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TENSE LOGICS AND THE THESIS OF DETERMINISM

The **logical determinism**[1] is a point of view, that for prove the thesis of determinism only logical principles are sufficient. Logical determinists says, that the principle of bivalency and the excluded middle low are sufficient – without adduction on other principles – for construction of proof of an argument on determinism. Arguments on determinism were considered in antiquities already. The problem was clearly formulated by Aristotle in IX chapter of *Hermenetutica*. Aristotle assumes that sentences on the past and the presence are true or false. In his opinion, the assumption that sentences on the future are true or false is sufficient for a construction of an argument of determinism. If all sentences on the future are true or false, then events described by these sentences are determined. If all future events are determined, then there are not accidental events and everything is necessary. Therefore, the thesis, that sentences on the future events are true or false implies, that – apart from the past and the presence – the future is also logical determined.

Recently, the problem of the logical determinism was considered by Jan Łukasiewicz[2]. In the article *On determinism* he gives the following interpretation of determinism:

> If an object A has a property b in a some moment of time t, then in every moment earlier than t, it is true, that the object A has the property b in the moment t.

[1] Apart from the logical determinism the physical determinism is considered. The **physical determinism** is a point of view, that every fact has immemorial causes in other earlier facts. The physical determinism is connected with the principle of causality.

[2] J. Łukasiewicz, *On determinism*, Selected Works, edited by L. Borkowski, Warszawa, 1970.

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It is, so called, the principle of determinism formulated semantically. Zbigniew Jordan wrote, that this principle not run on events, but it run on sentences which describes events. Jordan also wrote, that this principle says about particular properties of true sentences. In accordance with this principle predicative word "true" is an absolute predicative word. If a sentence is true, then it is true irrespective of a person, a place and a time in which says about it, it is true. True sentences are atemporal, but events described by these sentences have temporal component.

Constructions of proofs of arguments on determinism from the principle of bivalency and the excluded middle law were considered in [10]. However, the argument on determinism may be reconstructed from the principle of causality, too.

The principle of causality

> Each event Z in a moment t has a cause in a some event Z_1 in a moment t_1, earlier than t. In each moment later than t_1 and earlier than t, there is an event, which is a consequence of Z_1 and a cause of Z.

Therefore, if there is a cause of some event, then it is inescapable. But actual inescapability of some event, does not mean, that actual exists a cause of it. For example, the event *a man X will be dead* is inescapable, but perhaps not exists immediate cause of the man X death.

Some events creates causal-effect association with other events in a set of events. Usually, we assume, that this association is transitive. It means, if an event Z_1 is a cause of an event Z_2 and the event Z_2 is a cause of an event Z_3, then Z_1 is a cause of the event Z_3. Therefore, events Z_1, Z_2, Z_3 are creating a causal-effect chain. In a set of events, ordered with causal-effect association, every event has a cause in other events, which are preceding it in a causal-effects chain. Then, causal-effects chains are infinite. The infinity of causal-effect chains is not adequate condition, that each event has immemorial causes (is determined). Causal-effects chains may be infinite and limited in time. A necessary condition of infinity and limitation of causal-effect chains is density of causal-effect association. Each event is located in a some moment of time. The principle of causality postulates the existence of cause for each event. Therefore, each event in a causal-effect chain is located in a moment of time different from remaining moments of time. If we assume, that time is a measure of change (from cause to effect), then necessary condition for existence of infinite and limited in time causal-effect chains is density of time. Them, if we assume density of time, the principle of causality not implies of the thesis of determinism.

It was proved by Jan Łukasiewicz. We usually assume, that the physics time is continuous. This assumption implies that time is dense. The thesis of determinism is not a result of the principle of causality if we accept properties of time usually assumed. For prove, that the thesis of determinism is the result of the principle of causality, the assumption *time is discreet* is necessary. If we assume, that time is discreet, then causal-effect chains are infinite in the past. It is interpreted, that there are immemorial causes for each event. If each event has immemorial causes, then all events are determined.

Determination should be considered in a temporal context. It is realized in a some systems of tense logic.

The precursor of tense logic was A. N. Prior. One of the basic Prior conception was a temporal interpretation of modal operators. The tense operators are interpreted in the usual way:

F – at least once in the future,
G – it is always going to be the case,
P – at least once in the past,
H – it has always been the case.

