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FROM THE GENEALOGY OF MATHEMATICAL ECONOMICS: WALRAS, PARETO AND LANGE

All models are wrong, only some of them are useful.
William Edwards Deming

1. Mathematics allows us to solve various complicated issues concerning numerous aspects of life. Are mathematical methods universal enough to be used to study the dynamism of economic life? Economic life has both its quantitative and qualitative aspects and its phenomena are highly complex and changeable. The subject of research being relatively stable forms of the occurrence of economic phenomena and the search itself concerning calculation of objectively existing relations among different quantities, it is undoubtedly possible to achieve the desired results through certain mathematical methods (e.g. appropriate algorithms and techniques). However, the scope and the sort of methods to be applied to economic aspects, where the qualitative aspects or mechanisms of the examined economic problem are not well enough known or recognized, remains a question. How strong should the assumptions be for the mathematical method to strike equilibrium? What is the relation of abstract and generalizing models to existing reality? Nicolas Kaldor ([2]) claimed that *any theory must be based on all sorts of abstracts, however, the abstracts must not be selected in a vacuum space: they must be related to the characteristic features of economic process we experience.* Irrespective of an answer to the above questions, it may be stated that modern economy is highly mathematical. Growing formalization of economy introduces to the existing theory new methods (e.g. set, game or chaos theory) thus extending its scope and precision and comprising more and more aspects of economic life (such as analysis of the problem of choice in uncertainty conditions). Despite the fact that modern economics is unquestionably abstract and thus hardly representative for the existing world, it has managed to develop a number of practical mathematical techniques, such as linear, web, dynamic or genetic programming.

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2. From the historical point of view, mathematical methods have always played a bigger or smaller part in economic consideration. Italian monetarists in their works (B. Davanzatti – 1588 and G. Ceva – 1711) already postulated and attempted at wider use of mathematical methods in economics. Since late XVII c. attempts at employing mathematics for economic consideration purposes have become more frequent. The following ought to be mentioned:

– C. Beccaria – 1765 – consideration concerning pathology of economic life based on calculation and algebra,
– D. Bernoulli – 1781 – applying theory of probability to calculating the chances of gaining certain profit,
– N. F. Canard – 1801 – attempt at arriving at mathematic formula to determine demands and purchasing capabilities of buyers,
– J. H. Thunen – 1826, 1850 – consideration carried out partly in mathematical form,
– A. Cournot – 1838 – mathematic expression of correlations between demand, supply, prices, costs and incomes under different competitive conditions and monopolization degrees.

The last of the above-mentioned is considered to be among the first pioneers of the so-called formalistic revolution in economics, i.e. a precursor of general application of mathematics to the analysis of economic phenomena. Acknowledged as a great mathematician, he in a way anticipated Walras' ideas. Embracing correlations within economy, he claimed that in order to fully and rigorously solve the problems concerning respective parts of economic system, it is necessary to take into consideration the entire system. Cournot might have felt that the mathematical analysis was not adequately developed at the time. The reason that he did not deal with complex correlations of the theory of overall equilibrium may have been the numerous assumptions that had to be made to analyze the problem. Or he simply may not have had enough courage to deal formally with such a complicated problem. As it is, the first step was taken by Walras, who modeled a system of overall equilibrium through a clear and concise formal description with the use of mathematical notation.

3. **Walras Leon Marie Esprit (1834-1910)** with his 1873 work *Elements d'Economie Pure ou Theorie de la Richesse Sociale* ([6]) gave rise to the so-called Lausanne School in political economics. Another representative of this school was **Vilfredo Pareto (1848-1923)**, Walras' disciple who continued working on his conception and later succeeded him to the chair at Lausanne University.

Walras' interests included mainly overall equilibrium of goods on the market assuming that economy possessed self-driving mechanisms that restored the equilibrium of economic system upset by external stimuli (such as change in technology or consumer tastes). He focused on the exchangeability of economic goods. He studied the relations between demand and supply given a price that ensured their equality and lead to equilibrium. Walras believed that, since those relations concerned quantities, they had best to be presented through mathematical equations. Thus, knowing such parameters as for example consumer tastes or production technology, it is possible to mathematically establish optimum quantitative proportions among respective elements of economy, i.e. determine economic equilibrium. What takes place spontaneously through adjustment trial-and-error processes can be calculated with the use of algorithms provided that as many equations can be created as there are unknowns.

Let us assume that:
- economy consists of two sectors: enterprises and households,
- enterprises do not buy goods from one another,
- household preferences do not change,
- the level of technology does not change,
- there is a state of full employment,
- all industrial branches remain competitive.

