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THE PRISONER'S DILEMMA AND THE GAME OF LIFE

This paper aims at simulating social processes with the use of cellular automata[1]. At first glance, the theories of games and cellular automata seem to be unrelated despite the fact that they were created by the same person (John von Neumann) at the same time. It was not until recently that the two disciplines have been brought together. Here I intend to simulate social processes with the use of automata widely known as the Game of Life.

Let us start with a game between two players each having two strategies: to cooperate (C) and to defect (D). The player using C receives a payoff R (the reward) if the co-player uses C, and S (the sucker's payoff) if the co-player uses D. The player using D obtains the payoff T (the temptation) against the C-player, and P (the punishment) against the D-player (Tab. 1).

	cooperate	defect
cooperate	R, R	S, T
defect	T, S	P, P

Table 1. The game matrix

We assume that the payoff R for the two C-players is larger than the payoff P for two D-players. The game's satisfying conditions: $T > R > P \geq S$ and $2R > S + T$ are called the Prisoner's Dilemma.

Imagine that both players are prisoners who have been accused of a crime and are being interrogated in separate rooms. They can choose

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either of two options: to admit themselves guilty or not. If both maintain their innocence, they will be released for the lack of evidence or will go to jail for a short time. If A claims innocence but B says A is guilty, then A goes to jail and B gets a reward. A symmetric result follows if B claims innocence and A says B is guilty. If each says the other is guilty, both go to jail. The outcome for each player is called the player's payoff. Maintaining the innocence of both is called cooperating, saying that the other person is guilty is called defecting.

Why must the Prisoners Dilemma satisfy the conditions recalled before? – Because the sucker's punishment must be suitably high and the sum of acquittal must be greater then the sum of the full punishment and betrayal.

Let us analyze the payoff matrix (Tab. 2).

	cooperate	defect
cooperate	4,4	0,5
defect	5,0	2,2

Table 2. The payoff matrix

The amount of payoffs is for instance a number of years at liberty during the next 5 years.

In Nowak and May's version, when two cooperators play, both receive a payoff 1; when two defectors play, both receive 0; when a defector and a cooperator play, the cooperator receives 0 and the defector receives b. The dynamics of the interactions depends on the value of b (Tab. 3).

	cooperate	defect
cooperate	1,1	0,b
defect	b,0	0,0

Table 3. The payoff matrix

For a single game there is no reasonable solution for the Prisoner's Dilemma. From psychological regards, all players choose to testify against her or his companion (because every player is afraid of betrayal) and, as a result, all players loose. D is the dominant strategy and the C-players are doomed to extinction.

However, it is not the case when we consider players meeting for another round. This time each player plays according to a certain strategy and has some knowledge about the strategy of his enemy. We call such a game the Iterated Prisoner's Dilemma (IPD).

There are a lot of possible strategies in the IPD:

- Random (RAND) – makes random moves – defect or cooperates with 1/2 probability,
- Always Defect (AD) – always plays defection,
- Always Cooperate (AC) – always plays cooperation,
- Grim Trigger (GRIM) – starts with cooperation, but after one defection plays 'always defect',
- Tit-for-Tat (TFT) – starts with cooperation, then repeats opponent's moves,
- Tit-for-Two-Tats (TFTT) – like TFT, but forgives one defection.

Let us consider now the Prisoner's Dilemma game played on a square grid (Spatialized Prisoner's Dilemma). Each cell is occupied by one player. Each player plays one round of the Prisoner's Dilemma game against his/her eight nearest neighbours. The sum of payoffs from these eight games is the payoff of each player. After each iteration, each player looks at his/her neighbours and switches his/her strategy to the strategy that has obtained the highest score. As a result of the repeated games only the best strategies survive – the ones which give the greatest payoffs. There are some characteristic states of the configuration. Usually strategies group in clusters.

Notice that in the Spatialized Prisoner's Dilemma a further dimension (a spacial dimension) is added. It forms a bridge between the theory of games and the theory of cellular automata. The fields of strategies in the Spatialized Prisoner's Dilemma evolve in the manner of cellular automata.