The basic deduction system of tense logic is K_t[3]. K_t is the system of tense logic based on classical logic. It is a minimal tense logic. Formally it means, that ever other system of tense logic (based on classical propositional logic) is richer than K_t. Semantical considerations in K_t are based on point structure of time[4].

The system K_t was intended as a formal system coding aid of operators G, H, F, P reasonings about world taking temporal aspects of world into consideration There are no assumes for time structure in K_t semantics.

However, we usually assume, that the real time is linear, continuous, non-ending and non-beginning. If we accept such structure of time, a lot of philosophical and physics considerations are simpler. Of course, not all considerations are more simple. Some considerations are more complicated. For example the rejection of arguments on determinism. Some sentences, which not express of determinism in minimal tense logic, express

[3] This is equivalent of minimal deduction system K for modal logics. See. J. F. A. K. van Benthem, *The Logic of Time*, D. Reidel Publishing Company, Dordrecht, Holland, 1983.

[4] There are tense logic systems, such that semantical considerations are based on period time structure. (See. J. F. A. K. van Benthem, *The Logic of Time*, D. Reidel Publishing Company, Dordrecht, Holland, 1983, p. 193-218.)

determinism in tense logic of non-ending linear time. For example the sentence:

$$\alpha \Rightarrow HF\alpha.$$

According to understanding tense operators accepted in K_t (and K_t extensions which semantics are based on linear time) the sentence $\alpha \Rightarrow HF\alpha$ is reading as:

> *If it is true, that α, then it is true, that it has always been the case, that at least once in the future α.*

If we assume, that time is linear, then if at present there is event Z, which is a semantical correlate of sentence α, then it has always been, that the event Z will be in the future. Since, in every past moment of time it was known that the event Z will be in the future, then we can to say, that the event Z is determined.

If a tense logic of non-ending linear time is based on classical logic, then

$$F\alpha \vee F{\sim}\alpha$$

is a tautology of this logic.

According to understanding of F tense operator, $F\alpha \vee F{\sim}\alpha$ we read as follows:

> *At least once in the future α or at least once in the future $\sim\alpha$.*

Since, for any α, the sentence $F\alpha \vee F{\sim}\alpha$ is a tautology of this logic, then in a language of tense logic of non-ending linear time (based on classical logic) $F\alpha \vee F{\sim}\alpha$ express the thesis of determinism. Its consequence is a statement, that all future events are determined and there are not future accidental events. By means of a language of the tense logic of non-ending linear time is possible a description of only such that world, where future is determined.

Indeterminists usually assumes, that past is determined, but future – not. We should to consider a difference between the past and the future. One of arguments to accentuate of this difference is McTaggart paradox on unreality of time. This paradox is connected with two ways of understanding of time.

From one hand, time is understood as a dynamic process. Events are arranging itself according to distinguish between past, presence and future. This is the dynamic conception of time, called A-theory.

In other, these same events, temporal characteristics of which are changing with reference to the past, the presence and the future, are

arranged in order given by an earlier-later relation. This is the static conception of time, called B-theory. According to the B-theory of time, temporal characteristic of all events exist as data.

It seems, that the A-theory of time should be derived from the B-theory, and the B-theory should be derived from the A-theory. However, apparently the static conception of time is derived from the dynamic conception, but not inversely. Some philosophers says, that this is an argument, that the dynamic conception of time is contradictory and time is unreal and illusory. Followers the dynamic conception of time says, that there are ontological differences between past and future events. Past events already have been real events, whereas future events are only possible. The difference can be explained by – for example – logical asymmetry between the past and the future. All expressions on past events are true or false, whereas some expressions on future events are neither true nor false. We can to say, that each past event is determined, but we can not to say, that determined are all future events.

In a certain state of development of world the past may be unknown, but events, which has been, are not changing. The past of world is unique. However, future development of the world is indetermined and may be a lot of way to occur of it.

This point of view was a base to considerations on branching time structure.