Households are characterized by certain preferences and limited income. They enter the commodity market expressing a demand for goods offered by enterprises. Prices and amount of goods are established on this market. The market is in the state of equilibrium when the amount of particular goods offered and demanded is equal. On the market of manufacturing factors the situation is reversed. Enterprises make demands on households for production factors. According to supply of production factors by households, the prices of these factors are established. Equilibrium is struck when the offered amount equals the demanded amount in relation to each production factor. Households achieve their income on the market of production factors and spend it on the commodity market. The flow of income between enterprises and households represents the national economic income. For it to strike equilibrium, households must spend all their income. Enterprises, based on prices (of goods and production factors) and accessible technology, manufacture products in a way that maximizes their profits (a particular product ought to be manufactured with minimum costs and maximum profit). After a longer period of time, competition forces will lead to a situation where the price of goods will equal average production costs. To balance the level of national income, enterprises have to spend on production factors market all they acquired on commodity market. It should be mentioned that respective parts of the abstract model of economy are interrelated: a change in price of one product leads to repercussions in entire system.

Let us introduce the following symbols:
1) Let us assume that we have
   H – number of households
   F – enterprises
   n – products
   m – production factors.
2) Products:
   x^h – vector of household h demand for products
   x^f – vector of products offered by enterprise f
   p – vector of product prices.
3) Production factors:
   v^h – vector of production factors offered by household h
   v^f – vector of enterprise f demand for production factors
   w – vector of production factor prices.
4) Private property:
   π^f – enterprise f income
   σ^{hf} – part of enterprise f income held by household h.

Each enterprise will be in the state of equilibrium if:

$$\max \pi^f = px^f - wv^f$$

$$\Phi^f(x^f, v^f) = 0$$

i.e. if it maximizes its profit, on condition that together with the growth of amount of produced goods increases a demand for production factors.

Each household will be in the state of equilibrium if:

$$\max U^h = U(x^h, v^h)$$

$$px^h = wv^h + \sigma^h \pi$$

i.e. if the usefulness function representing household h preferences and its supply of production factors is maximized, provided that the cost of goods purchased by household h equals the sum of income acquired through selling production factors and the part of enterprise profits that belongs to household h.

Commodity market will be in the state of equilibrium if

$$\sum_{h=1}^{H} x_i^h = \sum_{f=1}^{F} x_i^f \qquad \text{where } i = 1, \dots, n$$

i.e. if the sum of all products purchased by households equals the amount of all products manufactured by enterprises.

The productionfactors market will be in the state of equilibrium if

$$\sum_{h=1}^{H} v_j^h = \sum_{f=1}^{F} v_j^f \qquad \text{where } j = 1, \ldots, m$$

i.e. if the sum of all production factors offered by households equals the sum of all production factors purchased by enterprises.

According to Walras and Pareto, the overall economic equilibrium can be assessed through such equations if only the number of those equations equals the number of appearing variables. Thus, as far as the number of equations is concerned, we have:

- $nF + mF + F = (n + m + 1)F$ equations in the enterprise equilibrium system
- $nH + mH + H = (n + m + 1)H$ equations in the household equilibrium system
- n equations in the commodity market equilibrium system
- m equations in the production factors market equilibrium system,

which gives us a total of $(n+m+1)(F+H)+n+m$ equations. Since one of them will be dependent, the number is reduced by one to $(n + m + 1)(F + +H) + n + m - 1$ equations. As far as the number of unknowns is concerned, we have:

- $n+m+1$ variables on the part of h^{th} household (x_i^h – products where $i = 1, \ldots, n$; v_j^h – production factors, where $j = 1, \ldots, m$; μ^h – Lagrange's multiplier), which, the number of households being H, gives us $(n+m+ +1)H$ unknowns
- $n + m + 1$ variables on the part of f^{th} enterprise (x_i^f – products where $i = 1, \ldots, n$; v_j^f – production factors where $j = 1, \ldots, m$; μ^f – Lagrange's multiplier), which, the number of households being F, gives us $(n+m+ +1)F$ unknowns
- $n + m$ variables on the part of price equilibrium (p_i – product prices where $i = 1, \ldots, n$; w_j – production factors prices where $j = 1, \ldots, m$)

which gives us a total of $(n+m+1)(F+H)+n+m$ unknowns. Since one of the goods is taken as unit of account (*numeraire*), their number is reduced by one to $(n + m + 1)(F + H) + n + m - 1$ unknowns.

Thus, the number of equations equals the number of unknowns. However, Pareto himself recognized the calculations to be complicated. Let us assume that we deal only with 100 households, 70 enterprises, 700 products and 3 production factors; in such far from realistic case we would have to solve 120 382 equations.

4. Pareto's contribution ([5]) consisted mainly of extending the application of mathematical methods, developing the concept of overall equilibrium and on reformulating the idea of usefulness. Pareto observed that usefulness is a relative value (which is more useful – one or two glasses of beer? – it depends on, for example, whether we are thirsty or not and whether it is warm or cold). Thus, he parted with the theory of usefulness measurability (which underlined previous thinking in Lausanne School) and based what became the so-called theory of choices on certain data observable in one's management.