A cellular automaton (CA) has the following features:

- It consists of a number of identical cells (often several thousand or even millions) arranged in a regular grid. The cells can be placed in a long line (a one-dimensional CA), in a rectangular array or even occasionally in a three-dimensional cube. In social simulations, cells may represent individual or collective actors such as countries.
- Each cell can be in one of a few states, for example, 'on' or 'off', 'alive' or 'dead'. We shall encounter examples in which the states represent attitudes (e.g. supporting one of several political parties), individual characteristics (e.g. racial origin), or actions (e.g. co-operating or not co-operating with others).
- Time advances through the simulation in discrete steps. After each time step, the state of each cell may change.

- The state of a cell at any time step is determined by a set of rules which specifies how that state depends on the previous state of that cell and the states of the cell's immediate neighbours. The same rules are used to update the state of every cell in the grid. The model is therefore homogeneous with respect to the rules.
- Because the rules only make reference to the states of other cells in a cell's neighbourhood, cellular automata are best used to model situations where the interactions are local. For example, if gossip spreads orally and individuals only talk to their immediate neighbours, the interaction is local and can be modelled with a cellular automaton.

To summarize, cellular automata model a world in which space is represented as a uniform grid, time advances by steps, and the 'laws' of the world are represented by a uniform set of rules which compute each cell's state from its own previous state and those of its close neighbours.

Cellular automata have been used as models in many areas of physical science, biology and mathematics, as well as social science. As we shall see, they are good at investigating the outcomes at the macro scale of millions of simple micro-scale events. One of the simplest, and certainly the best known example of cellular automata is Conway's Game of Life.

This is a two-dimensional cellular automaton. Each cell has one of two possible states: 'live' or 'dead'. Each cell has eight neigbours. The states of cells are changing according to certain rules:

- a dead cell with exactly three live neighbors becomes a live cell,
- a live cell with two or three live neighbors stays alive,
- a live cell with one or no neighbours dies (the case of loneliness),
- a live cell with four or more neighbours dies (the case of over population).

These specific rules were selected in 1970 by the mathematician J. H. Conway to guarantee that the cellular automaton is on the boundary between unbounded growth and decay into dullness. It was proved that its chaotic behaviour is unpredictable and it could be used to build a universal Turing-machine and even a universal constructor. The contrast between the simplicity of this rule and the complexity of the behaviour it produces is a constant source of wonder.

Note that each cell acts independently based on the old arrangement to produce a new one. The number of neighbours is counted from the old arrangement only. Therefore, if a dead cell has 3 neighbours, the cell will be alive in the next generation, even if those neighbors die.

Let us compare the Prisoner's Dilemma and the Game of Life:

the Prisoner's Dilemma	**the Game of Life**
the future of any player depends on the strategy of his/her neighbours	the future of any cell is determined by the state of its neighbours
the players are changing their own strategies in the way determined by the strategies of their enemies	the cells are changing colours in the way determined by the colours of their neighbours
the player can choose one of the two options: to cooperate or to defect	the cell has one of two states: live or dead
strategies	rules

Because of these similarities I should think that it is possible to define the strategies in the Prisoner's Dilemma in such a way that fields of strategies in the Spatialized Prisoner's Dilemma evolve in the manner of the Game of Life.

I claim that strategies in the Prisoner's Dilemma correspond with the rules in the Game of Life. There are four rules, so we have to group all strategies into four groups:

- strategies which are cooperators and were cooperators in the previous generation (blue)
- strategies which are defectors and were defectors in the previous round (red)
- strategies which are cooperators but were defectors in the previous round (green)
- strategies which are defectors but were cooperators in the previous round (yellow)

If we compare the colours of strategies with the colours of cells, it is possible to reach our target:

- the cells which are now dead and were dead in the previous generation are red
- the cells which are now live and were live in the previous generation are blue
- the cells which are now dead and were live in the previous generation are yellow
- the cells which are now live and were dead in the previous generation are green

Let us analyze some more examples. When we choose a low value for p (p is fraction of defectors in the first round) and b value about 1.85 (b is advantage for defection when opponent cooperates), we can see the gradual defector invasion pattern. Starting from a field of all cooperators with a single defector (with b = 1.85) we can observe a dynamical fractal or a kaleidoscope.

We can observe such configurations playing the Game of Life. If we start with a relatively simple initial condition, the emerging configuration can lead to wonderful kaleidoscopic patterns, periodic blinkers and gliders moving through a sea of dead cells or their gradual death.

References

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