The idea of a tense logic of branching time was given by A. N. Prior[5]. One of the main motivation to construction of the tense logic of branching time was attempt at doing an indeterministic tense logic. In the tense logic of branching time, arguments on determinism are rejected by modification of structure of time. If we consider a branching time structure, then apart from moments of linear time, we have to consider moments of time where time is branching. In these moments there are alternative possibilities of realization of the world. Since, realization of the world taking place according to one of the alternative, then if we do something, then it may be to cause, that future will be realized according to another branch of branching time structure.

In the branching time structure the past has no alternatives, however there are a lot of way of a realization of the future. Alternative possibilities of a realization of the future are called **possible futures**. However, usually we assume, that among all possible futures only one is realized. It is, so called, **actual future**.

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[5] A. N. Prior, *Past, Present and Future,* Oxford University Press, 1967.

There are logicians, which do not agree with an opinion, that tense logic of branching time is an indeterministic tense logic. P. Yourgran, for example, says, that regardless of which branch is actual future, events occurs in moments of time such that, these moments creates linear time structure[6]. If we choose actual future, then we have only one branch and real time structure is reduced to linear structure. However, if structure of real time is reduced to linear structure, then all events are determined[7].

It seems, that modalization of temporal operators is a solution of reduction of branching time to linear time. Tense logic system with modal--tense operators was given by R. P. McArthur[8]. There are no "actual futures" in a tense logic system with tense-modal operators. All branch of branching time structure McArthur calls **accessible future**. McArthur gives the following questions: how we can to distinguish "tomorrow will be...", "tomorrow should be...", "it is possible, that tomorrow will be...". In models based on linear time all sentences are equivalent. However, in models based on branching time it does not hold. In these models there is hidden modalization of future tense operators.

McArthur introduces the following modal-tense operators: $F^{\Diamond}$, $F^{\Box}$, $G^{\Diamond}$, $G^{\Box}$. Operators $F^{\Diamond}$, $F^{\Box}$ are interpreted as follows:

$F^{\Diamond}$ – it is possible, that at least once in the future...,

$F^{\Box}$ – it is necessary, that at least once in the future...

Aid of operators $F^{\Diamond}$, $F^{\Box}$ are defined the operators $G^{\Diamond}$ and $G^{\Box}$:

$$G^{\Diamond}\alpha \equiv \sim F^{\Box} \sim \alpha,$$

$$G^{\Box}\alpha \equiv \sim F^{\Diamond} \sim \alpha.$$

Axiomatization of minimal tense logic[9] with modal-tense operators was given by W. A. Smirnow[10]. However, if we would like to reject of arguments on determinism, we need semantics based on branching time. Branching time tense logic with operators $F^{\Diamond}$, $F^{\Box}$, $G^{\Diamond}$, $G^{\Box}$, H, P (operators H and P are interpreted as usually) was created by J. Burgess[11] and called $K_b^{\Box}$.

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[6] P. Yourgran, *On the logic of indeterministic time*, The Journal of Philosophy, Vol. 82, 1985, p. 548-559.

[7] A. Karpienko, *Fatalizm i słuczajność buduszczego*, Moskwa, 1990.

[8] R. P. McArtur, *Factuality and modality in the future tense*, Nous Vol. 8, 1974, p. 283-288.

[9] The are no conditions upon structure of time.

[10] W. A. Smirnow, *Logiczeskije sistiemy z modalnymi vriemiennymi operatorami*, Modalnyje i vriemiennyje logiki, Moskwa 1979, p. 89-98.

[11] J. P. Burgess, *Decidability for branching time*, Studia Logica, Vol. 39, 1980, p. 203-218.

The sentence

$$\alpha \Rightarrow HF^{\Diamond}\alpha$$

is a tautology of system $K_b^{\Box}$. But it does not express of determinism. We read this sentence as follows:

If it is true, that α, then it is true, that always has been, that α is possible in the future.

Then, in conclusion, α is only possible in the future, but it does not necessary.

In the $K_b^{\Box}$, the sentence $\alpha \Rightarrow HF^{\Box}\alpha$ express determinism. We read the sentence $\alpha \Rightarrow HF^{\Box}\alpha$ as follows:

If it is true, that α, then it is true, that always has been, that α is necessary in the future.

However, the sentence

$$\alpha \Rightarrow HF^{\Box}\alpha$$

is not a tautology of $K_b^{\Box}$. Because the thesis of determinism is not a tautology of $K_b^{\Box}$, we can to say, that the logic $K_b^{\Box}$ is an indeterministic tense logic.

