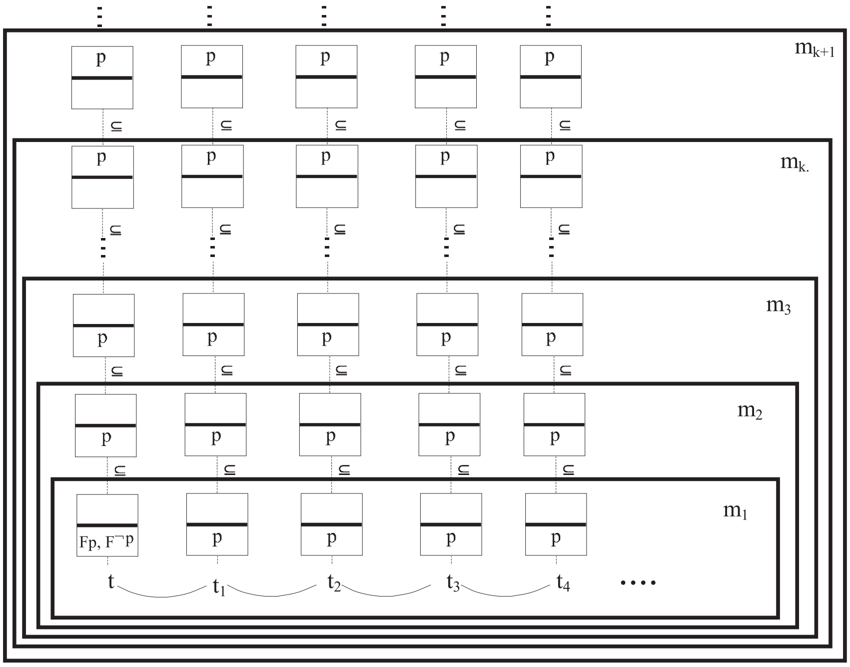

Fig. 8.

These formulas characterize the class of time branching in the future in terms of intuitionistic temporal logic[15].

We can imagine that in the terms of intuitionistic temporal logic we analyze a structure of time very similar to the structure of the infinite extensive game. For example:

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[14] If a formula is above the horizontal line in the square mapped for a moment t in a given state of knowledge, then we interpret that the formula is true at a moment t in this state of knowledge. Otherwise, we mean that the formula is false at this moment in this state.

[15] The various properties of the earlier-later relation in the intuitionistic temporal logic are discussed in Surowik D., *Some Remarks about Intuitionistic Tense Logic*, On Leibniz's Philosophical Legacy, Białystok 1997.

Example 7

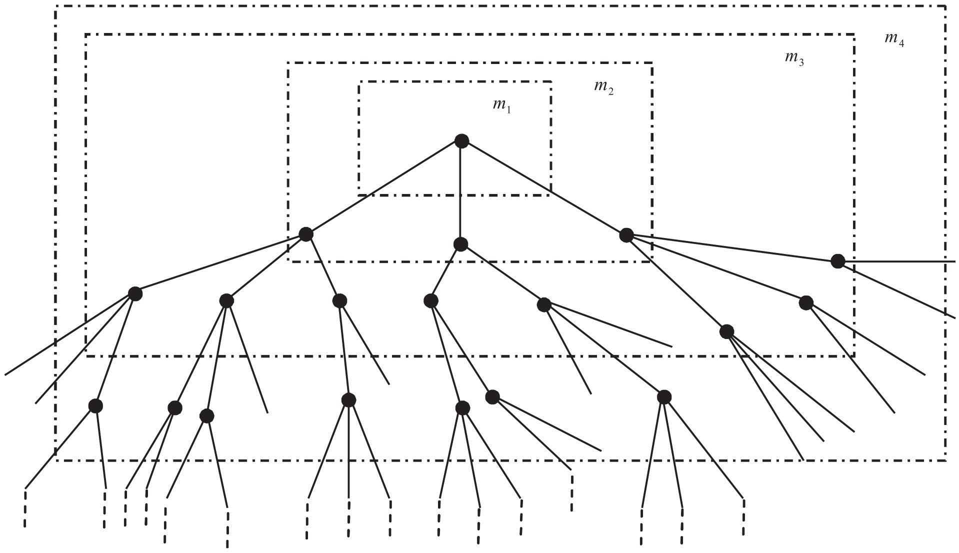

Fig. 9.

The open problem is:

> *Is a modification of intuitionistic temporal logic of branching time (by adding agents and payoffs to the semantics) possible for the analyses of infinite indeterministic extensive games?*

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