Pareto used the notion of indifference curves formulated by another economist, F. Edgeworth which visualized the scale of consumer preferences in relation to a given pair of goods. A consumer can acquire those goods in different quantitative combinations. Indifference curves are not measured. They are based on the choice one makes overcoming the difficulties in satisfying one's tastes. The choice is made in certain conditions on the basis of rational incentives in accordance with the economy principle. Needs are approached praxeologically. Tastes are identified with needs. As a result, when satisfying one's tastes, one makes choices according to preferences scale which, as Pareto initially believed, is statistically estimable. Indifference curves visualize consumer preferences scale. Respective indifference curves bring together all the possible combinations representing the same level of needs satisfaction. The higher the curve, the higher the level of needs satisfaction. Consumer possibilities determined by income are presented in the so-called price path. This method is based on the idea that desires (i.e. consumer preferences and possibilities) can be presented in two-dimensional space and that they visualize one's actions determined by external factors. Similar reasoning applies to a producer who decides on a certain combination of production factors. Given the indifference and production curves (visualizing the quantitative combinations of goods lost by the producer in relation to the gained ones), Pareto created a price theory where there is equilibrium between tastes and obstacles (between consumption and production), i.e. a price theory that, according to him, ensures overall equilibrium. Pareto tried to extend Walras' theory of overall equilibrium to the sphere of politics. In his studies he considered the actions of economic units not only in case of free competition but also of monopolization, such as, for example, socialist economy.

5. Walras' and Pareto's ideas greatly influenced the shape of other economists' viewpoint. For example, the thesis that socialism was theoretically and practically capable of rational income sharing was accepted by the majority of economists between the thirties and seventies. Neither Mises (it is impossible under the socialist system to rationally share income, since if there exists no free market, there are no independent prices, which makes it impossible to make rational decisions concerning income sharing) nor Hayek (the solution of the problem of income sharing in socialist system is possible theoretically but not practically, since socialist planners would not be able to gather enough necessary data, let alone the need to solve a system of millions of equations connected with the problem) were able to effectively convey their critical comments concerning socialism. In fact, some of those comments concerned Walras' equilibrium model on which economic analysis was based back then, in particular, its static character which did not explicitly allow for any individual enterprise activity and which did not deal formally with an adjustment processes in case of upset equilibrium.

Oscar Lange (1904-1965) ([4]) also refuted Hayek's arguments proving that it is possible to apply a market mechanism to socialist economy, which would lead to solving simultaneous equations through an empiric procedure of trial-and-error. The starting point of Lange's considerations is any given price system. When demand exceeds supply, prices are increased, whereas when supply exceeds demand, prices decrease. Final equilibrium is gradually achieved through such a trial-and-error process (first described by Walras). The prices satisfy the system of linear equations. Lange assumed that the process is convergent and goes in the direction of a price equilibrium system.

The market mechanism and trial-and-error process suggested by Lange served as a calculating apparatus for solving a system of equations. The solution was found through a convergent iteration process. Iteration worked on the basis of feedback which was supposed to gradually eliminate deviations. Lange imagined the process as a certain mechanism which (thanks to back coupling) automatically eliminates disturbances. According to Lange, such a mechanism stimulated market functioning whereas the market itself was one of the oldest recognized tools of solving simultaneous equations. Fascinated by electronic mathematical techniques, he wrote in 1965: *What is the problem? Let us make the computer solve simultaneous equations system and we will have the results in less than a second. Market process and trial-and-error equation procedure turn out to be old-fashioned. As a matter of fact, they should be treated as specific pre-electronic era calculating apparatus* while simultaneously expecting of these techniques the following kind of help: making it possible to change and re-schedule a plan, preparing alternative plans that would be adjusted to various objective situations that may crop up, and enabling the choice of optimum plan.

6. Economic models are idealistic and simplified forms of reality. They are for us in order to better understand complicated processes of economic life and to achieve certain goals (e.g. decision making). The question remains to what degree they comply with existing reality. Walras' and Pareto's mathematically expressed models gave rise to the so-called mechanistic paradigm in the analysis of economic processes. This paradigm, rooted in the mechanistic vision of the world, assumed as the theoretical basis of economy ideas borrowed its form classical mechanics. Such an approach is nowadays questioned in a number of basic assumptions ([1],[3]):

– *the analysis of economic processes concerns the state of equilibrium*; when monitoring real processes, it is obvious that economic development never reaches the state of equilibrium (cases of reappearing innovations) but only heads in the direction of the state which, on the other hand, constantly changes;
– *the knowledge of the process is full, which allows to make optimum choices*; limited calculative capacities and time limits force the decisions to be made based on the simplified models of reality and be far from optimum;
– *the only competition is the price competition which acts gravitation-like leading towards the state of relative equilibrium*; competition is a form of rivalry, it is won by those economic entities which manage their resources most effectively;
– *the criterion of economic entities activity is maximum profit*; real-life decision-making processes show that maximization possibilities are very limited if knowledge is limited: apart from trying to maximize profits, one also strives to ensure further development of the enterprise;
– *time is an absolute (the way it is understood in Newtonian mechanics): enterprises always remain in the state of equilibrium and the results of decision making are immediate*; in real-life processes the equilibrium time is usually much shorter that time of transition from one state of equilibrium to another; transition periods are of great importance.

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