In the literature is considered the problem of construction of many--valued tense logic was. It is connected with possibility of the rejection of arguments of determinism from the principle of bivalency. Many-valued tense logic were crated for example by A. N. Prior[12], N. Rescher and A. Urquhart[13], K. Trzęsicki[14]. However some sentences, which express determinism (for example $\alpha \Rightarrow PF\alpha$, $\alpha \Rightarrow H{\sim}G{\sim}\alpha$), are tautologies of these logics.

There is a question: is possible a construction of an indeterministic tense logic satisfying the following conditions:

- there are two logical values,
- there are no condition upon structure of time,
- there are no specific operators apart from tense operators.

It seems, that all conditions are fulfilled by an intuitionistic tense logic. In the system of tense logic based on intuitionistic propositional logic

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[12] Por. A. N. Prior, *Time and Modality*, Calendron Press, Oxford 1957.

[13] N. Rescher, A. Urquhart, *Temporal Logic*, Wien New York 1971, p. 219-224.

[14] K. Trzęsicki, *Logika operatorów czasów gramatycznych a problem determinizmu*, Białystok 1986, p. 298-328.

is rejected the excluded middle law. Then, in the intuitionistic tense logic, we can not to reconstruct of argument on determinism based on the excluded middle law. Intuitionistic tense logic systems were considered in a literature[15].

The sentence

$$\alpha \Rightarrow HF\alpha$$

is a tautology of intuitionistic tense logic. However, the meaning of tense operators is such that, the $\alpha \Rightarrow HF\alpha$ does not express of determinism.

Moreover, the sentence $F\alpha \vee F{\sim}\alpha$ (which express determinism) is not a tautology of intuitionistic tense logic.

And finally, the thesis of determinism is not a tautology of intuitionistic tense logic, even if we assume, that time is linear and non-ending[16].

References

[1] J. F. A. K. van Benthem, *The Logic of Time*, D. Reidel Publishing Company, Dordrecht, Holland, 1983.

[2] J. P. Burgess, *Decidability for branching time*, Studia Logica, Vol. 39, 1980.

[3] A. Karpienko, *Fatalizm i słuczajność buduszczego*, Moskwa 1990.

[4] J. Łukasiewicz, *On determinism*, Selected Works, edited by L. Borkowski, Warszawa, 1970.

[5] R. P. McArtur, *Factuality and modality in the future tense*, Nous Vol. 8, 1974.

[6] A. N. Prior, *Time and Modality*, Calendron Press, Oxford 1957.

[7] A. N. Prior, *Past, Present and Future*, Oxford University Press, 1967.

[8] N. Rescher, A. Urquhart, *Temporal Logic*, Wien New York 1971.

[9] W. A. Smirnow, *Logiczeskije sistiemy z modalnymi vriemiennymi operatorami*, Modalnyje i vriemiennyje logiki, Moskwa 1979.

[10] D. Surowik, *Some remarks about intuitionistic tense logic*, On Leibniz's Philosophical Legacy in the 350$^{\text{th}}$ Anniversary of His Birth, Białystok, 1997.

[15] K. Trzęsicki, *Intutionism and Indeterminism (Tense logical Considerations)*, Jan Woleński, Philosophical Logic in Poland, Kluwer Academic Publishers, 1994, D. Surowik, *Tense logic without the principle of the excluded middle*, Topics in Logic, Informatics and Philosophy of Science, 1999.

[16] D. Surowik, *Tense logic without the principle of the excluded middle*, Topics in Logic, Informatics and Philosophy of Science, 1999, p. 115.

[11] D. Surowik, *Tense logic without the principle of the excluded middle*, Topics in Logic, Informatics and Philosophy of Science, 1999.
[12] K. Trzęsicki, *Logika operatorów czasów gramatycznych a problem determinizmu*, Białystok 1986, s. 296.
[13] K. Trzęsicki, *Intutionism and Indeterminism (Tense logical Considerations)*, Jan Woleński, Philosophical Logic in Poland, Kluwer Academic Publishers, 1994.
[14] P. Yourgran, *On the logic of indeterministic time*, The Journal of Philosophy, Vol. 82, 1985.

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